## Comments

*Comments are short papers which comment on papers of other authors previously published in* **Physical Review C**. *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract and a keyword abstract.*

# Comments on "Time-symmetric, approximately relativistic particle interactions and radiation"

Roger D. Jones
*Applied Theoretical Physics Division, Los Alamos National Laboratory, University of California, Los Alamos, New Mexico 87545*
(Received 11 February 1981)

It is argued that recent conclusions concerning the Čerenkov effect in a slightly relativistic plasma are incorrect. It is shown that the error resides in the use of the Darwin Hamiltonian to describe interactions in the medium. When the modifications to the Darwin Hamiltonian by the medium are taken into account, the reported effect disappears.

In a recent paper,[1] it was concluded that electromagnetic Čerenkov emission should occur in a slightly relativistic plasma, i.e., one in which the electron thermal speed $v_e$ is a substantial fraction of the speed of light. I believe this conclusion is incorrect. The error resides in the use of the Darwin Hamiltonian[2] to describe the particle interactions in a medium. It will be argued that the Darwin Hamiltonian is not valid in a medium and that, when a Hamiltonian that is valid in a medium is used, the effect reported in Ref. 1 disappears.

The history of the controversy over the use of the Darwin Hamiltonian in a plasma is a long one. In 1962 Krizan and Havas[3] used the Darwin Hamiltonian to obtain the thermodynamic and kinetic properties of a plasma. Trubnikov and Kosachev redid the calculations in 1968 using the Darwin Lagrangian.[4] The two methods did not give the same result. Trubnikov attributed this to the fact that, in a plasma, the electron canonical momentum did not bear the simple relationship with the electron velocity that it did in a vacuum. Thus, he concluded that the traditional Darwin Hamiltonian was not valid in a plasma. In 1980, Jones and Pytte[5] derived a Hamiltonian that was valid in a plasma. They were able, using this Hamiltonian, to recover the earlier Trubnikov-Kosachev results.[6] They were also able to recover the results from a simple Fermi golden rule calculation.[7] The results of Ref. 1 are inconsistent with both the Trubnikov-Kosachev and the Jones-Pytte results.

It is easy to see why the Darwin Hamiltonian is not valid in a medium. The Darwin interaction term is given by[2]

$$H_D=\frac{-e^2}{2m^2c^2}\sum_{i<j}\vec{p}_i\cdot\left[\frac{(\vec{r}_i-\vec{r}_j)(\vec{r}_i-\vec{r}_j)}{|\vec{r}_i-\vec{r}_j|^3}+\frac{1}{|\vec{r}_i-\vec{r}_j|}\right]\cdot\vec{p}_j$$
$$=-\sum_{i<j}\frac{1}{V}\sum_{\vec{k}}\frac{4\pi e^2}{k^2}\frac{\vec{p}_i\cdot(\overleftrightarrow{I}-\hat{k}\hat{k})\cdot\vec{p}_j}{m^2c^2}\exp[i\vec{k}\cdot(\vec{r}_i-\vec{r}_j)]\,, \qquad (1)$$

where $-e$, $m$, and $c$ are the electron charge, mass, and the speed of light, respectively, $V$ is the volume of the normalization box, and $\vec{r}_i$ and $\vec{p}_i$ are the position and canonical momentum of the $i$th particle. The second line of Eq. (1) is simply the first line written as a Fourier transform. Here, $\hbar\vec{k}$ can be interpreted as the momentum transferred between two particles interacting with this potential. The summation over $i$ and $j$ are over all pairs of particles. Equation (1) can be derived by a number of techniques, but every one involves an expansion in $c^{-1}$ and retention of terms through order $c^{-2}$.

There are two dimensionless parameters associated with this expansion. To see this, consider $N$ electrons in a box of volume $V$. Assume that the parameters, the two dimensionless numbers containing the speed of light are $v_e/c$ and $k/\lambda_c$, where $\lambda_c \equiv c/\omega_p = c(mV/4\pi Ne^2)^{1/2}$. Here, $\lambda_c$ will be recognized as the collisionless skin depth. It is the distance which an electromagnetic wave of frequency $\omega < \omega_p$ can propagate in a plasma. Thus, since Eq. (1) results from an expansion in $c^{-1}$, it is an expansion in *both* the parameters $v_e/c$ and $1/(k\lambda_c)$ and consequently, Eq. (1) is only valid if the dimensions of the box are small compared with $\lambda_c$. If the dimensions of the box are large, then Eq. (1) must be modified to read[5]

$$H_D' = -\sum_{i<j} \frac{1}{V} \sum_{\vec{k}} \frac{4\pi e^2}{k^2+\lambda_c^{-2}} \frac{\vec{p}_i\cdot(\overleftrightarrow{I}-\hat{k}\hat{k})\cdot\vec{p}_j}{m^2c^2} \times \exp[i\vec{k}\cdot(\vec{r}_i-\vec{r}_j)] \ . \qquad (2)$$

The only difference between Eqs. (1) and (2) is the term $\lambda_c^{-2}$ in the denominator. It can be seen that if $k$ is large compared with $\lambda_c^{-1}$, then Eq. (2) reduces to Eq. (1). Equation (2) was obtained from the total particle-radiation Hamiltonian. The derivation was microscopic and the Darwin Lagrangian was not used in obtaining Eq. (2).

We can see that terms to infinite order in $1/(k\lambda_c)$ are retained, but only terms to first order in $p^2/(m^2c^2)$ are kept. It is necessary to keep terms to all orders in $1/(k\lambda_c)$ because, in a medium, glancing collisions, in which momentum total charge is neutralized by a positive smeared background. Then the parameters $N/V$, $m$, and $v_e$ completely determine the ideal thermodynamic properties of the system. If one wishes to examine radiation and particle interactions one needs also to consider the paramters $e$, $k$, and $c$. Using these six smaller than $\hbar\lambda_c^{-1}$ is transferred, are possible. It is just these collisions which contribute to the Čerenkov effect of Ref. 1. If one uses Eq. (2) in Ref. 1 instead of Eq. (1), the effect reported in that paper completely disappears.

It should also be mentioned that use of the usual Darwin Lagrangian[2] is acceptable for examining long-scale-length ($k^{-1} > \lambda_c$) phenomena. This is because the Darwin Lagrangian is an expansion in $v_e^2/c^2$, but not in $1/(k\lambda_c)$. This accounts for the agreement between the Jones-Pytte[6] results, which used Eq. (2), and the Trubnikov-Kosachev[4] results, which used a microscopic Lagrangian formalism.

It should also be noted that the macroscopic Maxwell equations obtained in Ref. 1 are pathological. The index of refraction is given by $n = \omega_p/\omega$. With this index of refraction, the macroscopic Maxwell equations do not reduce to the vacuum Maxwell equations as the number of electrons in the box vanishes.

In conclusion, the Darwin Hamiltonian cannot be used to describe a slightly relativistic plasma. If it is used, it leads to an incorrect version of the macroscopic Maxwell equation, with the consequence that surprising effects, such as are outlined in Ref. 1, are predicted.

This work was supported by the United States Department of Energy.

[1]J. E. Krizan, Phys. Rev. D 22, 3017 (1980).
[2]L. D. Landau and E. M. Lifshitz, *Classical Theory of Fields,* 4th edition (Pergamon, Oxford, 1975), p. 168; T. Itoh, Rev. Mod. Phys. 37, 159 (1965).
[3]J. E. Krizan and P. Havas, Phys. Rev. 128, 2916 (1962); J. E. Krizan, *ibid.* 140, A1155 (1965); 152, 136 (1966).
[4]B. A. Trubnikov, Nucl. Fusion 8, 51 (1968); 8, 59 (1968); Zh. Eksp. Teor. Fiz. 54, 939 (1968) [Sov. Phys.—JETP 27, 501 (1968)].
[5]R. D. Jones and A. Pytte, Phys. Fluids 23, 269 (1980).
[6]R. D. Jones, Ph.D. thesis, Dartmouth College, 1979 (unpublished).
[7]R. D. Jones and A. Pytte, Phys. Fluids 23, 273 (1980).