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Dynamic theory of quark and meson fields

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A new equation describing a confined fermion field is obtained via spinor factorization of the Klein-Gordon equation. It is shown that the particle-antiparticle bound states which follow from the appropriate Bethe-Salpeter equation are spin-0 and spin-1 meson fields. It is argued that the new equation describes a quark field.

INTRODUCTION

It is generally assumed that hadrons are composed of quarks.[1] Quarks can be described by the Dirac equation with some additional constraints imposed to achieve their confinement. Most models currently in use introduce the constraints in a rather *ad hoc* manner. The confining mechanism may also be provided by the very promising quantum-chromodynamic (QCD) theory[2] also based on the Dirac equation. The confining mechanism arises nontrivially in two dimensions due to gluon exchange. However, it is implicitly assumed that quarks and gluons are always coupled and hence by definition free quarks are not possible.

In an earlier paper we attempted to derive a quark equation, describing free quarks, from fundamental principles.[3] Let us recall that the Dirac equation can be derived via spinor factorization of the Klein-Gordon equation. Hence, by a similar technique, one can try to derive an equivalent of the Dirac equation which might be suitable for description of the free field and also interacting with Yang-Mills gluon quark fields.

Valuable insights into quark properties were provided by the light-cone approach.[4] For instance, various relations previously obtained in the $p\rightarrow\infty$ limit were rederived and apparently different models of current and constituent quarks were related.[4] Deep-inelastic electron scattering probes the small-distance (light-cone) structure of the hadron wave function.[4c] The most important part of the wave function is concentrated at almost light-cone distances between quarks and hence it is useful to use the light-cone variables $p^{\pm}=p^0\pm p^3$, $p_\perp=(p^1,p^2)$, instead of the four-vector p^m.

The success of the light-cone formalism suggests that it should be fruitful to work with the spinor components $p^{\alpha\dot\beta}=p^0\pm p^3$, $p^1\pm ip^2$, rather than in the mixed representations $(p^{\pm},p_\perp)$. Thus in this paper we shall use the spinor components $p^{\alpha\dot\beta}$ to factorize the Klein-Gordon equation.

The notation of Bjorken and Drell[5] is adopted. Indices $\alpha,\beta=1,2$; $m,n=0,1,2,3$; $P,Q,R=1,\ldots,8$ indicate spinor, four-vector, and SU(3)-vector components, respectively. The SU(3) matrices λ_P are consistent with the Gell-Mann convention.[6]

The outline of the paper is as follows. In Sec. I we rederive the fundamental equation[3] and also give its Hamiltonian form. In Sec. II the Lorentz covariance of the theory is discussed. In Sec. III the fundamental equation is investigated in some special frames of reference. In Sec. IV inner degrees of freedom of quark fields are studied. Finally, the quark confinement and the Bethe-Salpeter equation for quark-antiquark bound states are investigated from the QCD viewpoint in Sec. V. The last section contains conclusions and predictions as well as parallels with other quark models.

I. BASIC EQUATIONS

The spinor components $p^{\alpha\dot\beta}=(p^0+\vec\sigma\cdot\vec p)^{\alpha\dot\beta}$, where $\vec\sigma$ are the Pauli matrices, obey the identity

$$p^{1\dot1}p^{2\dot2}-p^{1\dot2}p^{2\dot1}=p_mp^m\ ,$$

so that the system of equations

$$\begin{aligned}&p^{1\dot1}\chi=M\xi^{1\dot1}\ ,\\&p^{2\dot1}\chi=M\xi^{2\dot1}\ ,\\&p^{2\dot2}\xi^{1\dot1}-p^{1\dot2}\xi^{2\dot1}=M\chi\end{aligned}\tag{1}$$

implies that for $M\neq0$ χ obeys the Klein-Gordon equation.

Equation (1) can be written in matrix form[3]

$$(\not p-M)\psi_q(x)=0,\quad \not p=\rho^np_n,\quad \psi_q(x)=\begin{bmatrix}\xi^{1\dot1}\\ \xi^{2\dot1}\\ \chi\end{bmatrix},$$
$$\rho_m=(\lambda_4,-i\lambda_7,-i\lambda_6,-i\lambda_5)\ .\tag{2}$$

The left and right solutions of Eq. (2) in the momentum representation,

$$(\not k-M)u_k=0,\quad \bar u_k(\not k-M)=0\ ,\tag{3}$$

$$u_k=\begin{bmatrix}k^0+k^3\\ k^1+ik^2\\ M\end{bmatrix},\quad \bar u_k=(k^0-k^3,\ -k^1+ik^2,M)\ ,\tag{4}$$

are independent in contrast to the standard equations (e.g., $\bar w=w^\dagger\gamma^0$ for the Dirac field).

We note that the Klein-Gordon equation can also be factorized to give a Hamiltonian form

$$p^0\psi_q = (p^1\lambda_7 + p^2\lambda_6 + p^3\lambda_5 + M\lambda_4)\psi_q \, , \tag{5}$$

where the identity

$$(p^0)^2 = (p^1 + ip^2)(p^1 - ip^2) + (p^3 + iM)(p^3 - iM)$$

was used instead of the spinor identity.

II. LORENTZ COVARIANCE

The Lorentz covariance of Eqs. (2) and (3) can be studied in the standard fashion.[3] The infinitesimal transformations $B = 1 + \vec{\lambda}\cdot\vec{n}\delta V$, $R = 1 - i\vec{\lambda}\cdot\vec{n}\delta\theta$, $\vec{n}\cdot\vec{n} = 1$, $\vec{\lambda} = (\lambda_1, \lambda_2, \lambda_3)$ represent boosts and rotations, respectively,[3] and correspond to the SU(2) subgroup of SU(3) degrees of freedom of $\psi_q(x)$. These Lorentz transformations are complex; they transform a general four-vector k^m into a complex one $k^{m'}$. However, the transformations in the x^0x^3 and x^1x^2 planes (λ_{03} and λ_{12}, respectively) can be made real if the following generators are introduced:

$$\begin{aligned} B(\tilde{\lambda}_{03}) &= 1 + \tfrac{1}{2}\delta V\left(\lambda_3 + \frac{1}{\sqrt{3}}\lambda_8 + c_1\right), \\ R(\tilde{\lambda}_{12}) &= 1 - \tfrac{1}{2}i\delta\theta\left(\lambda_3 - \frac{1}{\sqrt{3}}\lambda_8 + c_2\right), \end{aligned} \tag{6}$$

where the constants c_1, c_2 do not change the commutation relations of generators and will be determined in Sec. IV.

The rest-mass four-vector $k^{m(0)} = (M, 0, 0, 0)$ is transformed by Lorentz transformations into a general four-vector $k^m = (k^0, k^1, k^2, k^3)$ (Ref. 3):

$$k^{m(0)} \xrightarrow{\lambda^{01}} (k^{0'}, k^{1'}, 0, 0) \xrightarrow{\tilde{\lambda}^{12}} (k^{0'}, k^1, k^2, 0) \xrightarrow{\tilde{\lambda}^{03}} k^m \, . \tag{7}$$

The little group of $k^{m(0)}$ is O(2) and not O(3) as it is in the case of standard equations for $M \neq 0$. Hence Eqs. (3) are not covariant in frames with $p_\perp' \neq p_\perp$. On the other hand, the equation for the density matrix ρ_k,

$$\begin{aligned} \rho_k &= [(\not{k} + M) - M^{-1}(k^2 - \not{k}^2)]/2M = |u_k\rangle\langle\bar{u}_k|/2M \, , \\ &\mathrm{Tr}(\rho^m\rho_k) = k^m, \quad \rho_k^{\,2} = \rho_k \quad (k^2 = M^2) \, , \end{aligned} \tag{8}$$

can be written in the explicitly covariant form[3]

$$\rho_k(\not{k} - M) = (\not{k} - M)\rho_k = k^2 - M^2 = 0 \, . \tag{9}$$

To complete the discussion of relativistic covariance let us note that the matrices

$$\rho_5 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & +1 \end{bmatrix}, \quad \eta = \begin{bmatrix} +1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & +1 \end{bmatrix},$$

$$\begin{aligned} &[\rho_5, \rho^n]_+ = 0, , \; n = 0, 1, 2, 3, \\ &[\eta, \rho^l]_+ = 0 = [\eta, \rho^m]_-, \quad l = 1, 2, \quad m = 0, 3 \end{aligned} \tag{10}$$

correspond to the CPT and $P_\perp$ symmetries

$$\begin{aligned} CPT\psi_q(x) &= \rho_5\psi_q(-x), \\ P_\perp\psi_q(x) &= \eta\psi_q(x^0, -x^1, -x^2, x^3). \end{aligned} \tag{11}$$

The symmetry $P_\perp$ reflects the collinear symmetry of the quark field ψ_q. We note that CPT and $P_\perp$ (as well as their product) are the only discrete symmetries of Eq. (2).

It is important that Eq. (4) implies in the low-momentum limit that the second components of solutions are small. Thus the elimination of the small component ξ^{21} from Eqs. (1) and (2) gives

$$[\sigma^1 p^0 - (1 - \sigma^3)p_\perp^{\,2}/2M - (-i\sigma^2)p^3]\varphi = M\varphi \, , \tag{12}$$

or in the Hamiltonian form

$$p^0\varphi = [(\sigma^1 - i\sigma^2)p_\perp^{\,2}/2M + \sigma^3 p^3 + \sigma^1 M]\varphi \, . \tag{13}$$

In the nonrelativistic limit $|\vec{p}| \ll M$ or for $|p_\perp| \ll M$ Eq. (12) becomes the two-dimensional Dirac equation

$$(\gamma^0 p^0 - \gamma^3 p^3)\varphi = M\varphi, \gamma^0 = \sigma^1, \quad \gamma^3 = -i\sigma^2, \quad \gamma^5 = \sigma^3 \, . \tag{14}$$

It is interesting to compare Eq. (12) with the nonrelativistic (or $p_\perp = 0$) limit of the Dirac equation transformed nonlocally by the Melosh transformation.[4a,4e] The Melosh-transformed Dirac equation

$$\begin{aligned} &[\gamma^0 p^0 - \gamma^3 p^3 + \beta(p^2 + M^2)^{1/2}]\psi = 0 \, , \\ &\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix} \end{aligned} \tag{15}$$

reduces also for $|\vec{p}| \ll M$ or $|p_\perp| \ll M$ to the two-dimensional Dirac equation (14), the 4×4 Dirac matrices being the only difference (this accounts for the presence of spin which has been neglected in our model). The nonrelativistic ($|\vec{p}| \ll M$) and axial ($|p_\perp| \ll M$) consistency of Eqs. (12) and (15) is very satisfactory because the Melosh representation of the Dirac equation turned out to be very useful in relating current- and constituent-quark pictures.[4] The Melosh equation (15) has two disadvantages, however. First, it originated from the Dirac equation via nonlocal transformation; second, it is not linear with respect to p^m and hence cannot be treated as the fundamental quark equation. Accordingly, its utility is restricted to the frames with $|p_\perp| \ll M$.[4b,4e] On the other hand, the above-mentioned analogy suggests how to incorporate spin into our theory.

III. BASIC EQUATION IN SPECIAL FRAMES OF REFERENCE

It will be useful to study Eq. (2) in some special frames of reference. Let us first investigate the case of $p=(p^0,0,0,p^3)$ frame. The basic equations read

$$(\rho^0 p^0 - \rho^3 p^3)\psi_q = M\psi_q\ ,$$
$$\overline{\psi}_q(\rho^0\overleftarrow{p}^0 - \rho^3\overleftarrow{p}^3) = -M\overline{\psi}_q\ , \tag{16}$$
$$\overline{\psi}_q = \overline{\psi}_q^\dagger\tilde{\eta}\ ,\ \tilde{\eta} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

and are equivalent to the two-dimensional Dirac equations for fermion fields $\varphi, \overline{\varphi}$,

$$(\gamma^0 p^0 - \gamma^3 p^3)\varphi = M\varphi,\quad \overline{\varphi}(\gamma^0\overleftarrow{p}^0 - \gamma^3\overleftarrow{p}^3) = -M\overline{\varphi}\ ,$$
$$\overline{\varphi} = \varphi^\dagger\tilde{\eta},\quad \tilde{\eta} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \equiv \gamma^0\ . \tag{17}$$

In the momentum representation we have

$$(\rho^0 k^0 - \rho^3 k^3)u_k = M u_k\ ,$$
$$\overline{u}_k(\rho^0 k^0 - \rho^3 k^3) = M\overline{u}_k\ , \tag{18a}$$

$$u_k = \begin{pmatrix} k^0 + k^3 \\ 0 \\ M \end{pmatrix}(k^0 + k^3)^{-1/2}\ ,$$
$$\overline{u}_k = u_k^\dagger \eta = (M, 0, k^0 + k^3)(k^0 + k^3)^{-1/2}\ . \tag{18b}$$

The solutions are normalized as in the Dirac theory:

$$\overline{u}_k u_k = 2M = -\overline{u}_{-k}u_{-k}\ . \tag{19}$$

The second quantization of the field $\psi_q(x)$ gives

$$\psi_q(x) = \sum_k (a_k u_k e^{-ikx} + b_k^\dagger u_{-k} e^{ikx})\ , \tag{20}$$

where a_k, b_k have to obey the anticommutation fermion relations [note the equivalence of Eqs. (16) and (17)].

In the case of $p=(p^0, p_\perp, 0)$, $p_\perp = (p^1, p^2)$, we have

$$(\rho^0 p^0 - \rho^1 p^1 - \rho^2 p^2)\psi_q = M\psi_q\ ,$$
$$\overline{\psi}_q(\rho^0\overleftarrow{p}^0 - \rho^1\overleftarrow{p}^1 - \rho^2\overleftarrow{p}^2) = -M\overline{\psi}_q\ , \tag{21}$$
$$\overline{\psi}_q = \psi_q^\dagger\eta,\quad \eta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

If the second component of ψ_q is eliminated we get

$$p^0\chi = [(\sigma^1 - i\sigma^2)p_\perp{}^2/2M + \sigma^1 M]\chi\ . \tag{22}$$

We realize that Eq. (22) is the well-known Feshbach-Villars representation of the Klein-Gordon equation[7] describing a boson field $[\vec{p} = (p_\perp, 0)]$. Thus the second quantization has to be consistent with boson commutation rules. In the momentum representation Eq. (21) yields

$$(\rho^0 p^0 - \rho^1 k^1 - \rho^2 k^2)v_k = M v_k\ . \tag{23a}$$
$$\overline{v}_k(\rho^0 k^0 - \rho^1 k^1 - \rho^2 k^2) = M\overline{v}_k\ , \tag{23b}$$
$$\overline{v}_k v_k = 2M = +\overline{v}_{-k}v_{-k}\ , \tag{24}$$

different from the fermion ones, Eq. (19), so that the operators α_k, β_k,

$$\psi_q(x) = \sum_k (\alpha_k v_k e^{-ikx} + \beta_k^\dagger v_{-k} e^{ikx})\ , \tag{25}$$

obey the boson commutation rules and it can be shown that the Hamiltonian and conserved charge operator have proper form.

Now some meaning can be given to the nonphysical complex Lorentz transformations relating frames $(k^0,0,0,k^3)$ and $(k^0, k, 0)$. Such transformations would mix fermion and boson fields and thus are not allowed.

IV. INNER DEGREES OF FREEDOM

Results presented in this section have a rather preliminary character. A four-potential B_m, corresponding to the Yang-Mills field, is introduced into Eq. (1) or Eq. (2) in the usual fashion:

$$i\partial \to i(\partial_m - igB_m) \equiv \pi_m\ . \tag{26}$$

The presence of inner degrees of freedom can be detected by squaring Eq. (2) in analogy with the procedure used in the Dirac theory.[8]

Let us first assume that the four-potential has axial symmetry compatible with the axial symmetry of the field ψ_q, e.g.,

$$B_m = (-\tfrac{1}{2}Ex^3, 0, 0, -\tfrac{1}{2}Ex^0),\ E = \text{const},$$
or
$$B_m = (0, \tfrac{1}{2}Bx^2, -\tfrac{1}{2}Bx^1, 0),\quad B = \text{const}. \tag{27}$$

We have now

$$(\pi^0 + \pi^3)\psi_3 = M\psi_1\ ,$$
$$(\pi^1 + i\pi^2)\psi_3 = M\psi_2\ , \tag{28}$$
$$(\pi^0 - \pi^3)\psi_1 + (-\pi^1 + i\pi^2)\psi_2 = M\psi_3\ .$$

Equations (28) due to $[\pi^0 \pm \pi^3,\ \pi^1 \pm i\pi^2]_- = 0$ [cf. Eq. (27)] can be squared to yield

$$[(\pi_m\pi^m - M^2) + igF_{03}\tilde{\lambda}_{03} + gF_{12}\tilde{\lambda}_{12}]\psi_q = 0\ , \tag{29}$$

where

$$F_{03} = \partial_0 B_3 - \partial_3 B_0 + ig[B_3, B_0]_-\ ,$$
$$F_{12} = \partial_1 B_2 - \partial_2 B_1 + ig[B_1, B_2]_-\ , \tag{30}$$

and the constants c_1, c_2 in Eq. (6) are now determined and equal to $\frac{1}{3}$ and $-\frac{1}{3}$, respectively. We note that the form of Eq. (29) is in close analogy to the form of the squared Dirac equation.[8]

Equation (29) reveals thus the presence of inner degrees of freedom corresponding to the SU(3) generators. The squared equation, however, does not have the Hamiltonian form and we shall not interpret it further in this paper.

Let us now consider the case of a general four-vector B_m. We can gain some insight using the method of the small component[8] and thus working in the Hamiltonian formalism. The second component of ψ_q can be eliminated from Eq. (28) to give

$$[(\gamma^0\pi^0 - \gamma^3\pi^3) + (1-\gamma^5)(-\pi^1 + i\pi^2)(\pi^1 + i\pi^2)/2M]\chi = M\chi \ . \quad (31)$$

The Hamiltonian form of Eq. (31) reads

$$\pi^0\chi = \{[(\sigma^1 - i\sigma^2)\pi_\perp^2/2M - (g/2M)B_3] + \pi^3\sigma^3 + M\sigma^1\}\chi . \quad (32)$$

Comparison of Eqs. (32) and (13) shows that the substitution

$$p_\perp{}^2/2M \rightarrow \pi_\perp{}^2/2M - (g/2M)\vec{m}\cdot\vec{B}, \quad \vec{m} = (0,0,1) \ , \quad (33)$$

should be made in Eq. (13). Thus the field χ has an additional degree of freedom giving rise to the change of its kinetic energy. It is interesting that the interaction in Eq. (33) has axial symmetry. The vector $\vec{m}$ corresponds to the direction of the (internal) particle Yang-Mills magnetic moment aligned along the distinguished axis x^3.

Equations (32) and (33) are in close analogy with the Pauli equation[8] obtained from the Dirac equation within the formalism of the small component.

V. THE BETHE-SALPETER EQUATION FOR BOUND PARTICLE-ANTIPARTICLE STATES AND THE FERMION CONFINEMENT

It follows from the discussion of the Lorentz covariance of Eq. (3) that the density matrix $\rho_k \sim |u_k\rangle\langle\bar{u}_k|$ is covariantly defined in contradistinction to u_k (or $\bar{u}_k$). The density matrix is the Bethe-Salpeter (BS) amplitude for the particle-antiparticle pair ($q\bar{q}$) in absence of a binding potential.[3]

This situation is somewhat analogous to the Maxwell theory of electromagnetic fields. Electric or magnetic fields are not covariantly defined and hence have to be united in the Maxwell tensor $(\vec{E},\vec{B}) \leftrightarrow F^{mn}$. Similarly, the $q\bar{q}$ amplitude $\rho_k \leftrightarrow (u_k, \bar{u}_k)$ turns out to be the fundamental physical quantity in our theory. This fact can be interpreted as the quark confinement of the relativistic origin. Quarks can be detected in $(p^0,0,0,p^3)$ frames, i.e., in high-energy scattering conditions,[4] in which they should obey the two-dimensional Dirac equation, and in these frames our theory predicts fermions. On the other hand, in $(p^0, p_\perp, 0)$ frames there are only boson degrees of freedom and in general frames (p^0,p^1,p^2,p^3) only $q\bar{q}$ pairs, represented by the density matrix, have physical meaning and thus free fermions disappear from the theory. The lack of relativistic covariance of the basic Eq. (2) suggests that the fermion quarks can show up in high-energy scattering only.

The density matrix ρ_k can be given more precise interpretation on the basis of the BS equation. The BS equation, based on Eq. (2), should have the following structure[9]:

$$(M - \rho^n k_n)\phi(k,0)(M - \rho_m k^m) = \frac{\lambda}{\pi^2 i}\int d^4k' \, \frac{g_{mn}}{(k-k')^2 - i\epsilon}\Gamma^m\phi(k',0)\Gamma^n \ , \quad (34)$$

so that the BS equation for Dirac particles in two dimensions is obtained for $|\vec{k}| \ll M$ or $k_\perp \cong 0$. We shall compare the solutions of Eq. (34) with analogous results for the BS equation, based on the four-dimensional Dirac equation[9] in the simplest case $\lambda \rightarrow 0$.

In the Dirac case there are three sectors in the BS amplitude: P, AT, SV,[9] which for $\lambda = 0$ correspond to the solutions of the Klein-Gordon, the Duffin-Kemmer spin-1, and the Duffin-Kemmer spin-0 meson equations, respectively. Similar analysis for the Bogoliubov model[10] (based on BS-type equations) yielded the same results.[3] This shows that the BS formalism, based on the four-dimensional Dirac equation, leads to meson states.

The BS amplitude of the $q\bar{q}$ pair, based on Eq. (3), can be decomposed in terms of the SU(3) matrices (in analogy with the procedure used in the Dirac BS equation[9]):

$$\phi(k,0) = \sum_P \lambda_P\phi_P(k) + \phi_9(k) \ , \quad (35)$$

and the substitution of Eq. (35) to Eq. (34) gives for $\lambda = 0$ the standard spin-0 (and spin-1) meson solutions. Therefore, the density matrix ρ_k corresponds to quark-antiquark pair mesons.

It thus seems necessary to study the BS equation for quark-antiquark bound pairs in the presence of the Yang-Mills fields [i.e., based on Eq. (28)]. It follows from our results of Sec. II that the quark fields at low momentum or in the frame $p_\perp = 0$ obey the two-dimensional Dirac equation. Hence it is natural to start with the two-dimensional theory. We find it particularly useful that there was a tremendous work done in the field of two-dimensional theories.[2] Now, it is convenient to summarize the results of 't Hooft for the two-dimensional $q\bar{q}$ BS amplitude within the QCD formalism.[11] The theory is described by the Lagrangian density[2,11]

$$L = \tfrac{1}{4}F_{mn,}{}^j_i F^{mn,}{}^i_j + \bar{q}^{ai}(i\gamma^m D^j_{ni} - m_a\delta^j_i)q^a_j \ , \quad (36)$$

where $m,n = 0,1$; $i,j = 1,\ldots,N$; a correspond to the space-time, color, and flavor degrees of freedom, respectively.

The Yang-Mills tensor and the covariant derivative have the form

$$F_{mn,}{}^{j}_{i} = \partial_m A_{n,}{}^{j}_{i} - \partial_n A_{m,}{}^{j}_{i} + g[A_m, A_n]^j_i\,,$$
$$D_{m,}{}^{j}_{i} = \delta^j_i \partial_m + g A_{m,}{}^{j}_{i}\,,\quad A_{m,}{}^{j}_{i} = (-i/2) A_m{}^k (\lambda_k)^j_i\,, \tag{37}$$

where A_m is the Yang-Mills potential, $m = 0, 1$.

't Hooft found that the Bethe-Salpeter equation satisfied by the ladder diagrams for the quark-antiquark scattering amplitude reduced to an eigenvalue condition for the two-particle spectrum. The spectrum was found to be discrete only and thus the theory does not contain states with two free quarks.

VI. CONCLUSIONS AND PREDICTIONS

Let us summarize the results of our model. It follows that (i) the Hamiltonian and the Lorentz generators are built from the SU(3) matrices; (ii) it has been shown that if a four-potential is introduced into Eq. (2), the interaction terms which arise are proportional to the hypercharge and isotopic spin operators; (iii) in the low-momentum limit and in the axial momentum frame we get the two-dimensional Dirac equation corresponding to a fermion field; (iv) in the $(p^0, p_\perp, 0)$ frame there are only boson states and in a general momentum frame only meson $q\bar{q}$ states have physical meaning; (v) the quark field ψ_q has collinear symmetry; (vi) solutions of the BS equation for the bond particle-antiparticle states contain spin-0 and spin-1 meson fields.

Results (i)–(iii) show that Eq. (2) describes a fermion field [in the $(p^0, 0, 0, p^3)$ frame] having SU(3) degrees of freedom that suggests a quark field. One of the most important results is that Eq. (14), the low-momentum limit (or axial momentum limit) of Eq. (2), is consistent with several models of the quark field. The string models,[12] the Gross-Neveu approach[13] (based on the Dirac equation in two dimensions), the parton model[14] successful in the region $|p^3|/|p_\perp| \gg 1$, and the MIT bag model[15] (at high angular momenta) can be considered to be of two-dimensional space-time character. In the light-cone approach the frame $p_\perp = 0$ was found to be of great importance and collinear symmetry of the quark field was established.[4] Also QCD suggests a tubelike character of quark fields.[16] Our theory predicts quasi-two-dimensional behavior of the quark field at low momentum and demonstrates the collinearity of quark fields [point (v)]. Point (vi) provides an important argument. It shows that particle-antiparticle states correspond to meson fields and hence ψ_q can be interpreted as a meson component. Furthermore, it follows that only $q\bar{q}$ states, mesons, have physical meaning in all momentum frames.

The picture emerging from points (iii) and (vi) supports the quark-parton model of hadrons[4c,14] in which the fermion partons (valence quarks) are submerged in the cloud of boson partons (quark-antiquark pairs) and gluons. It follows from our theory that fermion partons correspond to the $p \cong (p^0, 0, 0, p^3)$ frames while the boson partons correspond to the $p \cong (p^0, p_\perp, 0)$ frames of reference. Therefore, the boson partons, obeying the Feshbach-Villars equation in our theory, can be identified with so-called wee partons[4c,14a] for which the ratio $|p^3|/|p_\perp|$ is small.

It is interesting that the theory yields some new phenomenological predictions. First, the present approach shows directly that quarks at low momentum or $p_\perp = 0$ obey the two-dimensional Dirac equation (8). Second, the wee partons are shown to obey the Feshbach-Villars equation $[p \cong (p^0, p_\perp, 0)]$. Third, it follows from Eq. (2) that quark fields are CPT and $P_\perp$ invariant only. However, quarks are detected in experimental conditions corresponding to $p \cong (p^0, 0, 0, p^3)$ frames and obey approximately the Dirac equation so that the C or CP violation should be small.

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