Reply to "Comments on 'Time-symmetric, approximately relativistic particle interactions and radiation'"

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(Received 16 March 1981)

The inclusion of screening effects, as argued by Jones, effectively brings in terms of higher order than c^{-2}. The Darwin Hamiltonian, like the Darwin Lagrangian, usually is written in perturbative expansion, to order c^{-2}. The physical effects of the Čerenkov radiation (order c^{-2}) and the Bohr polarization effect result from a weak-damping analysis with the Darwin Hamiltonian; contact has been made also in previous agreement with quantum field-theoretic calculations for degenerate systems, with the same Hamiltonian. The significance of adding screening should be examined within an approximation which includes c^{-3} terms and higher—so neither the Darwin Hamiltonian nor the Darwin Lagrangian is sufficient to analyze these situations. The medium modifications of Jones involve macroscopic considerations and higher orders and so the criticism of the microscopic Darwin Hamiltonian is misplaced.

The use of the Darwin Lagrangian in some treatments[1,2] raises questions because the methods essentially go beyond the c^{-2} order (the Darwin Lagrangian is relativistically correct to this order) in formulating an effective, coarse-grained particle Hamiltonian; this Hamiltonian is subsequently used in statistical-mechanical calculations for a plasma. However, since the Darwin Lagrangian itself is only correct to order c^{-2}, any results obtained with it must be compared with those obtained with the Darwin Hamiltonian (similarly correct to the same order).

In addition, the introduction of elements which are macroscopic at an early stage is questionable, particularly when a comparison with a microscopic treatment via the Darwin Hamiltonian is made. To clarify further, the *a priori* assumption by Jones and Pytte of the dispersion relation $\omega^2=\omega_0{}^2+l^2c^2$, where $\omega_0{}^2=4\pi ne^2/m$ and n is the density, introduces macroscopic elements into the basic Hamiltonian.

Effectively, terms of order c^{-3} and higher are introduced into the theory.[3] In arguing[4] that new terms of all orders in the parameter $l/(k\lambda_c)=\omega_0/kc$ are retained, Jones is restating the point that all orders of c^{-1} are indeed involved. The justification of glancing collisions is not sufficient as an argument, since it is precisely these small-angle collisions that are treated by the long-range Lenard-Balescu c^{-2} generalization as treated elsewhere.[5,6]

The Darwin Hamiltonian has been used to deduce physical results in a number of contexts: application in calculating relativistic corrections in atomic physics have been made, and it has also been used for nuclear two-body calculations. In charged systems such as plasmas the use of the Darwin Hamiltonian for calculation of quantum-electrodynamic effects (without renormalization) for a degenerate system leads to agreement with quantum field-theoretic calculations carried out to the same order.[7] This is in contrast to the procedure of Jones and Pytte[1]; they argue that one must alter the quantum-electrodynamic c^{-2} limiting form of Itoh,[8] before using the Hamiltonian in subsequent statistical calculations.

That a physical effect of order c^{-2} in a plasma, namely the Čerenkov effect, vanishes in the Jones-Pytte[1] and Trubnikov-Kosachev[9] treatments should not be viewed as a strength. In the paper under discussion,[5] the Čerenkov effect does emerge in the weak-damping situation, as does the Bohr polarization term. Indeed, agreement between Jones's result and Trubnikov's indicates that a problem of consistency exists in extension of terms beyond c^{-2} in arriving at a fundamental Hamiltonian; in a recent paper Lapiedra and Santos[10] conclude that the Trubnikov results to higher order than c^{-2} are meaningless.

Note that the effect of introducing higher-order effects (than c^{-2}) is evident in H_D' in Jones's Eq. (2) if $[k^2+(\omega_0/c)^2]^{-1}$ is expanded.[4] Even if one were to accept the description of the derivation of (2) as "microscopic," the problem of including terms beyond c^{-2} would remain. Also, the claim by Jones and Pytte[11] that "to treat plasmas of higher temperature, or include terms of higher order than T/mc^2 . . . we must include the radiation degrees of freedom in the Hamiltonian for the system" is incompatible with the introduction of higher orders as above. Note that radiation effects other than Čerenkov are at least of order c^{-3}.

The comment on the pathology of the index relation is the result of Jones's applying an inadmissible limiting procedure to a weak-damping result ($\omega_0 \to 0$, ω fixed). As stated in Ref. 5, the Čerenkov condition $\omega < \omega_0$ must be satisfied.

Finally, there is the question of whether radiation can arise from a time-symmetric microscopic form, using statistical mechanics. Although Wheeler and Feynman[12] demonstrated that classical electrodynamics, including radiation, can be derived from a time-symmetric theory, a statistical-mechanical connection has never been explicitly demonstrated; it is indicated[5] in a limited sense (order c^{-2}) that this may be possible. The usual derivations of the Čerenkov effect employ retardation from the outset.[13] In fact, in a test-particle treatment of a plasma, using retardation, Shafranov[14] obtained the Čerenkov effect.

The Wheeler-Feynman treatment considers a universe which absorbs radiation. One may also use an interaction-constraint condition to derive quantum-electrodynamic results, in the spirit of the Wheeler-Feynman treatment.[15] From this perspective, Jones's suggestion for limitation of box size to dimensions of the order of λ_c, for one to apply the Darwin Hamiltonian, is too restrictive. In addition, it is necessary to investigate such questions in connection with the thermodynamic limit for charged relativistic systems. In practice one is forced to conclude that with systems such as terrestrial plasmas, there will be some radiation, although the extent to which this affects questions such as stability is an open question.

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[1]R. D. Jones and A. Pytte, Phys. Fluids 23, 269 (1980).

[2]B. A. Trubnikov, Nucl. Fusion 8, 51 (1968); 8, 59 (1968).

[3]The possibility of deriving the results of Trubnikov by an *ad hoc* use of a screening factor was noted earlier [J. Krizan, Phys. Rev. 177, 376 (1969)]; whether one uses more elaborate procedures to justify the screening is beside the point. These introduce effects higher than order c^{-2}.

[4]R. D. Jones, preceding paper, Phys. Rev. D 25, 591 (1982).

[5]J. Krizan, Phys. Rev. D 22, 3017 (1980).

[6]J. Krizan, Phys. Rev. 140, A1155 (1965).

[7]T. E. Dengler and J. E. Krizan, Phys. Rev. A 2, 2388 (1970).

[8]T. Itoh, Rev. Mod. Phys. 37, 159 (1965).

[9]B. A. Trubnikov and V. V. Kosachev, Zh. Eksp. Teor. Fiz. 54, 939 (1968) [Sov. Phys.—JETP 27, 501 (1968)].

[10]R. Lapiedra and E. Santos, Phys. Rev. D 23, 2181 (1981).

[11]R. D. Jones and A. Pytte, Phys. Fluids 23, 273 (1980).

[12]J. A. Wheeler and R. P. Feynman, Rev. Mod. Phys. 17, 157 (1945).

[13]V. P. Zrelov, *Cherenkov Radiation in High-Energy Physics* (Israel Program for Scientific Translations, Jerusalem, 1970), Part I.

[14]V. D. Shafranov, *Review of Plasma Physics* (Consultants Bureau, New York, 1967), Vol. 3, p. 104.

[15]J. Krizan, Phys. Rev. D 1, 2772 (1970).