Behavior of non-Abelian magnetic fields at high temperature

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We investigate the properties of non-Abelian magnetic fields at high temperature using Monte Carlo simulation and finite-size scaling on SU(2) lattice gauge theory. Magnetic flux is introduced using twisted boundary conditions. We find that color magnetic fields are screened, not confined, at long distances, and measure the screening length.

In this paper we investigate the behavior of non-Abelian magnetic fields at high temperature using Monte Carlo methods. Recently, Monte Carlo work has confirmed that a high temperature non-Abelian gauge theories undergo a transition to a phase in which electric test charges are liberated.[1] Here we examine the behavior of magnetic test charges in this high-temperature phase of an SU(2) lattice gauge theory. This subject is of interest not only because of our desire to understand non-Abelian gauge theories in all possible limits, but particularly with regard to cosmology. The lightest topological monopoles in grand unified theories generally have non-Abelian magnetic charges as well as their topological U(1) magnetic charge.[2] In the low-temperature phase it is generally believed from duality arguments that these color magnetic fields are screened,[3] so that the color magnetic charge cannot play an important role in monopole dynamics. However, these duality arguments cannot be used in the high-temperature phase. Linde has raised the possibility that non-Abelian magnetic flux is excluded from the high-temperature vacuum and focused into color magnetic flux tubes, leading to a linear (confining) potential between colored monopoles.[4]

Perturbative calculations of the spatial components of the gluon propagator on the other hand suggest that the gluon acquires a "magnetic mass" at high temperatures, i.e., the magnetic fields of test charges are screened.[5] The mass, or inverse screening length, first appears in two-loop diagrams. Therefore, we expect the mass to be

$$m(T)=T[C_2 g^2(T)+C_4 g^4(T)+\cdots\ ] ,$$

where $g^2(T)$ is the QCD coupling constant renormalized at energy T and the C's are constants. However, infrared divergences make perturbative computation of even the lowest-order coefficient C_2 impossible. This makes a nonperturbative study of non-Abelian magnetic fields attractive.

While this work was in progress, Billoire, Lazarides, and Shafi reported a calculation very similar to ours.[6] Our results are consistent with theirs. We will comment later on the differences between the two calculations.

We perform Monte Carlo simulation on an SU(2) lattice gauge theory, using a variation of the Metropolis method to carry out the updatings of the links. In order to simulate the high-temperature behavior of the theory, the size of the lattice in the time direction is small compared to the size in the spatial directions. To investigate the behavior of magnetic fields, we measure the effect of imposing a twist in one of the spatial planes (the xy plane). As discussed by 't Hooft[3] and by Groneveld, Jurciewicz, and Korthals-Altes,[7] the dependence of the free energy and its derivatives on the twist probes the behavior of the magnetic field.

To understood what is meant by twisted boundary conditions and how a twist affects the energy, it is helpful to proceed using an indirect construction. Srednicki and Susskind have shown how monopoles with charges in the center of the gauge group can be embedded in a lattice gauge theory.[8] Figure 1 shows a single time slice of a lattice gauge theory containing a monopole-antimonopole pair, which can be thought of as living inside the three-cubes of the lattice. A Dirac string runs from the monopole to the antimonopole, passing through the plaquettes shown in the figure. To hide this Dirac string we multiply the product of the link variables around each of these plaquettes by an element of the center of the gauge group. In the case of SU(2), the center is $Z(2)$ and the only nontrivial element is -1 times the identity matrix.

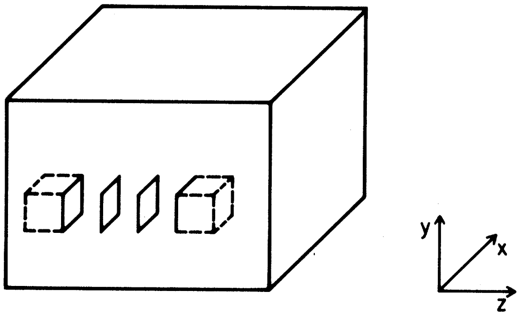

FIG. 1. A monopole-antimonopole pair in a lattice. The plaquettes drawn in solid lines have a modified action $S_p = \prod U_l \sigma_p$, where σ_p is an element of the center of the group. The lattice is translationally invariant in the time direction, which is not shown here.

As discussed by Srednicki and Susskind, this string may be moved around the lattice by multiplying plaquettes by elements of the center of the group. The physical magnetic field lines emerge from the string at the monopole end, and return through the antimonopole.

Now let us imagine pulling the monopole and antimonopole far apart in the z direction while keeping the size of the lattice in the x and y directions fixed (Fig. 2). Then a segment of the lattice between the monopole and antimonopole will just be a lattice with twisted boundary conditions. The twist, or string, can be moved around the lattice by redefining variables, but cannot be removed entirely. The point is that we can think of twisted boundary conditions as a relic of a Srednicki-Susskind monopole passing through the lattice. The expectation value of the internal energy depends on the behavior of the monopoles' magnetic field as it interacts with the restricted size of the lattice in the xy plane. Three possibilities exist: magnetic flux tubes are formed (Meissner effect), magnetic fields have Coulomb behavior, or magnetic charges are screened.

If magnetic flux tubes are formed one such flux tube will be left passing through the lattice. In this case we expect a contribution to the total energy (extensive) which is independent of the size of the lattice in the x and y directions. This means that intensive quantities such as the average plaquette should show an effect proportional to $(N_x N_y)^{-1}$, where N_i is the size of the lattice in the ith direction. If magnetic fields are Coulombic, the flux will spread out over the available area and the extensive energy differences will be proportional to $(N_x N_y)^{-1}$. Finally, if magnetic fields are screened the effect of the twist will fall off exponentially with the size of the lattice. From heuristic arguments such as this we are unable to deduce the exact form of the effect of the twist in the case. The analysis of 't Hooft indicates that a "zero" temperature, when the lattice is space-time symmetric, the effect of the twist on the free energy per plaquette F is

$$F(\text{twist}) - F(\text{no twist}) \propto \frac{1}{N_x N_y} e^{-K N_x N_y} \tag{2}$$

and hence the effect on the internal energy $\partial F / \partial \beta$ is

$$\frac{\partial}{\partial \beta}[F(\text{twist}) - F(\text{no twist})] \propto \frac{\partial K}{\partial \beta} e^{-K N_x N_y} , \tag{3}$$

where $1/\sqrt{K}$ is the magnetic mass, here expressed in dimensionless (lattice) units. The arguments used to derive this form depend on the ability to interchange the space and time axes, which is absent in high-temperature lattices. Although we cannot give a rigorous justification, we will nevertheless adopt Eq. (2) as our ansatz for the high-temperature case. In any case, our data are not good enough to distinguish between this form and exponentials multiplied by other powers of the area.

At zero temperature these space-time duality arguments indicate that if electric test charges are confined than magnetic test charges are screened, and *vice versa*. This relation has been checked in a Monte Carlo study by Mack and Pietarinen.[9] At high temperatures, because the lattice is not space-time symmetric, it is possible for both electric and magnetic test charges to be screened.

We used the Wilson action for lattice gauge theory

$$S = \sum_{\text{plaquettes}} \beta \operatorname{Tr} \left[\prod_{\substack{\text{links in} \\ \text{plaquetts}}} U_l \sigma_p \right] , \tag{4}$$

where $\sigma_p = -1$ for the plaquettes in the Dirac string and 1 otherwise. The lattice was translationally invariant in the time direction (suppressed in Figs. 1 and 2) so, to be precise, σ_p was -1 for plaquettes in the xy plane with $x = 0$, $y = 0$, for all z and t. The lattice sizes used were $4^3 \times 2$, $5^3 \times 2$, and $6^3 \times 2$ with twisted boundary conditions and ordinary periodic (although skewed[10]) boundary conditions at $\beta = 4/g_0{}^2 = 2.6$. We measured the difference of the average plaquette between the twisted and untwisted lattices, and the difference

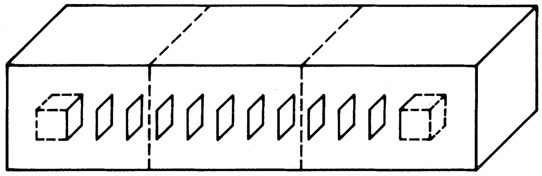

FIG. 2. The monopoles have been pulled apart. The center section of this lattice is a lattice with twisted boundary conditions.

between the xy plaquettes and the plaquettes in the other two spatial planes of the twisted lattice, as suggested in Ref. 7. (Because our lattice was asymmetric, we could not use the xt, yt, and zt plaquettes in this measurement.) The results of these measurements showed that the difference in internal energies arises mostly from the plaquettes in the xy plane, so we also measured the effect of the twist on the average value of these plaquettes. The results shown here required about 25 000 Monte Carlo passes for each size lattice, and were performed using the CYBER 175 at Fermilab. To update one link on this machine takes about 0.2 msec. Limited computing resources prevented us from working at several values of $\beta=4/g_0^2$. We used β of 2.6 because it is large enough to be in the scaling regime (above the crossover) but not so large that correlation lengths should be larger than our lattices.

We now investigate the different hypothesis for the behavior of the magnetic fields. If a flux tube is formed, the effect on the intensive quantities evaluated here should be proportional to $1/N^2$, where $N=4,5,6$ is the spatial size of the lattice. If the magnetic field has Coulomb behavior, the effects of twist should go as $1/N^4$. In Table I we show N^2 times effect and N^4 times effect for the various measured quantities. It can be seen that the flux-tube hypothesis is completely inconsistent with our data and the Coulomb hypothesis is ruled out to a high degree of confidence. We are left with the screening hypothesis. In Fig. 3 we plot the logarithms of the various quantities as a function of the area N^2. It can be seen that all the quantities are consistent with an exponential fall-

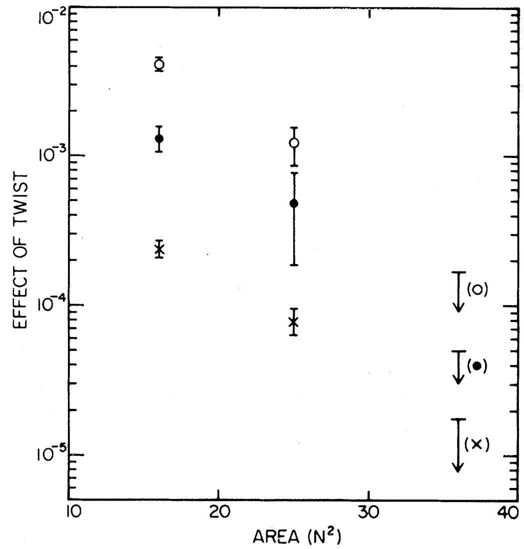

FIG. 3. The effect of twist as a function of lattice size. The solid circles are the change in the average plaquette when a twist is imposed. The open circles are the change in the plaquette in the xy plane. The crosses are the difference between the xy plaquettes in the twisted lattice and the other spatial plaquettes in the twisted lattice. This last quantity has been moved down one decade to improve readability.

TABLE I. Measured quantities on $N^3\times 2$ lattices. The subscripts "tw" or "no" label twisted and untwisted lattices. $\langle U_{ij}\rangle$ is the expectation value of plaquettes with an ij orientation while $\langle U\rangle$ is the average value of the plaquette.

Quantity	N	Quantity	$N^2\times$quantity	$N^4\times$quantity
$\langle U_{xy}-\frac{1}{2}(U_{xz}+U_{yz})\rangle_{\rm tw}$	4	$(2.38\pm0.29)\times10^{-3}$	$(3.8\pm0.17)\times10^{-2}$	0.609 ± 0.074
	5	$(0.787\pm0.163)\times10^{-3}$	$(0.20\pm0.04)\times10^{-2}$	0.492 ± 0.102
	6	$(0.013\pm0.159)\times10^{-3}$	$(0.05\pm0.06)\times10^{-2}$	0.017 ± 0.206
$\langle U\rangle_{\rm tw}-\langle U\rangle_{\rm no}$	4	$(1.31\pm0.27)\cdot10^{-3}$	$(2\pm0.4)\cdot10^{-2}$	0.33 ± 0.07
	5	$(0.44\pm0.24)\cdot10^{-3}$	$(1.1\pm0.6)\cdot10^{-2}$	0.27 ± 0.15
	6	$-(1\pm1.5)\cdot10^{-4}$	$-(0.36\pm0.54)\cdot10^{-2}$	-0.13 ± 0.19
$\langle U_{xy}\rangle_{\rm tw}-\langle U_{xy}\rangle_{\rm no}$	4	$(4.09\pm0.43)\cdot10^{-3}$	$(6.5\pm0.7)\cdot10^{-2}$	1.0 ± 0.1
	5	$(1.22\pm0.35)\cdot10^{-3}$	$(3.0\pm0.9)\cdot10^{-2}$	0.76 ± 0.21
	6	$-(0.7\pm2.4)\cdot10^{-4}$	$-(0.28\pm8.6)\cdot10^{-3}$	-0.09 ± 0.31

TABLE II. Fits to quantities to a/N^2, b/N^4, or Ce^{-kN^2}, with χ^2.

Quantity	a	χ^2	b	χ^2	c	k	χ^2
$\langle U_{xy}-\frac{1}{2}(U_{xz}+U_{yz})\rangle_{\rm tw}$	0.0216 ± 0.0027	26.5	0.53 ± 0.06	7.5	0.021	0.136 ± 0.021	1.1
$\langle U\rangle_{\rm tw}-\langle U\rangle_{\rm no}$	0.0112 ± 0.0028	12.6	0.28 ± 0.06	5.1	0.0187	0.165 ± 0.054	1.3
$\langle U_{xy}\rangle_{\rm tw}-\langle U_{xy}\rangle_{\rm no}$	0.0367 ± 0.045	38.6	0.89 ± 0.09	12.3	0.0507	0.157 ± 0.031	1.5

off, and all the slopes are approximately equal. Fits to the data for the three hypothesis are given in Table II. Notice that the values for the $6^3\times2$ lattice are only upper limits. For the effect of twist on the xy plaquette, the best fit to our data of the form

$$\langle U_{xy}\rangle_{\rm untwisted}-\langle U_{xy}\rangle_{\rm twisted}=Ce^{-kN^2} \tag{5}$$

has $k=0.156\pm0.031$ ($C=0.051$). χ^2 for this fit is 1.5 (one degree of freedom). In contrast, the best fit to this quantity using the Coulomb ansatz has $\chi^2=12$ and the confinement ansatz gives $\chi^2=38$ (each with two degrees of freedom).

We expect the temperature to set the scale of physical masses, so if we write $k=m_{\rm mag}{}^2=A^2T^2$ with $T=\frac{1}{2}$ we find $A=0.7\pm0.08$ for $\beta=4/g_0{}^2=2.6$.

We can now make contact with the work of Billoire, Lazarides, and Shafi. These authors assume that $m_{\rm mag}$ is proportional to the lowest order in g^2 found in perturbation theory: $m=Bg^2(T)T$, where $g^2(T)$ is the coupling constant renormalized at momentum scale T. They then use the one-loop renormalization-group equation and the relation between the lattice renormalization scale and the continuum renormalization scale[11] to compute $g^2(T)$ from the bare lattice coupling $g_0{}^2$. The result of these approximations is an equation predicting the effect of the twist as a function of the bare coupling $g_0{}^2$ for constant lattice size. Using Monte Carlo data from $4^3\times2$ lattices at several values of the bare coupling, they measure the difference in the total energy between twisted and untwisted lattices, finding a good fit with $B=0.24$. If we remove a factor of $g^2(T)_{\rm one\ loop}=2.959$ from our A, we find $B=0.27\pm0.03$, in good agreement. Although both investigations measured the effect of a twist on the lattice, they are in a sense complementary. In this work we varied the spatial size of the lattice, which allowed us to demonstrate magnetic screening directly, using only a few assumptions. Billoire, Lazarides, and Shafi, on the other hand, fixed the lattice size and varied the bare coupling. When the bare coupling is changed, the renormalized coupling, the physical (dimensional) temperature, and the spatial size of the lattice in dimensional units all change, so some additional assumptions about the physics must be included in order to interpret the results. The agreement of the result of these assumptions with the data, however, is evidence that the magnetic screening length, a dimensionful quantity, is varying with the bare coupling in the manner demanded by the renormalization group. The agreement beween the results of these two different methods is grounds for having more confidence in the results than we would have from either investigation alone.

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