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Nucleon-energy correlations in $\nu d \rightarrow \nu n p$

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The nucleon-energy correlation $\sigma(K_1,K_2)$, where K_1 and K_2 are the kinetic energies of the outgoing nucleons, is studied in the weak neutral disintegration of the deuteron, $\nu + d \rightarrow \nu + n + p$. The studies are made in all five (S, P, T, A, and V) variants of the neutral-current weak-interaction Lagrangian. The study in the region of low kinetic energies of the nucleons provides means to distinguish between the axial-vector and tensor couplings.

I. INTRODUCTION

The existence of neutral currents in $\Delta S = 0$ processes has been established in high-energy neutrino reactions but its space-time and isospin structure is still a subject of considerable theoretical study. While most of the theoretical analysis of the experimental results has been done in $V-A$ theory,[1] there have been some attempts to analyze various experiments in S, P, T interactions.[2] These theories are not ruled out from the data in particle-physics processes. It has been stressed[3] that nuclear-physics processes can play a decisive role in elucidating the space-time and isospin structure of these fundamental interactions. It is in this connection that much emphasis has been recently placed on the process $\nu(\bar{\nu}) + d \rightarrow \nu(\bar{\nu}) + n + p$, both theoretically and experimentally.[4] The experiments of Pasierb *et al.* at reactor energies[5] provide a clear signature for the presence of either axial-vector or tensor interactions. However, the Los Alamos Meson Physics Facility (LAMPF) and ANL experiments planned at intermediate energies[6] would provide information about the isospin structure of neutral currents. This is because at these energies the final dinucleons can be produced in various isoscalar and isovector states as emphasized by Ali and Dominguez.[4] In a recent experiment Pasierb *et al.*[5] have found

$$\sigma_{\mathrm{expt}} = (3.8 \pm 0.9) \times 10^{-45}\ \mathrm{cm}^2/\bar{\nu}\,.$$

This result is in fair agreement with the Weinberg-Salam theory. It can also be explained in the helicity-flipping theories with tensor interactions. The strength of the tensor coupling constant derived from this process[7] is however smaller than the coupling constant derived by Adler *et al.*[8] but seems consistent with the strength implied by a study of earlier data in astrophysics and particle-physics processes.[9]

Since this reaction has been observed, further analysis of this experiment can be made. With this motivation we have presented a discussion of another observable $\sigma(K_1, K_2)$, i.e., the energy correlation of the outgoing nucleons which can be useful in determining the structure of neutral currents. This correlation at low energies has been earlier discussed by Frahm[10] in $V-A$ theories of weak interactions. We have extended his work by giving a complete theoretical formulation of this problem in all the variants of the weak-interaction theory, i.e., V, A, S, P, and T. The expressions for $\sigma(K_1, K_2)$ are derived in this paper, which can be used to study this process at higher energies and momentum transfer relevant to the experiments planned at ANL and LAMPF. The general expressions for $\sigma(K_1, K_2)$ which include the effect of final-state interactions have been derived in all cases. The numerical calculations have, however, been made only at low energies corresponding to the experiments of Pasierb *et al.*,[5] where only S waves are produced in the final state. The numerical results in the helicity-conserving theories with $V-A$ currents are evaluated in the Weinberg-Salam model. In Sec. II, we describe our formulation. In Sec. III we discuss the numerical results and compare them with the work of Frahm.[10]

II. FORMULATION

In the following we calculate the matrix elements for the process $\nu d \rightarrow \nu n p$ in the helicity-conserving V, A theories as well as in the helicity-flipping S, P, T theories.

A. Helicity-conserving (V, A) theories

The matrix element for the process is

$$\mathfrak{M} = \bar{u}(k')\gamma_\lambda(1-\gamma_5)u(k)\langle np\,|J_0^\lambda|\,d\rangle\,, \tag{2.1}$$

where k and k' are the initial and final lepton (neutrino) momenta measured in the deuteron rest frame and $\langle np\,|J_0^\lambda|\,d\rangle$ is the hadronic matrix element derived in the impulse approximation[11]

$$\langle np\,|J_0^\lambda|\,d\rangle = \frac{G}{\sqrt{2}}\int \phi_f^*(\vec{r})(\Lambda_\lambda^p e^{i\vec{q}\cdot\vec{r}/2} + \Lambda_\lambda^n e^{-i\vec{q}\cdot\vec{r}/2})\phi_i(\vec{r})d^3r\,, \tag{2.2}$$

with $\vec{q}=\vec{k}-\vec{k}'$ as the three-momentum transfer $\phi_i(\vec{r})$ is the initial deuteron wave function and $\phi_f(\vec{r})$ is the final dinucleon wave function where we are considering only the singlet proton-neutron state. Λ_λ^N $(N=p,n)$ is the nonrelativistic reduction of the single-nucleon operator T_λ^N, given by

$$T_\lambda^N=\left[F_1^N(q^2)\gamma_\lambda+\frac{i}{2M}\sigma_{\lambda\eta}q_\eta F_2^N(q^2)\right.$$
$$\left.-g_A^N(q^2)\gamma_\lambda\gamma_5-h_A^N(q^2)\gamma_5 q_\lambda\right], \qquad (2.3)$$

where

$$F_{1,2}^N(q^2)=\tfrac{1}{2}(\tfrac{2}{3})^{1/2}g_{V0}F_{1,2}^{(0)}(q^2)+\tfrac{1}{2}\epsilon g_{V3}F_{1,2}^{(3)}(q^2)+\tfrac{1}{2}(\tfrac{1}{3})^{1/2}g_{V8}F_{1,2}^{(8)}(q^2),$$
$$g_A^N(q^2)=\tfrac{1}{2}(\tfrac{2}{3})^{1/2}g_{A0}g_A^{(0)}(q^2)+\tfrac{1}{2}\epsilon g_{A3}g_A^{(3)}(q^2)+\tfrac{1}{2}(\tfrac{1}{3})^{1/2}g_{A8}g_A^{(8)}(q^2). \qquad (2.4)$$

$\epsilon=+1$ describes the process $\nu+p\to\nu+p$ and $\epsilon=-1$ describes the process $\nu+n\to\nu+n$.

The nonrelativistic reduction of the matrix element described by Eq. (2.3) is derived as[12]

$$\Lambda_i^N=\left\{F_E^N(q^2)\frac{p_i'}{M}+iF_M^N(q^2)\epsilon_{ijk}\frac{q_j\sigma_k}{2M}-g_A^N(q^2)\left[\left(1+\frac{\vec{q}^{\,2}}{8M^2}\right)\sigma_i+\frac{p_i'p_j'\sigma_j}{2M^2}-\frac{q_iq_j}{8M^2}\sigma_j\right]\right\},$$
$$\Lambda_0^N=\left[F_E^N(q^2)\left(1+\frac{\vec{p}'^{\,2}}{2M^2}\right)-g_A^N(q^2)\frac{\vec{\sigma}\cdot\vec{p}'}{M}+i\epsilon_{ijr}F_R^N(q^2)\frac{q_jp_k'}{4M^2}\sigma_i\right], \qquad (2.5)$$

where

$$F_E^N(q^2)=F_1^N(q^2)+\left(\frac{q^2}{4M^2}\right)F_2^N(q^2),$$
$$F_M^N(q^2)=F_1^N(q^2)+F_2^N(q^2), \qquad (2.6)$$
$$F_R^N(q^2)=F_1^N(q^2)+2F_2^N(q^2),$$

and $\vec{p}'=\vec{p}+\vec{q}/2$, where $\vec{p}$ is the relative momentum of two nucleons inside a deuteron.

B. Helicity-flipping (*S,P,T*) theories

The matrix element for the process in S, P, T theories of the neutral current can be written in the impulse approximation as[8]

$$\mathfrak{M}=\frac{G}{\sqrt{2}}[\bar{\nu}(1-\gamma_5)\nu\langle np|\Lambda|d\rangle+\bar{\nu}\sigma_{ij}\nu\langle np|\Lambda_{ij}|d\rangle-i\bar{\nu}\sigma_{0i}\nu\langle np|\Lambda_{0i}|d\rangle], \qquad (2.7)$$

where

$$\langle np|\Lambda|d\rangle=\int\phi_f^*(\vec{r})[\Lambda^p e^{i\vec{q}\cdot\vec{r}/2}+\Lambda^n e^{-i\vec{q}\cdot\vec{r}/2}]\phi_i(\vec{r})d^3r,$$
$$\langle np|\Lambda_{ij}|d\rangle=\int\phi_f^*(\vec{r})[\Lambda_{ij}^p e^{i\vec{q}\cdot\vec{r}/2}+\Lambda_{ij}^n \bar{e}^{i\vec{q}\cdot\vec{r}/2}]\phi_i(\vec{r})d^3r,$$
$$\langle np|\Lambda_{0i}|d\rangle=\int\phi_f^*(\vec{r})[\Lambda_{0i}^p e^{i\vec{q}\cdot\vec{r}/2}+\Lambda_{0i}^n \bar{e}^{i\vec{q}\cdot\vec{r}/2}]\phi_i(\vec{r})d^3r \qquad (2.8)$$

and

$$\Lambda^N=F_S^N(q^2)\left(1+\frac{\vec{q}^{\,2}}{8M^2}-\frac{i\vec{\sigma}\cdot\vec{q}\times\vec{p}}{4M^2}\right)+F_p^N(q^2)\frac{\vec{\sigma}\cdot\vec{q}}{2M},$$
$$\Lambda_{ij}^N=T_1^N(q^2)\epsilon_{ijk}\sigma_k\left(1+\frac{\vec{q}^{\,2}}{8M^2}\right)-\frac{T_1^N(q^2)}{4M^2}\{i(q_ip_j-q_jp_i)+\vec{\sigma}\cdot\vec{p}[2i(p_i\sigma_j-\sigma_ip_j)-\epsilon_{ijk}q_k]+\vec{\sigma}\cdot\vec{q}[2i(p_i\sigma_j-\sigma_ip_j)+\epsilon_{ijk}p_k]\}$$
$$+i\frac{T_2^N(q^2)}{2M^2}\vec{\sigma}\cdot\vec{q}(\sigma_iq_j-\sigma_jq_i), \qquad (2.9)$$
$$\Lambda_{0i}^N=T_1^N(q^2)\frac{\vec{\sigma}\cdot\vec{q}}{2M}\sigma_i+iT_2^N(q^2)\left(\frac{q_i}{M}-\delta_{ij}\frac{p_jq_0}{M^2}-\frac{\vec{\sigma}\cdot\vec{q}}{2M^2}\sigma_iq_0\right),$$

where the total nucleon form factors $F_S^N(q^2), F_P^N(q^2)$, and $T_{1,2}^N(q^2)$ are defined as[8]

$$F_S^N(q^2)=\tfrac{1}{2}(\tfrac{2}{3})^{1/2}g_{S0}F_S^{(0)}(q^2)+\tfrac{1}{2}\epsilon g_{S3}F_S^{(3)}(q^2)+\tfrac{1}{2}(\tfrac{1}{3})^{1/2}g_{S8}F_S^{(8)}(q^2),$$
$$F_P^N(q^2)=\tfrac{1}{2}(\tfrac{2}{3})^{1/2}g_{P0}F_P^{(0)}(q^2)+\tfrac{1}{2}\epsilon g_{P3}F_P^{(3)}(q^2)+\tfrac{1}{2}(\tfrac{1}{3})^{1/2}g_{P8}F_P^{(8)}(q^2), \qquad (2.10)$$
$$T_{1,2}^N(q^2)=\tfrac{1}{2}(\tfrac{2}{3})^{1/2}g_{T0}F_{T_{1,2}}^{(0)}(q^2)+\tfrac{1}{2}\epsilon g_{T_3}F_{T_{1,2}}^{(3)}(q^2)+\tfrac{1}{2}(\tfrac{1}{3})^{1/2}g_{T_8}F_{T_{1,2}}^{(8)}(q^2).$$

Again $\epsilon = +1$ describes the process $\nu + p \to \nu + p$ and $\epsilon = -1$ describes the process $\nu + n \to \nu + n$.

In order to calculate the matrix element from Eqs. (2.2) and (2.8) we use the Hulthen wave function for the deuteron, i.e.,

$$\phi_i(\vec{r}) = \left[\frac{\alpha\beta}{2\pi(\alpha+\beta)}\right]^{1/2}\left(\frac{\alpha+\beta}{\beta-\alpha}\right)\left(\frac{e^{-\alpha r}-e^{-\beta r}}{r}\right),$$

with (2.11)

$\alpha = 46$ MeV, $\beta = 237$ MeV.

Using a plane wave for the final-state wave function, $|\mathfrak{M}|^2$ is calculated in V, A and S, P, T theories and the expressions thus obtained for $|\mathfrak{M}|^2$ are given in Appendix A (1). The effect of final-state interactions is studied by calculating the matrix elements with the final-state wave function which takes into account the rescattering effects of outgoing nucleons. To do this $\phi_f(\vec{r})$ in Eqs. (2.2) and (2.8) has been expanded in terms of the angular momentum wave functions and the final-state interaction has been incorporated through the phase shifts following standard methods.[13]

$|\mathfrak{M}|^2$ is then calculated in V, A and S, P, T theories and the expressions thus obtained for $|\mathfrak{M}|^2$ are given in Appendix A (2).

C. Nucleon-energy correlation function $\sigma(K_1, K_2)$

The differential cross section for this process is given as

$$d\sigma = \frac{(2\pi)^4}{4E_\nu M_d}\delta^4(p_1' + p_2' + k' - k - d)|\mathfrak{M}|^2 \times \frac{d^3p_1'}{(2\pi)^3 2E_1'}\frac{d^3p_2'}{(2\pi)^3 2E_2'}\frac{d^3k'}{(2\pi)^3 2E_\nu'} \qquad (2.12)$$

where $(E_\nu, \vec{k})$ is the four-momentum of incident neutrino, M_d is the deuteron mass, and $(E_1', \vec{p}_1')$, $(E_2', \vec{p}_2')$, and $(E_\nu', \vec{k}')$ are the four-momentum of outgoing proton, neutron, and neutrino, respectively.

The correlation function $\sigma(K_1, K_2)$ is obtained by integrating Eq. (2.12) over the final neutrino momentum

$$\sigma(K_1, K_2) = \frac{p_1'p_2'}{(4\pi)^5 E_\nu M_d} \times \int d\Omega_1 d\Omega_2 \frac{\delta(E_1' + E_2' + E_\nu' - E_\nu - M_d)}{E_\nu'}|\mathfrak{M}|^2, \qquad (2.13)$$

where E_ν' is calculated using conservation of momentum. After performing the angular integrations with the help of the δ function in Eq. (2.13), the correlation function $\sigma(K_1, K_2)$ is derived to be

$$\sigma(K_1, K_2) = \frac{p_1'}{(4\pi)^4 E_\nu^2 M_d}\int_{\omega_1}^{\omega_2} d(\cos\theta_{12})\int_{v_1}^{v_2} d(\cos\theta_1)\frac{|\mathfrak{M}|^2}{\sqrt{c}}, \qquad (2.14)$$

where

$$c = 1 - \cos^2\theta_1 - \cos^2\theta_2 - \cos^2\theta_{12} + 2\cos\theta_1\cos\theta_2\cos\theta_{12} \qquad (2.15)$$

and

$$\cos\theta_{12} = \frac{1}{2p_1'p_2'}[E_\nu'^2 - E_\nu^2 - p_1'^2 - p_2'^2 + 2E_\nu(p_1'\cos\theta_1 + p_2'\cos\theta_2)], \qquad (2.16)$$

and the values ω_1, ω_2, and v_1, v_2 are given in Appendix B.

III. NUMERICAL RESULTS AND DISCUSSION

General expressions for the nucleon-energy correlation function $\sigma(K_1, K_2)$ are given in Eq. (2.14) for $V-A$ and S, P, T theories. Numerical evaluation of Eq. (2.14) has, however, been made at low energies where only S states in the final state are produced. This corresponds to the low-energy experiments of Pasierb *et al.*,[5] where some events have been observed. We defer the numerical evaluation of these equations at higher energies until results from the ANL experiments, etc., become available. In the following we give numerical results for the nucleon-energy correlation for the cases discussed in Secs. II A and II B.

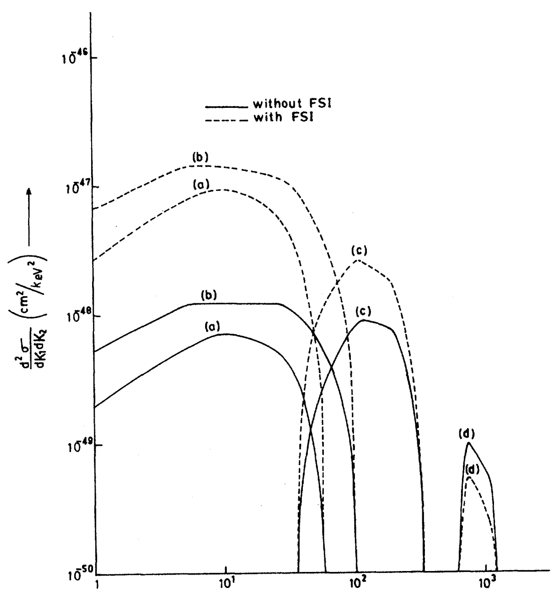

FIG. 1. $d^2\sigma/dK_1dK_2 \sim K_2$, for $^3S \to {}^1S$ transitions in the Weinberg-Salam model in helicity-conserving theories without final-state interactions (FSI) and with FSI at proton kinetic energies (a) $K_1 = 1.5$ keV, (b) $K_1 = 15$ keV, (c) $K_1 = 150$ keV, and (d) $K_1 = 900$ keV, for fixed incident neutrino energy $E_\nu = 5.5$ MeV.

A. Without final-state interactions

In the Weinberg-Salam model the dominant contribution comes from the isovector axial-vector currents and corresponds to the transition from the ^3S deuteron state (neglecting D states) to the singlet S neutron-proton state. The $^3S \to {}^3S$ transition is not allowed due to the absence of isoscalar axial-vector current in this model. The contribution from the isoscalar vector current to this transition is highly suppressed at low energies due to CVC (conserved vector currents). The correlation function is therefore calculated in the Weinberg-Salam model taking the axial-vector coupling constant $g_A' = 1.24$. The results are shown in Fig. 1 for various values of the nucleon kinetic energies.

To evaluate the correlation function in helicity-flipping (S, P, T) theories, only the form factor for the tensor coupling is required since the process under consideration takes place predominantly through tensor coupling. It is found that the transition from triplet S state to singlet S state is achieved through the isovector tensor coupling (G_{T_1}'). In these theories the triplet-S-to-triplet-S transition is also possible through the isoscalar coupling (G_{T_1}). The numerical estimates are made for these transitions using G_{T_1} and G_{T_1}' given by Adler *et al.*[8] The results are presented in Fig. 2 for the $^3S \to {}^1S$ transition. For the $^3S \to {}^3S$ transition the results are identical to these curves but are multiplied by a constant factor $(G_{T_1}/G_{T_1}')^2$ (Ref. 14).

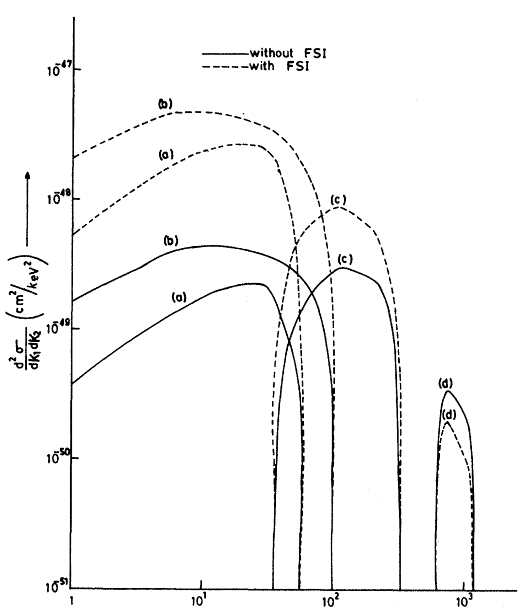

FIG. 2. $d^2\sigma/dK_1dK_2 \sim K_2$, for $^3S \to {}^1S$ transitions in the quark model in helicity-flipping theories, without FSI and with FSI at proton kinetic energies (a) $K_1 = 1.5$ keV, (b) $K_1 = 15$ keV, (c) $K_1 = 150$ keV, and (d) $K_1 = 900$ keV, for fixed incident neutrino energy $E_\nu = 5.5$ MeV.

B. Final-state interactions

The effect of final-state interactions is evaluated from Eq. (2.14) only for S states. The following two forms of the wave function have been used[15]:

$$\text{(a)}\ F_{S,T}(p_c r) = [\sin(p_c r + \delta_{S,T}) - e^{-\lambda^a_{S,T} r}\sin\delta_{S,T}]\,, \tag{3.1}$$

$$\text{(b)}\ F_{S,T}(p_c r) = (1 - e^{-\lambda^b_{S,T} r})\sin(p_c r + \delta_{S,T})\,, \tag{3.2}$$

where $\delta_{S,T}$ are the singlet and triplet n-p scattering lengths. The parameters $\lambda^a_{S,T}$ and $\lambda^b_{S,T}$ are given by[15]

$$\lambda^a_{S,T} = \frac{3}{2r_{S,T}}\left[1 + \left(1 - \frac{16 r_{S,T}}{9 a_{S,T}}\right)^{1/2}\right], \quad \lambda^b_S = 1.215\ \text{fm}^{-1}, \quad \lambda^b_T = 1.244\ \text{fm}^{-1}. \tag{3.3}$$

Using

$a_S = -23.678$ fm, $r_S = 2.51$ fm,

$a_T = 5.396$ fm, $r_T = 1.724$ fm,

we find

$$\lambda^a_S = 1.249\ \text{fm}^{-1} \text{ and } \lambda^a_T = 1.447\ \text{fm}^{-1}. \tag{3.4}$$

Using the above values for $a_{S,T}$ and $r_{S,T}$, the phase shifts of these low energies are calculated from the equation

$$\cot\delta_{S,T} = -\frac{1}{a_{S,T}\,p_c} + \tfrac{1}{2} r_{S,T}\,p_c\,. \tag{3.5}$$

Using these forms for the wave functions, the radial matrix elements K_S, K_T defined in Eqs. (A10) and (A11) are easily evaluated. For the $^3S \to {}^1S$ transition the results are shown in Figs. 1 and 2. The two wave functions given in Eqs. (3.1) and (3.2) give essentially the same result. For the $^3S \to {}^3S$ transition which is possible in helicity-flipping theories, the results are found to be small as compared to the $^3S \to {}^1S$ transition in both cases discussed in Sec. III B.[14] The dominant transition is therefore the $^3S \to {}^1S$ transition even in these theories.

C. Discussion and comparison with Frahm's[10] work

The results presented in this paper can be effective in determining the tensor current couplings as the energy correlation function offers a distinction between the axial-vector and tensor case. While the present experimental result on $\bar\nu_e + d \to \nu_e + n + p$ can be explained by a reduced tensor coupling,[7] the determination of the energy correlation function would clearly confirm its presence or absence. For a fixed proton kinetic energy the $\sigma(K_1, K_2)$ falls off rapidly with the neutron kinetic energy. The fall is more rapid in the case of tensor coupling than in the case of axial-vector coupling. The overall difference in the numerical values of $\sigma(K_1, K_2)$ in these two cases is, however, more pronounced at small nucleon kinetic energies. It is at these kinetic energies that the results are most affected by inclusion of final-state interactions. Theoretically, this is the region of low kinetic energies, which offers a clear distinction between two theories. Experimentally it would be presumably very difficult to perform experiments at such small kinetic energies. It would be interesting to find out the range of nucleon kinetic energies which are observed in the experiments of Pasierb *et al.*[5] In order to get information on the isospin structure of the neutral currents, one needs to go to higher energies where higher states are produced in the final state due to the transition induced by isoscalar currents. The formulation developed in Sec. II would be useful in analyzing the experiments at higher energies.

In an earlier work Frahm[10] has calculated the correlation function $\sigma(K_1, K_2)$ for this transition. He has considered $V-A$ theory and has used the following wave functions for the initial and final states:

$$\phi_i(\vec{r}) = (\gamma/2\pi)^{1/2}\frac{e^{-\gamma r}}{r},$$

$$\phi_f(\vec{r}) = \sin(p_c r + \delta_S)/p_c r.$$

If we take the limits of $\alpha \to \gamma$, $\beta \to \infty$, $q \to 0$, and $\lambda_S^{a,b} \to \infty$ in our final expressions, the results of Frahm are reproduced. Our results are therefore more general and further extend the work of Frahm. We have used more appropriate wave functions for the initial and final states. The effect of using a Hulthen wave function instead of an exponential wave function is to soften the peak in $\sigma(K_1, K_2)$. The q dependence of the matrix elements occurring in Eqs. (A1), (A6), (A9), and (A13) which has been neglected by Frahm does not contribute at low energies but would show up at higher energies.

We have presented a complete discussion of the energy correlation function $\sigma(K_1, K_2)$ in all the five variants of the weak-interaction theory. At low energies relevant to the reactor antineutrino energies only the axial-vector and tensor couplings contribute. The correlation function is sensitive enough to distinguish between the helicity-conserving and helicity-flipping neutral-current couplings due to different energy dependence in the two cases and should be experimentally pursued. The results presented here can be easily extended to the higher energies and q^2 applicable to the ANL and LAMPF experiments, where higher waves can be produced. We have, at present, deferred such a study until further experimental results become available.

APPENDIX A: RESULTS FOR MATRIX ELEMENT SQUARED

1. Without final-state interactions

$|\mathfrak{M}|^2$ is calculated using the $\phi_i(\vec{r})$ given in Eq. (2.11) and plane waves for the final state. The following results are obtained for V, A and S, P, T cases.

(i) V, A case:

$$|\mathfrak{M}|^2 = 2^6\pi G^2 M_d E_1' E_2' E' E_\nu^2 \frac{\alpha\beta(\alpha+\beta)}{(\beta-\alpha)^2}\frac{1}{2p_c^2 q^2}\left[\ln\left(\frac{(A+p_c q)}{A-p_c q}\right) - \ln\left(\frac{B+p_c q}{B-p_c q}\right)\right]^2 \mathcal{G}_{S,T}, \tag{A1}$$

where

$$\mathcal{G}_S = \left\{ g_A'^2(q^2)\left[\left(1-\frac{\vec{k}\cdot\vec{k}'}{3E_\nu E_\nu'}\right) - \frac{\vec{p}_c^{\,2}}{3M^2}\left(1+\frac{\vec{k}\cdot\vec{k}'}{E_\nu E_\nu'}\right) - \frac{1}{2M^2}\frac{\vec{k}\cdot\vec{q}\,\vec{k}'\cdot\vec{q}}{3E_\nu E_\nu'} - \frac{2}{M}\frac{(E_\nu \vec{k}'\cdot\vec{p}_c + E_\nu'\vec{k}\cdot\vec{p}_c)}{3E_\nu E_\nu'} + \frac{2}{M^2}\frac{\vec{k}'\cdot\vec{p}_c\,\vec{k}\cdot\vec{p}_c}{3F_\nu E_\nu'}\right]\right.$$

$$+ F_M'^2(q^2)\frac{1}{2M^2}\frac{(q^2 E_\nu E_\nu' - \vec{k}\cdot\vec{q}\,\vec{k}'\cdot\vec{q})}{3E_\nu E_\nu'} + F_M'(q^2) g_A'(q^2)\left[\frac{2}{M}\frac{(F_\nu'\vec{k}\cdot\vec{q} - E_\nu \vec{k}\cdot\vec{q})}{3E_\nu E_\nu'} + \frac{1}{M^2}\frac{(\vec{k}\cdot\vec{p}_c\,\vec{k}\cdot\vec{q} - \vec{k}\cdot\vec{q}\,\vec{k}'\cdot\vec{p}_c)}{3E_\nu E_\nu'}\right]$$

$$\left. + F_R'(q^2) g_A'(q^2)\frac{1}{2M^2}\frac{(\vec{k}\cdot\vec{p}_c\,\vec{k}'\cdot\vec{q} - \vec{k}\cdot\vec{q}\,\vec{k}'\cdot\vec{p}_c)}{3E_\nu E_\nu'}\right\} \quad \text{for the } {}^1S \text{ state}, \tag{A2}$$

$$\mathcal{G}_T = \Bigg\{ F_1^{\,2}(q^2)\left(1+\frac{\vec{k}\cdot\vec{k}'}{E_\nu E_\nu'}\right) + 2g_A^{\,2}(q^2)\Bigg[\left(1-\frac{\vec{k}\cdot\vec{k}'}{3E_\nu E_\nu'}\right) - \frac{\vec{p}_c^{\,2}}{3M^2}\left(1+\frac{\vec{k}\cdot\vec{k}'}{E_\nu E_\nu'}\right) - \frac{1}{2M^2}\frac{\vec{k}\cdot\vec{q}\,\vec{k}'\cdot\vec{q}}{3E_\nu E_\nu'}$$

$$- \frac{2}{M}\frac{(E_\nu \vec{k}'\cdot\vec{p}_c + E_\nu'\vec{k}\cdot\vec{p}_c)}{3E_\nu E_\nu'} + \frac{2}{M^2}\frac{\vec{k}'\cdot\vec{p}_c\vec{k}\cdot\vec{p}_c}{3E_\nu E_\nu'}\Bigg]$$

$$+ 2F_M^{\,2}(q^2)\frac{1}{2M^2}\frac{(q^2E_\nu E_\nu' - \vec{k}\cdot\vec{q}\,\vec{k}'\cdot\vec{q})}{3E_\nu E_\nu'} + 2F_M(q^2)g_A(q^2)\left[\frac{2}{M}\frac{(E_\nu'\vec{k}\cdot\vec{q} - E_\nu\vec{k}'\cdot\vec{q})}{3E_\nu E_\nu'} + \frac{1}{M^2}\frac{(\vec{k}\cdot\vec{p}_c\vec{k}'\cdot\vec{q} - \vec{k}'\cdot\vec{p}_c\vec{k}\cdot\vec{q})}{3E_\nu E_\nu'}\right]$$

$$+ 2F_R(q^2)g_A(q^2)\frac{1}{2M^2}\frac{(\vec{k}\cdot\vec{p}_c\vec{k}'\cdot\vec{q} - \vec{k}\cdot\vec{q}\,\vec{k}'\cdot\vec{p}_c)}{3E_\nu E_\nu'}\Bigg\} \text{ for the triplet-}S\text{ state,} \tag{A3}$$

$$A = \alpha^2 + p_c^{\,2} + q^2/4, \quad B = \beta^2 + p_c^{\,2} + q^2/4\,, \tag{A4}$$

and

$$p_c^{\,2} = \left[\left(\frac{m_1 m_2}{m_1+m_2}\right)\left(\frac{\vec{p}_1'}{m_1} - \frac{\vec{p}_2'}{m_2}\right)\right]^2. \tag{A5}$$

(ii) (S, P, T) case:

$$|\mathfrak{M}|^2 = 2^8\pi G^2 M_d E_1'E_2'E_\nu'E_\nu\left(\frac{2\alpha\beta(\alpha+\beta)}{(\beta-\alpha)^2 2p_c^2q^2}\right)\left[\ln\left(\frac{A+p_cq}{A-p_cq}\right) - \ln\left(\frac{B+p_cq}{B-p_cq}\right)\right]^2\mathfrak{F}_{S,T}\,, \tag{A6}$$

where

$$\mathfrak{F}_S = \Bigg(2G_{T_1}'^{2}(q^2)\left\{\left(1+\frac{1}{3}\frac{\vec{k}\cdot\vec{k}'}{E_\nu E_\nu'}\right) - \frac{1}{3M}\left[3E_\nu' - E_\nu\left(2+\frac{\vec{k}\cdot\vec{k}'}{E_\nu E_\nu'}\right)\right]\right\} - G_p'(q^2)G_{T_1}'(q^2)\frac{1}{4M}\frac{(E_\nu'\vec{k}\cdot\vec{q} - E_\nu\vec{k}'\cdot\vec{q})}{3E_\nu E_\nu'}$$

$$+ G_p'^{2}(q^2)\frac{q^2}{4M^2}\frac{1}{3}\left(1-\frac{\vec{k}\cdot\vec{k}'}{E_\nu E_\nu'}\right)\Bigg) \text{ for the } {}^1S \text{ state} \tag{A7}$$

$$\mathfrak{F}_T = \Bigg(G_S^{\,2}(q^2)\frac{1}{8}\left(1-\frac{\vec{k}\cdot\vec{k}'}{E_\nu E_\nu'}\right) + 4G_{T_1}^{2}(q^2)\left\{\left(1+\frac{1}{3}\frac{\vec{k}\cdot\vec{k}'}{E_\nu E_\nu'}\right) - \frac{1}{3M}\left[3E_\nu' - E_\nu\left(2+\frac{\vec{k}\cdot\vec{k}'}{E_\nu E_\nu'}\right)\right]\right\}$$

$$- 2G_p(q^2)G_{T_1}(q^2)\frac{1}{4M}\frac{(E_\nu'\vec{k}\cdot\vec{q} - E_\nu\vec{k}'\cdot\vec{q})}{3E_\nu E_\nu'} + 2G_p^{\,2}(q^2)\frac{q^2}{4M^2}\frac{1}{3}\left(1-\frac{\vec{k}\cdot\vec{k}'}{E_\nu E_\nu'}\right)\Bigg) \text{ for the } {}^3S \text{ state.} \tag{A8}$$

2. With final-state interactions

$|\mathfrak{M}|^2$ is calculated using the $\phi_i(\vec{r})$ given in Eq. (2.11) and the final-state wave functions given in Eqs. (3.1) and (3.2). The following results are obtained for V, A and S, P, T cases.

(i) V, A case:

$$|\mathfrak{M}|^2 = 2^6\pi G^2 M_d E_1'E_2'E_\nu'E_\nu K_{S,T}^{\;2}\,\mathcal{G}_{S,T}\,, \tag{A9}$$

where $\mathcal{G}_{S,T}$ are given in Eqs. (A2) and (A3) and $K_{S,T}$ are given as

$$\text{(a) } K_{S,T} = \left[\frac{2\alpha\beta(\alpha+\beta)}{(\beta-\alpha)^2}\right]^{1/2}\frac{1}{p_cq}\Bigg(\sin\delta_{S,T}\left\{\left[\tan^{-1}\left(\frac{2\alpha q}{2A-q^2}\right) - \tan^{-1}\frac{q}{2\alpha_{S,T}'}\right] - \left[\tan^{-1}\left(\frac{2\beta_q}{2B-q^2}\right) - \tan^{-1}\frac{q}{2\beta_{S,T}'}\right]\right\}$$

$$+ \cos\delta_{S,T}\left[\tfrac{1}{2}\ln\left(\frac{A+p_cq}{A-p_cq}\right) - \tfrac{1}{2}\ln\left(\frac{B+p_cq}{B-p_cq}\right)\right]\Bigg), \tag{A10}$$

$$\text{(b) } K_{S,T}=\frac{2\alpha\beta(\alpha+\beta)}{(\beta-\alpha)^2}\frac{1}{p_c q}\left(\sin\delta_{S,T}\left\{\left[\tan^{-1}\left(\frac{2\alpha q}{2A-q}\right)-\tan^{-1}\left(\frac{2\alpha'_{S,T}q}{2A'_{S,T}-q^2}\right)\right]-\left[\tan^{-1}\left(\frac{2\beta_q}{2B-q}\right)-\tan^{-1}\left(\frac{2\beta'_{S,T}q}{2B'_{S,T}-q^2}\right)\right]\right\}\right.$$

$$\left.+\cos\delta_{S,T}\left\{\left[\tfrac{1}{2}\ln\left(\frac{A+p_c q}{A-p_c q}\right)-\tfrac{1}{2}\ln\left(\frac{A'_{S,T}+p_c q}{A'_{S,T}-p_c q}\right)\right]-\left[\tfrac{1}{2}\ln\left(\frac{B+p_c q}{B-p_c q}\right)-\tfrac{1}{2}\ln\left(\frac{B'_{S,T}+p_c q}{B'_{S,T}-p_c q}\right)\right]\right\}\right), \tag{A11}$$

with

$$\alpha'_{S,T}=\alpha+\lambda^{a,b}_{S,T}, \quad \beta'_{S,T}=\beta+\lambda^{a,b}_{S,T}, \tag{A12}$$

$$A'_{S,T}=\alpha'^{\,2}_{S,T}+p_c^2+q^2/4, \quad B'_{S,T}=\beta'^{\,2}_{S,T}+p_c^2+q^2/4,$$

and A, B are given in Eq. (A4).

(ii) S, P, T case:

$$|\mathfrak{M}|^2=2^8\pi G^2 M_d E'_1 E'_2 E'_\nu E_\nu K_{S,T}{}^2\mathfrak{F}_{S,T}, \tag{A13}$$

where $K_{S,T}$ are given in Eqs. (A10) and (A11) and $\mathfrak{F}_{S,T}$ are given in Eqs. (A7) and (A8).

APPENDIX B: LIMITS ON COSθ_{12} AND COSθ_1

The limits of integration in Eq. (2.14), where ω_1, ω_2 are the minimum and maximum values of $\cos\theta_{12}$ and v_1, v_2 are the minimum and maximum values of $\cos\theta_1$, are given as (see Frahm[10] for a detailed discussion)

(i) $\omega_1=\min(\cos\theta_{12})$

$$=-1, \quad \text{if } |p'_1-p'_2|+k'>k, \tag{B1}$$

(ii) $\omega_1=\min(\cos\theta_{12})$

$$=\frac{(k-k')^2-p'^{\,2}_1-p'^{\,2}_2}{2p'_1p'_2}, \quad \text{if } |p'_1-p'_2|+k'<k, \tag{B2}$$

(i) $\omega_2=\max(\cos\theta_{12})$

$$=+1, \quad \text{if } k'+k>p'_1+p'_2, \tag{B3}$$

(ii) $\omega_2=\max(\cos\theta_{12})$

$$=\frac{(k+k')^2-p'^{\,2}_1-p'^{\,2}_2}{2p'_1p'_2}, \quad \text{if } k'+k<p'_1+p'_2, \tag{B4}$$

and

$$v_1=\min(\cos\theta_1)=xy-[(1-x^2)(1-y^2)]^{1/2}, \tag{B5}$$

$$v_2=\max(\cos\theta_1)=xy+[(1-x^2)(1-y^2)]^{1/2}, \tag{B6}$$

where

$$x=(s^2+p'^{\,2}_1-p'^{\,2}_2)/2sp'_1, \tag{B7}$$

$$y=(s^2+k^2-k'^2)/2sk, \tag{B8}$$

and

$$s=(p'^{\,2}_1+p'^{\,2}_2+2p'_1p'_2\cos\theta_{12}). \tag{B9}$$

The quantities k, k', p'_1, and p'_2 are the magnitude of momenta $\vec{k}$, $\vec{k}$, $\vec{p}'_1$, and $\vec{p}'_2$, i.e.,

$$k=|\vec{k}|, \quad k'=|\vec{k}'|, \quad p'_1=|\vec{p}'_1|, \quad p'_2=|\vec{p}'_2|. \tag{B10}$$

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