Brief Reports

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Experimental bounds on the coupling of massless spin-1 torsion

Donald E. Neville
Physics Department, Temple University, Philadelphia, Pennsylvania 19122
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Agreement of reactor neutrino data with Weinberg-Salam neutral-current theory is shown to imply $(g_T{}^2/4\pi) < 10^{-10}$, where g_T is the coupling of a massless spin-1 torsion multiplet to fermions. If one demands that emission of massless spin-1 torsion quanta not destroy the energy balance inside helium-burning stars, one obtains a stronger limit $(g_T{}^2/4\pi) < 10^{-34}$. This bound is extremely strict, yet a torsion-balance experiment recently suggested by Newman should be able to improve even this bound by several orders of magnitude.

I. INTRODUCTION

The idea that torsion, the antisymmetric part of the affinity $\Gamma^{\lambda}{}_{[\mu\nu]}$, could play a role in gravity theory goes back at least to Cartan.[1] The idea that torsion should couple most naturally to the fermion axial-vector current, producing spin-spin forces, dates at least to Weyl.[2] For decades, however, little attention was paid to torsion theories, because the torsion field was thought to be nonpropagating. This is, it was thought to be nonzero except inside matter, therefore incapable of producing long-range $1/r$ forces between intrinsic spins.

Torsion is nonpropagating if one adopts the traditional ECSK (Einstein-Cartan-Sciama-Kibble) Lagrangian as the basis for torsion dynamics.[3] With the rise of gauge theories in the 1970's, various authors began experimenting with (field strength)^2 Lagrangians which go beyond ECSK theory and allow the torsion field to propagate.[4–6]

In a recent paper (hereafter referred to as I) I showed that the torsion-to-fermion coupling strength in a certain class of torsion theories could not be too strong, or production of the quanta associated with the torsion field would make helium-burning stars unstable.[7] Although the torsion quanta considered in I were massless, they produced no $1/r$ force because they were derivative coupled rather than minimally coupled to fermions. The derivative coupling was motivated less by gauge-theoretic ideas than by considerations relevant only within the framework of ECSK theory.

Reference I does not consider minimally coupled torsion, because at the time I wrote that paper I believed any $1/r$ force between oriented spins would be masked in all cases by the usual magnetic forces. However, recently I was informed by Riley D. Newman that magnetic forces can be shielded out by superconductors, so that a torsion-balance experiment involving shielded magnetized samples should be able to detect quite weak nonmagnetic spin-spin couplings.[8] Accordingly, in this paper the techniques of I are used to calculate a bound on the coupling of a massless axial-vector boson (the totally antisymmetric part of the torsion $\epsilon^{\alpha\lambda\mu\nu}\Gamma_{\lambda\mu\nu}$) to the fermion axial-vector current.

Section II places some preliminary bounds on the coupling by using data from neutrino scattering experiments, where magnetic interference is completely absent. These bounds are not so strong as the astrophysical bound, but they are independent of any assumptions about stellar structure. Also, the neutrino bound will be used later to show that certain mean-free-path corrections to the astrophysical bound are negligible. In Sec. III the astrophysical bound is calculated and it is shown that Newman's experiment should be able to improve on this bound by about four orders of magnitude.

II. BOUNDS ON THE COUPLING FROM NEUTRAL-CURRENT EXPERIMENTS

Consider the amplitude for $f_1+\nu_2\to f_3+\nu_4$ scattering, where fermions f_1,f_3 may be electrons or quarks (the conventions are those of Bjorken and Drell):

$$M=(g^2/8\cos^2\theta_W)\bar{f}_3\gamma_\lambda[T_3(1-\gamma_5)-2Q\sin^2\theta_W]f_1[\eta_{\lambda\lambda'}/(k^2-m_Z{}^2)]\,\bar{\nu}_4\gamma_{\lambda'}(1-\gamma_5)\nu_2$$
$$+g_T{}^2\bar{f}_3\gamma_\lambda\gamma_5 f_1[\eta_{\lambda\lambda'}/k^2]\bar{\nu}_4\gamma_{\lambda'}\gamma_5[(1-\gamma_5)/2]\nu_2\,. \quad (2.1)$$

The first term represents the effect of Z^0 exchange, the second term of massless torsion exchange. I neglect the neutrino mass, hence ignore possible $k_\lambda k_{\lambda'}$ terms in propagators. I assume only left-handed neutrinos are available experimentally; accordingly, the second term contains a $(1-\gamma_5)/2$ projection operator.

Evidently the net effect of torsion is to change the "C_A" coefficient $(-T_3)$ by an amount δ:

$$-T_3\to -T_3+(g_T{}^2/k^2)/(g^2/4\cos^2\theta_W m_Z{}^2)$$
$$\equiv -T_3+\delta\,. \quad (2.2)$$

In I the ratios of neutral- to charged-current total cross sections (for scattering of ν and $\bar{\nu}$ off isoscalar nucleon targets) were calculated as a function of δ, and present experimental data were shown to be compatible with $\delta\le 0.1$. For the model analyzed in I, δ was a constant, whereas here δ is a function of (momentum transfer)^2 k^2. Since experiments typically are not conducted in the exact direction where $1/k^2$ blows up, it will be adequate to replace $|k|$ by $\langle E_\nu\rangle$, the average lab neutrino energy. For reference, I give the exact formula connecting k^2 to E_ν and θ_L, the lab scattering angle:

$$k^2=\frac{-2E_\nu(1-\cos\theta_L)m_f E_\nu}{E_\nu(1-\cos\theta_L)+m_f}\,. \quad (2.3)$$

In a heavy-liquid bubble chamber (Gargamelle)[9] with $E_\nu=2$ GeV $<<m_f$, evidently $|k|\cong\langle E_\nu\rangle$ is a good approximation. Therefore,

$$\delta\equiv(g_T{}^2/k^2)/(g^2/4\cos^2\theta_W m_Z{}^2)$$
$$\cong(g_T{}^2/\langle E_\nu\rangle^2)/[e^2/(37.3\text{ GeV})^2]<0.1$$

which implies

$$(g_T{}^2/4\pi)<10^{-5}\,. \quad (2.4)$$

In the previous paragraph the Gargamelle group was quoted because their $\langle E_\nu\rangle$ is the lowest of any experiment scattering neutrinos off hadrons and the lowest $\langle E_\nu\rangle$ gives the best bound. If one considers neutrino-electron scattering involving reactor neutrinos (1.5 MeV $<E_\nu<3$ MeV), rather than neutrino-hadron scattering, then it is possible to improve the bound by a factor of 10^{-6}, solely because of the smaller $\langle E_\nu\rangle$.[10] If one is perhaps overly cautious and demands only $\delta<1$ (because the reactor data is subject to heavy corrections for background effects), then the resulting bound is

$$(g_T{}^2/4\pi)<10^{-10}\,. \quad (2.5)$$

III. ASTROPHYSICAL BOUND ON g_T

This section calculates the rate of stellar energy loss due to production of massless torsion quanta. To do this one must first calculate the rate at which they are produced in the reaction

$$\gamma(p_1,\epsilon_1)+e^-(p_2)\to\text{torsion}(p_3,\epsilon_3)+e^-(p_4)\,. \quad (3.1)$$

In lowest order, there are two diagrams contributing to process (3.1), one with an electron in the direct (s) channel, another with an electron in the crossed (u) channel, exactly as for Compton scattering. The corresponding amplitudes are

$$M_d=g_T e\bar{u}_4\epsilon_3\gamma_5(\not{p}_1+\not{p}_2+m_e)\epsilon_1 u_2$$
$$\times[m_e{}^2-(p_1+p_2)^2]^{-1}\,, \quad (3.2a)$$

$$M_c=g_T e\bar{u}_4\epsilon_1(\not{p}_3-\not{p}_2+m_e)\epsilon_3\gamma_5 u_2$$
$$\times[m_e{}^2-(p_3-p_2)^2]^{-1}\,. \quad (3.2b)$$

Since the electrons will be nonrelativistic $(kT/m_ec^2)=\frac{1}{40}$, $p_2>>p_1,p_3$, and the Feynman propagators will simplify, e.g.,

$$(\not{p}_1+\not{p}_2+m_e)[m_e{}^2-(p_1+p_2)^2]^{-1}$$
$$\cong-(\not{p}_2+m_e)/2p_{30}m_e\,. \quad (3.3)$$

After the results (3.3) are inserted into Eqs. (3.2) and a spin average is performed one gets

$$\sum \tfrac{1}{4} \mid M_d + M_c \mid^2 = e^2 g_T^2 (2m_e)^{-2} \sum_{\epsilon} \{ [(2\epsilon_1 \cdot p_2)^2 (p_2 \cdot p_4 + 2\epsilon_3 \cdot p_4 \epsilon_3 \cdot p_2) + m_e^2 (2\epsilon_1 \cdot p_2)^2] / (2p_{10} m_e)^2$$
$$+ [(2\epsilon_3 \cdot p_2)^2 (p_2 \cdot p_4 + 2\epsilon_1 \cdot p_4 \epsilon_1 \cdot p_2) + m_e^2 (2\epsilon_3 \cdot p_2)^2] / (2p_{30} m_e)^2$$
$$+ 2 [(2\epsilon_1 \cdot p_2)(2\epsilon_3 \cdot p_2)(\epsilon_3 \cdot p_4 \epsilon_1 \cdot p_2 + \epsilon_1 \cdot p_4 \epsilon_3 \cdot p_2 - p_2 \cdot p_4 \epsilon_1 \cdot \epsilon_3$$
$$- m_e^2 \epsilon_1 \cdot \epsilon_3)] / 4 p_{10} p_{30} m_e^2 \} . \tag{3.4}$$

The quantities $\epsilon_i \cdot p_4 \epsilon_j \cdot p_2$ can be neglected compared to $p_2 \cdot p_4$, and the latter can be set equal to m_e^2. The torsion energy p_{30} can be eliminated using

$$1 = [(p_1 + p_2)^2 - m_e^2] / [(p_3 + p_4)^2 - m_e^2] = p_1 \cdot p_2 / p_3 \cdot p_4 \cong p_{10} / p_{30} . \tag{3.5}$$

One can anticipate an eventual thermal average over initial directions and phase-space average over final directions, and make the replacements

$$\sum_{\epsilon_i} (\epsilon_1 \cdot p_2)(\epsilon_3 \cdot p_2)(\epsilon_1 \cdot \epsilon_3) \rightarrow -4 \vec{p}_2^{\,2} / 9 , \tag{3.6a}$$

$$\sum_{\epsilon_3} (\epsilon_3 \cdot p_2)^2 = \sum_{\epsilon_1} (\epsilon_1 \cdot p_2)^2 \rightarrow 4 \vec{p}_2^{\,2} / 3 . \tag{3.6b}$$

Equation (3.4) now simplifies to

$$\tfrac{1}{4} \sum \mid M_d + M_c \mid^2 = e^2 g_T^2 (2 m_e p_{10})^{-2} \{ \tfrac{8}{3} + \tfrac{8}{3} + \tfrac{16}{9} \} (\vec{p}_2)^2 . \tag{3.7}$$

The three numbers in the curly brackets come from direct, crossed, and interference terms, respectively.

The torsion energy p_{30} produced per cm^3 per sec follows from the following thermal average:

$$\int dn_{\gamma 1} dn_{e2} p_{30} V_{\rm rel} d\sigma = \int dn_{\gamma 1} dn_{e2} p_{30} \sum \tfrac{1}{4} \mid M_d + M_c \mid^2 d^3 p_3 d^3 p_4 (2\pi)^4 \delta^{(4)} (4 m_f^2 / 16 p_{10} p_{20} p_{30} p_{40})$$
$$= (N_e / V)(32 e^2 g_T^2 / 3 m_e)(kT)^3 \zeta(2) . \tag{3.8}$$

The electron density factor (N_e/V) comes from the average dn_{e2} over the initial (Maxwellian) electron distribution and the ζ function comes from the average $dn_{\gamma 1}$ over the photon blackbody distribution.

I now estimate result (3.8) roughly, to verify that it is correct in order of magnitude. $V_{\rm rel} d\sigma$ is of order matrix element squared times phase space. Since the initial state is P-wave and there is a bremsstrahlung denominator,

$$V_{\rm rel} d\sigma = O((e g_T \vec{p}_2 / p_{10} m_e)^2 \delta^{(4)} d^6 p m_e^2 / p_0^4)$$
$$= (e^2 g_T^2 \vec{p}_2^{\,2} / p_{10}^2 m_e^2)(1)$$
$$\rightarrow e^2 g_T^2 m_e kT / (kT)^2 m_e^2 .$$

The quantities, $dn_{\gamma 1}$ and dn_{e2} are of order $(kT/\hbar c)^3$ and (N_e/V) particles/cm^3, respectively, and p_{30} is of order kT. Multiplying all this together, I get exactly result (3.8) up to factors of order unity.

Numerical evaluation of Eq. (3.8) now gives an energy production of $(g_T^2/4\pi)(1.0 \times 10^{38}$ ergs/cm^3 sec). (As in I, the star is assumed to be a 15 $M_\odot$ red giant with a $4 M_\odot$ helium core at $T = 1.7 \times 10^8$K, density $= 1.14 \times 10^3$ gm/cm^3.) This rate must be below the rate of nuclear energy loss of 10^4 ergs/cm^3 sec, which gives

$$(g_T^2 / 4\pi) < 1 \times 10^{-34} . \tag{3.9}$$

At this point one should check that the mean free path l of a torsion quantum is much greater than the core radius $R = 10^{10}$ cm; otherwise most of the $(g_T^2/4\pi)(1.0 \times 10^{38}$ ergs/cm^3 sec) will not escape, and estimate (3.9) is inaccurate. A formula for l was developed at Eqs. (4.5)–(4.7) of I. Only one change is necessary. One must use the new production rate $(g_T^2/4\pi)(1.0 \times 10^{38})$ in place of Eq. (4.5) of I. One gets

$$l \cong 3.6 \times 10^{-9} \text{cm} / (g_T^2 / 4\pi) . \tag{3.10}$$

If $l < R$, then from a random-walk estimate only a fraction $(l/R)^2$ of the torsion quanta will actually escape, leading to the bound

$$(l/R)^2 (g_T^2 / 4\pi)(1.0 \times 10^{38}) < 10^4 \text{ergs/cm}^3\text{sec} \tag{3.11}$$

(if $l < R$). Substituting Eq. (3.10) for l, one gets

$$(g_T{}^2/4\pi) > 10^{-3} \quad (\text{ if } l < R) \; . \tag{3.12}$$

This is inconsistent with the neutral-current bounds, Eqs. (2.4) and (2.5), so that l must be greater than R. Therefore, the correction term $(l/R)^2$ in Eq. (3.11) is not necessary, and the bound (3.9) is correct as it stands.

The coupling of Eq. (3.9) is extremely small, yet Newman's torsion-balance experiments should be able to detect couplings even smaller than this. Newman defines a "figure of merit" ratio α for his apparatus. α is the ratio (smallest detectable anomalous torque)/(magnetic torque). For two electrons in two magnetized samples a distance r cm apart we estimate

$$\alpha = \frac{g_T{}^2 \langle \sigma_1 \rangle \langle \sigma_2 \rangle / 4\pi r}{e^2 \langle \sigma_1 \rangle \langle \sigma_2 \rangle (\hbar/m_e c)^2 / 4\pi r^3} < (10^{-11}\text{cm}^{-2}) r^2 \; , \tag{3.13}$$

where $\langle \sigma_i \rangle$ refers to electron spin, the factors of $(\hbar/mc)$ come from the Bohr magnetons, and the numerical upper limit follows from the upper limit (3.9) on $g_T{}^2$. This value for α is well within Newman's target sensitivity, $\alpha = 10^{-15}$.

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