Impossibility of eliminating the axion by means of the 't Hooft mechanism

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The Peccei-Quinn mechanism leads to axions or massless quarks. Recently, a scheme was proposed by which it was hoped that the presence of axions might be avoided altogether by means of the so-called 't Hooft mechanism. In this paper, we shall show that the price which must be paid for this, necessarily, is the existence of massless quarks.

Several mechanisms[1] have been proposed in order to solve the strong CP puzzle—the puzzle of the smallness of the strong CP violations. One of them is the Peccei-Quinn mechanism. The idea of Peccei and Quinn involves imposing on the Lagrangian a U(1) global symmetry which has a gauge anomaly. This allows one to "rotate away" the θ parameter (in the sense that θ is not a physically observable quantity). This U(1) global symmetry can be realized either in the Wigner-Weyl mode—which leads to the existence of massless quarks—or in the Nambu-Goldstone mode—which leads to the presence of a pseudo-Goldstone boson called the "axion."[2] (Because the Peccei-Quinn symmetry is anomalous, the axion is not exactly massless.) Both of these alternatives seemed to be phenomenologically unsatisfactory.[3] Thus, the Peccei-Quinn mechanism came across problems. Several solutions[4] to these problems have been suggested. Some authors have constructed models in which axions do exist but they are harmless and invisible. Recently, an idea[5] was proposed by which it was hoped that the presence of axions might be avoided altogether. According to this idea, even though the Peccei-Quinn symmetry is broken, no axion would result if some linear combination of the PQ generator and a gauge generator remained unbroken (this is the so-called 't Hooft mechanism). We shall show that the price that must necessarily be paid in such a scheme is the existence of massless quarks. Thus, the alternative of massless quarks or axions cannot be escaped in this way.

Let us consider a renormalizable model with a gauge group G which possesses a Peccei-Quinn global symmetry with the corresponding generator expressed as $Q_{\rm PQ}$. Assume that this model has only one θ parameter. (Thus G may either have one non-Abelian factor, or the non-Abelian part of G may be a power of a simple group, S^n, with the non-Abelian couplings forced to be equal by a discrete reflection symmetry which interchanges the factor groups.) Let us then denote the Lagrangian by $\mathscr{L}=\mathscr{L}_0+\mathscr{L}_\theta$ where $\mathscr{L}_\theta=(\theta/32\pi^2)\sum_a F^a_{\mu\nu}{}^*F^{a\mu\nu}$ and where the index a runs over all the non-Abelian generators. By assumption, the PQ symmetry is anomalous so that the $\mathscr{L}_\theta$ term can be "rotated away." Now let $Q_{\rm PQ}$ and a gauge generator Q_g be spontaneously broken but a linear combination $Q_f=\alpha Q_{\rm PQ}+\beta Q_g$ remain unbroken. Then, there is only one Goldstone boson. But as this is absorbed to give mass to the gauge boson associated with Q_g there is no axion associated with the breaking of the PQ symmetry.

We will prove a simple theorem, namely, that some massless quarks must exist in such a model.

The proof of this theorem is quite simple. The current associated with $Q_{\rm PQ}$ has a $\sum_a F^a_{\mu\nu}{}^*F^{a\mu\nu}$ anomaly by assumption. But $\sum_a F^a_{\mu\nu}{}^*F^{a\mu\nu}$ contains $\sum_a G^a_{\mu\nu}{}^*G^{a\mu\nu}$ where $G^a_{\mu\nu}$ is the gluon field strength of SU(3) of color. So clearly, the PQ current has a gluon-gluon anomaly. We can denote this symbolically by writing

$$A(Q_{\rm PQ};\lambda^a,\lambda^a)\neq 0,\quad \lambda^a\in {\rm SU}(3)_c\ . \tag{1}$$

Now, the current associated with the gauge generator Q_g has no gluon-gluon anomaly by renormalizability. That is,

$$A(Q_g;\lambda^a,\lambda^a)=0;\quad \lambda^a\in {\rm SU}(3)_c\ . \tag{2}$$

So, the current of the exact symmetry $Q_f=\alpha Q_{\rm PQ}+\beta Q_g$ *has* a gluon-gluon anomaly:

$$\begin{aligned} A(Q_f;\lambda^a,\lambda^a)&=\alpha A(Q_{\rm PQ};\lambda^a,\lambda^a)+\beta A(Q_g;\lambda^a,\lambda^a)\\ &=\alpha A(Q_{\rm PQ};\lambda^a,\lambda^a)\neq 0\ . \end{aligned} \tag{3}$$

Now, only colored fermions can contribute to $A(Q_f;\lambda^a,\lambda^a)$ as λ^a is a color generator. Furthermore only massless fermions can contribute. For, consider a massive fermion with mass term $m\bar{\psi}_L\psi_R$ + H.c. Since Q_f is exact, $m\xrightarrow{Q_f}m$. Thus ψ_L and ψ_R transform in the same way under Q_f. Therefore ψ cannot contribute to the anomaly. Since the anomaly $A(Q_f;\lambda^a,\lambda^a)$ is indeed nonvanishing, and only massless colored fermions can contribute to it, the theorem is proved.

APPENDIX

Recently, an SU(8)×SU(8) grand unified model has been proposed (we shall call it the 8×8 model).[5] This model has a PQ symmetry and no axion. According to the theorem, there must be massless quarks in the model. We will show, through explicit calculations, how the theorem works in this model.

First, let us give a simplified description of the 8×8 model.

The local gauge group is $SU(8)_L\times SU(8)_R$. And there is a discrete symmetry which interchanges the two SU(8) factor groups. The fermion content is

$$\psi^{(I)\alpha}\in(8,1)^L_{(I)}=\begin{pmatrix}\nu\\ q_{a_i}\\ e\\ q_{b_i}\end{pmatrix}_{L,(I)} \tag{A1}$$

$$q_{a_i}=\begin{cases}u_R, & i=1\ ,\\ u_G, & i=2\ ,\quad I=1\ ,\\ u_B, & i=3\ ,\end{cases}$$

$$q_{b_i}=\begin{cases}d_R, & i=1\ ,\\ d_G, & i=2\ ,\\ d_B, & i=3\ ,\end{cases}$$

$I=1,2,3,4$ four generations,

$$\psi^{(I)}_\mu\in(1,8^*)^L_{(I)}\ , \tag{A2}$$

$$\psi_{[\alpha\beta]}\in(28^*,1)^L\ , \tag{A3}$$

$$\psi^{[\mu\nu]}\in(1,28)^L\ . \tag{A4}$$

The Higgs particle content is

$$\rho^{(\alpha\beta)}{}_{(\mu\nu)}\in(36,36^*)\ , \tag{A5}$$

$$\omega^{[\alpha\beta]}{}_{[\mu\nu]}\in(28,28^*)\ , \tag{A6}$$

$$\sigma_R^{(\mu\nu)}+\sigma_L^{(\alpha\beta)}\in(1,36)+(36,1)\ , \tag{A7}$$

$$\chi^\alpha_\mu\in(8,8^*)\ . \tag{A8}$$

The $SU(8)_L\times SU(8)_R$-invariant Yukawa couplings are

$$\begin{aligned}&(a_{IJ}\psi^{(I)}_\mu\psi^{(J)}_\nu\sigma_R^{(\mu\nu)}+\text{H.c.})\\ &\quad+(b_{IJ}\psi^{(I)\dagger}_\alpha\psi^{(J)\dagger}_\beta\sigma_L^{(\alpha\beta)}+\text{H.c.})\\ &\quad+(C_{IJ}\psi^{(I)\alpha}\psi^{(J)}_\mu\chi^{\dagger\mu}_\alpha+\text{H.c.})\\ &\quad+(d\,\psi_{[\alpha\beta]}\psi^{[\mu\nu]}\omega^{[\alpha\beta]}{}_{[\mu\nu]}+\text{H.c.})\ .\end{aligned} \tag{A9}$$

The $SU(8)_L\times SU(8)_R$-invariant Higgs potential includes the following terms which are significant for studying the global symmetries:

$$\begin{aligned}&\chi^\alpha_\mu\chi^\beta_\nu\rho^{\dagger(\mu\nu)}{}_{(\alpha\beta)}+\text{H.c.}\ ,\\ &\sigma_R^{(\mu\nu)}\sigma^\dagger_{L(\alpha\beta)}\rho^{(\alpha\beta)}{}_{(\mu\nu)}+\text{H.c.}\ ,\\ &\rho^{(\alpha\beta)}{}_{(\mu\nu)}\omega^{\dagger[\nu\lambda]}{}_{[\beta\gamma]}\chi^{\dagger\mu}_\alpha\chi^\gamma_\lambda+\text{H.c.}\ ,\\ &\sigma_R{}^{(\mu\nu)}\sigma^\dagger_{L(\alpha\beta)}\chi^\alpha_\mu\chi^\beta_\nu+\text{H.c.}\ ,\\ &\omega^{[\alpha\beta]}{}_{[\mu\nu]}\omega^{[\alpha'\beta']}{}_{[\mu'\nu']}\omega^{[\alpha''\beta'']}{}_{[\mu''\nu'']}\omega^{[\alpha'''\beta''']}{}_{[\mu'''\nu''']}\\ &\qquad\times\epsilon^{\mu\nu\mu'\nu'\mu''\nu''\mu'''\nu'''}\epsilon_{\alpha\beta\alpha'\beta'\alpha''\beta''\alpha'''\beta'''}\ .\end{aligned} \tag{A10}$$

Let us consider a general global symmetry Q under which ψ^α, ψ_μ, $\psi_{[\alpha\beta]}$, and $\psi^{[\mu\nu]}$ have charges a_1, a_2, a_3, and a_4, respectively. (See Table I.) The charge assignment of the other fields are determined by the terms in Eqs. (A9) and (A10). The term $\rho^{(\alpha\beta)}{}_{(\mu\nu)}\,\omega^{\dagger[\mu\lambda]}{}_{[\beta\gamma]}\,\chi^{\dagger\mu}_\alpha\,\chi^\alpha_\lambda$ requires $2(a_1+a_2)+(a_3+a_4)=0$. If the last term in (A10), which is quartic in $\omega^{[\alpha\beta]}[\mu\nu]$, is present, then $(a_1+a_2)=(a_3+a_4)=0$ in which case there will be no PQ symmetry in the model. (This is easy to check.) So we will drop this term.

There are three independent global symmetries in this model (without the ω^4 term). We will write their charges as Q_{K_1}, Q_{K_2}, and Q_{PQ}. The corresponding charge assignments are shown in Table I. Q_{K_2} just measures the number of "mirror fermions," i.e., those in (28*,1) + (1,28). These are the fermions which had to be added to make the model anomaly free. This charge is exactly conserved and has no F^*F anomaly. Q_{K_1} is broken[6] but a linear combination of Q_{K_1} and a gauge generator Q_g is exactly conserved and corresponds to the vector baryon number on the ordinary fermions. It also has no F^*F anomaly:

$$Q_g=T_L\oplus T_R\ ,$$

$$T_L=\text{diag}(-\tfrac{1}{4},\tfrac{1}{12},\tfrac{1}{12},\tfrac{1}{12},-\tfrac{1}{4},\tfrac{1}{12},\tfrac{1}{12},\tfrac{1}{12})\ ,$$

$$T_R=\text{diag}(-\tfrac{1}{4},\tfrac{1}{12},\tfrac{1}{12},\tfrac{1}{12},-\tfrac{1}{4},\tfrac{1}{12},\tfrac{1}{12},\tfrac{1}{12})\ ,$$

TABLE I. The Q_{K_1}, Q_{K_2}, and Q_{PQ} are the charges associated with the three independent global rotations of the 8×8 model.

	Q	Q_{K_1}	Q_{K_2}	Q_{PQ}
ψ^{α}	a_1	$\frac{1}{4}$	0	$\frac{1}{4}$
ψ_{μ}	a_2	$-\frac{1}{4}$	0	$-\frac{1}{8}$
$\psi_{[\alpha\beta]}$	a_3	0	$\frac{1}{4}$	0
$\psi^{[\mu\nu]}$	a_4	0	$-\frac{1}{4}$	$-\frac{1}{4}$
$\sigma_R{}^{(\mu\nu)}$	$-2a_2$	$\frac{1}{2}$	0	$\frac{1}{4}$
$\sigma_L{}^{(\alpha\beta)}$	$2a_1$	$\frac{1}{2}$	0	$\frac{1}{2}$
$\chi_{\mu}{}^{\alpha}$	a_1+a_2	0	0	$\frac{1}{8}$
$\omega^{[\alpha\beta]}_{[\mu\nu]}$	$-(a_3+a_4)$	0	0	$\frac{1}{4}$
$\rho^{(\alpha\beta)}_{(\mu\nu)}$	$2(a_1+a_2)$	0	0	$+\frac{1}{4}$

"B_V"$=Q_{K_1}+Q_g$. Finally, Q_{PQ} does have an F^*F anomaly and hence can be used to rotate away θ at the grand-unification level. Q_{PQ} is broken spontaneously[6] but a linear combination of Q_{PQ} with a gauge generator Q_g' is exactly conserved (up to anomalies) and corresponds to the lepton number minus the right-handed neutrino number "$L_V-N_{\nu R}$" $=Q_{PQ}+Q_g'$.

$$Q_g'=T_L'\oplus T_R' \,,$$

$$T_L'=\mathrm{diag}(\tfrac{3}{4},-\tfrac{1}{4},-\tfrac{1}{4},-\tfrac{1}{4},\tfrac{3}{4},-\tfrac{1}{4},-\tfrac{1}{4},-\tfrac{1}{4})\,,$$

$$T_R'=\mathrm{diag}(-\tfrac{1}{8},-\tfrac{1}{8},-\tfrac{1}{8},-\tfrac{1}{8},\tfrac{7}{8},-\tfrac{1}{8},-\tfrac{1}{8},-\tfrac{1}{8})\,.$$

If we look at the values of $L_V-N_{\nu R}$ on the fermions in the $(28^*,1)+(1,28)$ we find that there are

TABLE II. The Peccei-Quinn charges of the mirror fermions in the $(28^*,1)+(1,28)$.

$\psi_{[\alpha\beta]}$	Color	$L_V-N_{\nu,R}$	$\psi^{[\mu\nu]}$	Color	$L_V-N_{\nu,R}$
$\psi_{[15]}$	1	$-\frac{3}{2}$	$\psi^{[15]}$	1	$+\frac{1}{2}$
$\psi_{[1a_i]}$	3*	$-\frac{1}{2}$	$\psi^{[1a_i]}$	3	$-\frac{1}{2}$
$\psi_{[1b_i]}$	3*	$-\frac{1}{2}$	$\psi^{[1b_i]}$	3	$-\frac{1}{2}$
$\psi_{[5a_i]}$	3*	$-\frac{1}{2}$	$\psi^{[5a_i]}$	3	$+\frac{1}{2}$
$\psi_{[5b_i]}$	3*	$-\frac{1}{2}$	$\psi^{[5b_i]}$	3	$+\frac{1}{2}$
$\psi_{[a_ia_i']}$	3	$\frac{1}{2}$	$\psi^{[a_ia_i']}$	3*	$-\frac{1}{2}$
$\psi_{[b_ib_i']}$	3	$\frac{1}{2}$	$\psi^{[b_ib_i']}$	3*	$-\frac{1}{2}$
$\psi_{[a_ib_i]}$	9=3+6*	$\frac{1}{2}$	$\psi^{[a_ib_i]}$	9=3*+6	$-\frac{1}{2}$
	$a_i,a_i'=2,3,4$				
	$b_i,b_i'=6,7,8$				

four color-triplet left-handed Weyl spinors that remain exactly massless due to $L_V - N_{\nu R}$ conservation. This is shown in Table II. Thus, this model has massless quarks.

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[6] The pattern of vacuum expectation values (VEV's) is given in Ref. 5. In our notation the following Higgs scalars get nonvanishing VEV's: $(\sigma_R)^{11}$, $(\chi)^5{}_5$, $(\chi)^{a_i}{}_{a_i}$, $(\chi)^{b_i}{}_{b_i}$, $(\omega)^{a_i b_i}{}_{a_i b_i}$, $(\omega)^{a_i b_j + a_j b_i}{}_{a_i b_j + a_j b_i}$, $(\rho)^{a_i 5 - b_i 1}{}_{a_i 5}$.