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SU(2) adjoint Higgs model

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An adjoint Higgs field with coupling β_H is added to the Wilson lattice SU(2) in order to study the interpolation between the pure SU(2) ($\beta_H=0$) and pure U(1) ($\beta_H=\infty$) gauge theories. Monte Carlo simulations support our theoretical expectation that the deconfinement transition of the U(1) theory connects to the O(3) Higgs transition to produce a two-phase structure in the $\beta-\beta_H$ plane. Implications for the physics of SU(2) confinement and for the continuum limit of the Georgi-Glashow model are discussed.

I. INTRODUCTION

As a prototype, SU(2) lattice gauge theory provides the simplest example in which to explore the physics of quark confinement. However, even this model exhibits complex behavior which has yet to be understood fully. Foremost among these is the rapid crossover between the strong- and weak-coupling regimes, with its concomitant sudden onset of asymptotically free scaling.[1] Thus, it is worthwhile to extend and modify the original SU(2) action to attempt to achieve further insight into the SU(2) dynamics.[2,3,4] In this paper, we undertake the study of a non-Abelian Higgs model that interpolates between the familiar terrritory of Abelian U(1) and the as yet not fully charted case of SU(2). In particular, this will allow us to examine whether the role of Abelian U(1) monopoles in generating confinement extends beyond U(1) into the non-Abelian context, as has been suggested by 't Hooft.[5]

To connect SU(2) to U(1), we introduce on the lattice an adjoint Higgs field (vector ϕ^α, $\alpha=1,2,3$) coupled to the adjoint link variable $\mathscr{O}_{\alpha\beta}$ with strength β_H,

$$\mathscr{O}_{\alpha\beta}=\tfrac{1}{2}\,\mathrm{Tr}(U\tau_\alpha U^\dagger\tau_\beta)\ , \qquad (1.1)$$

where U is the usual link variable in the fundamental representation. This is a lattice version of the Georgi-Glashow model[6] for the symmetry breaking SU(2)→U(1). The Georgi-Glashow model has two phases: a broken-symmetry phase with an unbroken U(1) subgroup and a massless photon, and a confining massive phase with a full unbroken SU(2) symmetry. Thus, we also expect two phases in the lattice model. We investigate this structure via a combination of analytic analysis and Monte Carlo calculations. We then conclude with some comments on the relevance of our lattice model for the broader issues of confinement and the nature of the continuum theory.

II. THEORETICAL CONSIDERATIONS

In this section, we will first set the notation and then proceed to study analytically the structure of the $\beta-\beta_H$ phase plane. Our theoretical work will enable us to formulate a rough picture of the two-phase (confinement and Higgs) structure of the plane.

In the Euclidean lattice formulation, the gauge-invariant contribution to the action $S_H(U,\phi)$ of the real unimodular vector Higgs field $\phi^\alpha(n)$ on each site n coupled to the SU(2) gauge matrices on the link l from site n is given by

$$S_H(U,\phi)=\sum_{n,l}\phi^\alpha(n)\mathscr{O}_{\alpha\beta}(U_l(n))\phi^\beta(n+l)\ , \qquad (2.1)$$

where $\mathscr{O}_{\alpha\beta}$ is the adjoint link variable (1.1).

The corresponding partition function is given by

$$Z(\beta,\beta_H) = \int \{d\phi\}\{dU_l\}\exp\left[\frac{\beta}{2}S_W(U) + \frac{\beta_H}{2}S_H(U,\phi)\right], \quad (2.2)$$

where $S_W(U)=\sum_{l\in P}\mathrm{Tr}U_p$ is the usual Wilson action, $\{dU_l\}$ is the integral with respect to the SU(2) Haar measure over all configurations of the U's and similarly $\{d\phi\}$ is the configuration integral over the ϕ's with the measure

$$d\phi(n)=\frac{d^3\phi(n)}{4\pi}\delta^3(\vec{\phi}^{\,2}(n)-1)\ . \quad (2.3)$$

This measure is that of the classical O(3) Heisenberg spin model, and (2.2) is the natural gauge-invariant generalization of that model.

The boundaries[7] of the $\beta-\beta_H$ plane can be easily recognized (see Fig. 1). At $\beta_H=0$, this is just the standard Wilson SU(2) model. According to standard lore and current numerical evidence,[1] it under goes no phase transitions. Thus, the Wilson loop,

$$W(C)=\left\langle \tfrac{1}{2}\mathrm{Tr}\prod_{l\in C}U_l\right\rangle , \quad (2.4)$$

shows confining (area law) behavior for all values of β. For $\beta=0$, we have a trivial model, with free energy per site

$$F=\frac{1}{L^4}\ln Z=\ln\left[\frac{1-e^{-\beta_H}}{\beta_H}\right]+\text{const.} \quad (2.5)$$

We therefore expect no phase boundaries along this axis, and the $\beta=0$ Wilson-loop area law persists for all β_H. As $\beta_H\to\infty$, we choose the unitary gauge $[\vec{\phi}(n)=(0,0,1)]$ giving $S_H=\mathscr{O}_{33}$, and we parametrize link variable by $U_l=k_0+i\vec{k}\cdot\vec{\sigma}$, with $k_0^2+|\vec{k}|^2=1$, making each link diagonal,

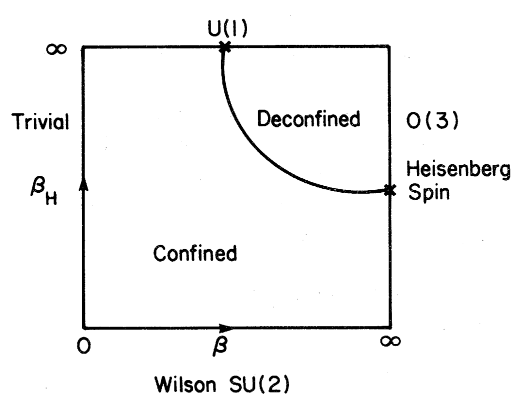

FIG. 1. Theoretical expectation for the $\beta-\beta_H$ phase plane.

$$U_l=k_0+ik_3\sigma 3=\begin{bmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta}\end{bmatrix} .$$

In this limit we then obtain a U(1) theory with $\beta=1/e_0{}^2$,

$$Z(\beta)=\int \{d\theta_l\}\exp\left[\beta\sum_P \cos\theta_P\right] . \quad (2.6)$$

It has been shown rigorously that the area law which occurs at small β fails at weak coupling, where the system is in a Coulomb, massless phase[8]. This phase transition has been observed in Monte Carlo simulations to be second order and to occur at $\beta\approx 1.0$.[9] Finally, at $\beta=\infty$, up to a gauge transformation the adjoint matrix becomes diagonal,

$$\mathscr{O}_{\alpha\beta}=\tfrac{1}{2}\mathrm{Tr}(U_l\tau_\alpha U_l^\dagger\tau_\beta)\to\delta_{\alpha\beta}\ .$$

The model then reduces to the O(3) Heisenberg spin model. This model has a well-studied second-order transition from a massive phase for small β_H to a Goldstone, massless phase at large β_H. Rigorous bounds on this transition exist. An upper bound given by Frölich, Simon, and Spencer[10] states that

$$\beta_H^* \le N\int \frac{d^dk}{(2\pi)^d}\frac{1}{4\sum_\mu \sin^2(k^\mu/2)} \quad (2.7)$$

for an O(N) spin model in d dimensions. The lower bound is given by mean-field theory[11]

$$\beta_H^* \ge \frac{N}{2d}\ . \quad (2.8)$$

For our case, these bounds translate into

$$0.75\le\beta_H^*\le 0.93\ . \quad (2.9)$$

We can extend our analysis away from the boundary through perturbation theory. First, we will consider the effects on the U(1) model of unfreezing the non-Abelian "charged" vector modes for small $1/\beta_H$. Consider the unitary gauge and parametrize the link variable by

$$U=\begin{bmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta}\end{bmatrix}(1-A_1{}^2-A_2{}^2)^{1/2} + i(A_1\sigma_1+A_2\sigma_2)\ , \quad (2.10)$$

where $A_1\pm iA_2$ are charged fluctuations. Expanding S_W and S_H, Eqs. (2.1) and (2.2), to quadratic order in $A_{1,2}$, we have

$$Z=\int\{d\theta_l\}\{d^2A_{i,l}\}\exp\left[-\beta_H\sum_l(A_1{}^2+A_2{}^2)_l\right]\exp\left[\beta\sum_P\cos\theta_P\left[1-\tfrac{1}{2}\sum_{l\in P}(A_1{}^2+A_2{}^2)_l\right]\right] \tag{2.11}$$

Performing the Gaussian integrations for $\beta_H >> \beta$ gives

$$Z=\int\{d\theta_l\}\exp\left[\beta_{\rm eff}\sum_P\cos\theta_P\right],\quad \beta_{\rm eff}=\beta\left[1-\frac{2}{\beta_H}\right]. \tag{2.12}$$

This result tells us that to first order in $1/\beta_H$, the non-Abelian theory gives an effective U(1) theory with coupling $\beta_{\rm eff}$. Thus, the phase transition moves off the U(1) line towards the O(3) line, We will verify this shortly in our Monte Carlo calculation.

Similarly, we can expand for small β_H around the pure SU(2) theory, obtaining the effective action

$$S_{\rm eff}=\frac{\beta}{2}{\rm Tr}U_P+\left[\frac{\beta_H}{6}\right]^4{\rm Tr}_A\,U_P+{\rm const}\times\beta_H^2\ , \tag{2.13}$$

where ${\rm Tr}_A(U)={\rm Tr}^2(U)-1$ is the trace in the adjoint representation. Thus, the first nontrivial correction is the addition of an adjoint [i.e., SO(3)] gauge term, identical to the one studied by Bhanot and Creutz.[2] They find a first-order line extending from $\beta_A=\infty$ that ends at $(\beta,\beta_A)=(1.6,0.9)$, where to lowest order $\beta_A=3(\beta_H/6)^4$ in our effective action. (Their modified action is $\frac{1}{2}\beta{\rm Tr}U_P+\frac{1}{3}\beta_A{\rm Tr}_A U_P$.) The ending of the first-order line implies at the least that the introduction of β_H does not cause a transition very close to the Wilson $\beta_H=0$ line. We shall discuss this further after analyzing the Monte Carlo results.

A similar perturbation expansion can be applied to the $\beta=\infty$ Heisenberg theory yielding

$$S_H(U,\phi)=\sum_{n,\mu}\left[1-\frac{\vec\theta_\mu{}^2(n)}{2}\right]\vec\phi(n)\cdot\vec\phi(n+\mu)-\vec\theta_\mu(n)\cdot[\vec\phi(n)\times\vec\phi(n+\mu)]+\tfrac{1}{2}[\vec\theta_\mu(n)\cdot\vec\phi(n)][\vec\theta_\mu(n)\cdot\vec\phi(n+\mu)]\ , \tag{2.14}$$

where U_l has been expanded as

$$U=e^{i\vec\theta_\mu\cdot\vec\tau/2}\approx1+\frac{i\vec\tau\cdot\vec\theta}{2}-\frac{\vec\theta_\mu{}^2}{2}\ .$$

We see that here the lowest-order brings in additional ϕ^4 interactions into the effective action. Thus, a finite $1/\beta$ does not just produce an effective renormalized O(3) model, so the behavior of the critical line is more difficult to predict.

From the above discussion, we are led to conjecture the phase structure depicted in Fig. 1. We have speculated that the U(1) transition at $\beta_H=\infty$ and the $\beta=\infty$ O(3) transition are connected to separate the Higgs phase from the confinement phase. We now turn to Monte Carlo simulations to test this expectation.

III. MONTE CARLO RESULTS

We have performed Monte Carlo calculations on periodic lattices of size 4^4 and 5^4 sites and measured the Wilson action per plaquette,

$$E_P=\frac{1}{6L^4}\frac{\partial}{\partial\beta}\ln Z=\langle\tfrac{1}{2}{\rm Tr}U_P\rangle \tag{3.1}$$

as well as the Higgs action per link

$$M=\frac{1}{4L^4}\frac{\partial}{\partial\beta_H}\ln Z=\langle\tfrac{1}{2}\phi^\alpha(n)\mathcal{O}_{\alpha\beta}\phi^\beta(n+\mu)\rangle\ , \tag{3.2}$$

employing the unitary gauge $[\phi^\alpha(n)=\delta_{\alpha3}]$.

The results for the phase boundaries are shown in Fig. 2. As anticipated in the previous section, we see that there is a second-order phase transition line connecting the U(1) and the Heisenberg transition points. The location of the U(1) critical point agrees with its value previously determined by Monte Carlo studies,[9] while the location of the Heisenberg point, although not determined by our data as accurately, is consistent with the theoretical bounds.

One should note that we found no evidence for a separate line pointing towards the SU(2) crossover at $\beta=2.2$, as seen by Bhanot and Creutz[2] in their model. However, this is not inconsistent with our result that for small β_H these models coincide. In fact, if one computes the β_H which corresponds to the critical β_A at which their first-order line ends, one obtains $\beta_H=4.4$. Not only is this clearly outside the perturbative regime where the identifica-

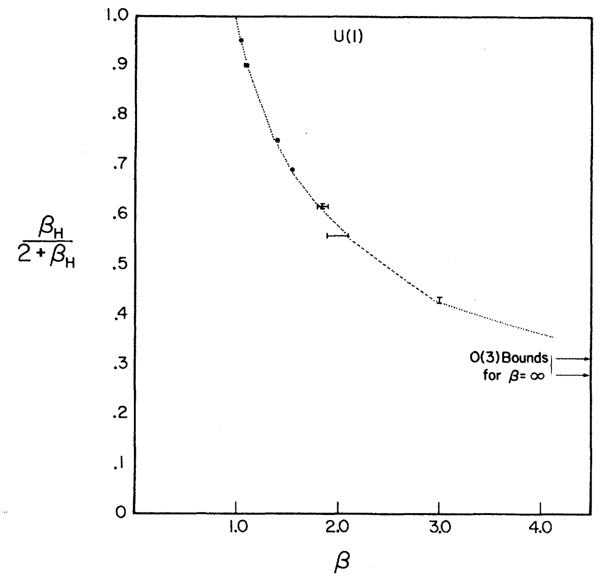

FIG. 2. Monte Carlo results for the phase plane.

tion of the two models is valid, but it actually lies in the vicinity of our second-order line. The lack of such a first-order line pointing to the crossover provides evidence for the conclusion that the U(1) behavior is irrelevant for confinement in SU(2). We will return to this point in the conclusion.

Finally, we check the validity of our first-order formula Eq. (2.12) for the transition near the U(1) line. Setting $\beta_{\rm eff}=0.98$ for a 5^4 lattice,[9] we obtain

$$\frac{1}{\beta_c}=\frac{1}{0.98}\left[1-\frac{2}{\beta_H}\right]. \tag{3.3}$$

To check this, it is convenient to plot our data on reciprocal axes, $1/\beta\equiv e_0{}^2$ and $1/\beta_H$ (Fig. 3). Indeed, we see the correct linear behavior for small $1/\beta_H$. Moreover, if we use the first-order formula,

$$E_P=\left[1-\frac{2}{\beta_H}\right]E_{U(1)}\left[\beta\left[1-\frac{2}{\beta_H}\right]\right], \tag{3.4}$$

where $E_{U(1)}$ is the U(1) energy per plaquette,[9] the agreement for $\beta_H=20$ is very good, while for $\beta_H=4.5$ the nonlinear corrections are clearly important (Fig. 4). Obviously, there are order-$1/\beta_H{}^2$ corrections visible even for $1/\beta_H=0.05$, so the range of validity of the first-order approximation is limited. On the other hand, it is remarkable to note the smooth behavior of the entire transition viewed in reciprocal couplings.

We thus see that our theoretical expectations are fulfilled by the Monte Carlo data. We now examine the relevance of this work to the broader questions of confinement and the continuum limit.

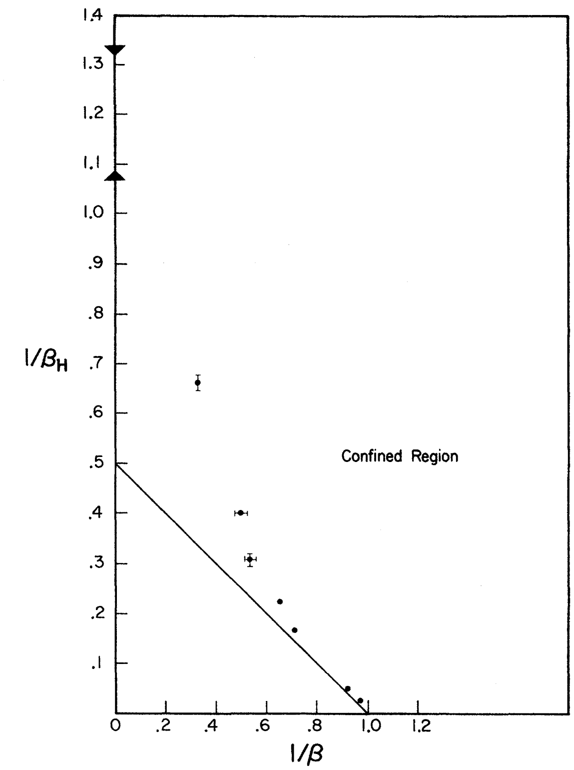

FIG. 3. Comparison of Monte Carlo results for a critical line with first-order perturbation theory in $1/\beta_H$. (The rigorous bounds are indicated on the $1/\beta=0$ axis.)

IV. DISCUSSION

As a generalization of the pure SU(2) theory, it might be hoped that an analysis of our model can

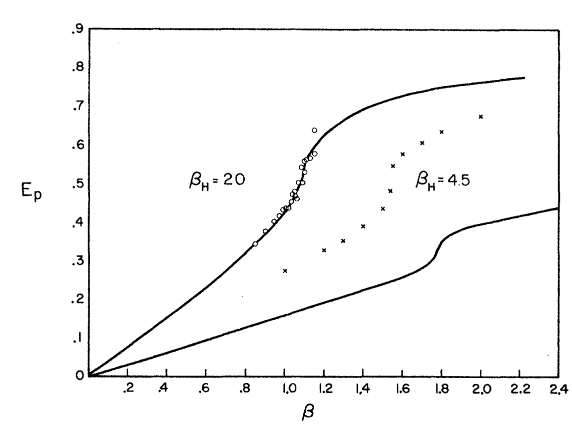

FIG. 4. Comparison of Monte Carlo data for E_p with the linear perturbation from the U(1) theory [Eq. (3.4)].

lead to more insight into the dynamics of confinement. In fact, 't Hooft[5] has suggested that confinement might be understood as a "condensation" of monopoles with U(1) charge. If this were to be the case, we would expect to see some evidence that the SU(2) crossover was connected to the U(1) dynamics. Our work, however, does not seem to support this idea. There is no hint of any relationship of the monopole-induced second-order phase transition and the SU(2) crossover.

A possible description for the behavior of the second-order phase transition is as follows. As β_H decreases from ∞, the effective mass of the monopole decreases due to the availability of non-Abelian paths in group space. The monopoles can then condense at a larger value of β, and the phase transition disappears. Thus, the monpoles exhibit trivial dynamics for small β_H, and so have no bearing on the SU(2) crossover and the problem of confinement.

All this is in contradistinction to the recent results[4] concerning the relation of Z_2 degrees of freedom to the crossover. There, the densities of the Z_2 monopoles were shown to undergo dramatic changes at the crossover. Furthermore, directly manipulating the Z_2 dynamics via a chemical potential for Z_2 monpoles changes the crossover to a first-order transition. Finally, even the Bhanot-Creutz model,[2] which superficially seems related to our Higgs model as shown in Sec. II, can be understood through its underlying Z_2 dynamics.

Apart from the question of confinement, our model possesses interesting physics in its own right. Specifically, it allows a rigorous definition of a fixed-length Higgs-field version of the continuum Georgi-Glashow model. To make contact with the continuum theory, it is necessary to examine the behavior of the lattice model under renormalization-group transformations. Under a chage of scale (such as block spinning), the fixing of a physical mass defines a flow in coupling-constant space. A second-order transition point, or critical point, is an unstable fixed point under such a transformation. That is, the flow on opposite sides of the point is towards two different limiting theories. The continuum theory is defined by shrinking the lattice spacing to zero. We can do this by going backwards along the flow lines, ending up, of course, at a critical point. There, the correlation length measured in lattice spacings diverges or, keeping the correlation length (the physical mass) fixed, the lattice spacing goes to zero.

Our work suggests the phase flow diagram schematically sketched in Fig. 5 for the adjoint Higgs model. The second-order points fall into two classes. First, a line of critical points whose flow on either side is to $\beta=0$, $\beta_H=0$ and to $\beta=\infty$, $\beta_H=\infty$, respectively; and second, three isolated critical points each with different flow structures. The first of these isolated points, point A, on the U(1) line defines on one side regular QED and, on the other, a confined phase of QED with a condensate of magnetic monopoles. The second, point B, on the O(3) line, is the usual critical point of a Heisenberg spin system. The third, at $\beta=\infty$, $\beta_H=0$, is the critical point of pure SU(2). The fact that each of these three critical points possesses only a single flow line associated with it allows the theory infinitesimally off these flow lines to differ from the theory on one of these flow lines. For example, it is known from Elitzur's theorem[12] that $\langle\phi\rangle$ is identically zero everywhere in the interior of the phase plane due to gauge invariance. For the Heisenberg model ($\beta=\infty$), on the other hand, it is clear that $\langle\phi\rangle$ is nonzero in the ordered phase.

The line of critical points, which will correspond to different versions of the Georgi-Glashow model, remains to be examined. Approaching the line from below, one obtains a confined, SU(2)-symmetric, massive theory by holding the string tension fixed. Incidentally, this theory provides a viable alternative to conventional QCD, where the Higgs field describes a massive scalar gluon in addition to the usual quark and vector-gluon constituents. Approaching from above, keeping the mass

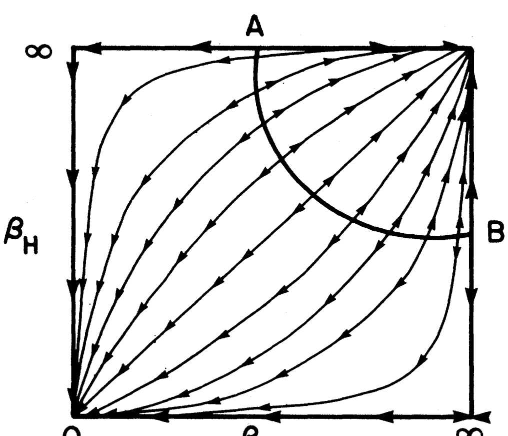

FIG. 5. Sketch of renormalization flow diagram in the $\beta-\beta_H$ plane.

of the vector meson fixed defines a broken SU(2) theory with a massless photon. Such a theory [generalized to SU(3)] was considered by DeRújula, Giles, and Jaffe[13] as a possible mechanism for quark liberation at long distances, but as pointed out by Georgi,[14] a second-order transition is necessary for such a scheme to be consistent with a linear potential at short distances. The resulting theory has a free parameter, denoting the location on the line. Our continuum theory is then defined by two parameters, the physical mass and a single coupling constant corresponding to the above free parameter. However, a naive continuum description of the standard Georgi-Glashow model contains three free parameters: the gauge coupling, the Higgs self-coupling, and the Higgs mass. It is not immediately clear, then, how to connect our theory with this one. There are two possibilities. It may be that a consistent, nonperturbative definition of the model does indeed contain only two parameters and cannot be studied in perturbation theory. The situation would then be similar to what is believed to occur in $\lambda\phi^4$ theory in four dimensions. There, a rigorous field theory can be defined by going to the Heisenberg transition corresponding to a nonlinear σ model, and it is believed that the one parameter (i.e., the mass scale) theory obtained is unique and is in fact free ($\lambda=0$). On the other hand, there may be an alternative lattice model which allows a more general continuum limit. Such a question is clearly worthy of further study.

During the final phase of this investigation we received a report describing related work by Lang, Rebbi, and Virasoro.[15] They performed Monte Carlo calculations on a finite-group analog of the adjoint Higgs model herein considered. The phase structure they obtained contains the transition line observed here, along with other transitions directly attributable to the finiteness of their group.

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