Derivation of the probability rule

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A derivation of the rule that states that $\psi^*(\bar{x})\psi(\bar{x})$ is the coordinate probability distribution at the space-time point $\bar{x}$ in quantum mechanics is presented. A coordinate-measuring experiment involving two light pulses that overlap at a prescribed space-time point is employed. A classical charged particle would reveal itself by absorbing energy from each pulse and reemitting a light wave whose energy and direction can be measured. In order to repeat this experiment a large number of times, the wave function must be separated into a number of identical copies. Since the (nonrelativistic) Schrödinger equation conserves $\int \psi^*(\bar{x})\psi(\bar{x})d^3x$, this separation is such that repeated measurements will give coordinates with a distribution proportional to $\psi^*(\bar{x})\psi(\bar{x})$.

I. INTRODUCTION

The mathematics of quantum mechanics is often connected to physical measurements through the postulate that $\psi^*(\bar{x})\psi(\bar{x})$ is the probability distribution for coordinate measurements where $\psi(\bar{x})$ is the wave function at the space-time point with coordinates that are components of the four-vector $\bar{x}$. This postulate is probably sufficient to determine, through the analysis of the associated thought experiments, the probabilities of measuring other quantities. For example, the momentum probability distribution is identified by this means in Sec. 8 of Ref. 1 (herein referred to as I) and in Ref. 2. We shall see here that it is not necessary to *postulate* that $\psi^*(\bar{x})\psi(\bar{x})$ is the coordinate probability distribution. This fact will be *derived* from the mathematics of quantum mechanics with the aid of a coordinate-measuring thought experiment.

The need for this derivation arose in the work of Everett[3] who proposed that this formula for the probability distribution arises from a Hilbert-space metric that weights the vectors in that space. DeWitt[4] and Hartle,[5] on the other hand, have gone further. They attempted to derive the probability formula from quantum mechanics without the introduction of a metric. They considered the sequence of measurements of a given quantity made on each of N identical systems in identical initial states. The overall supersystem including all the N systems, the measuring apparatus, and the observer has a final state (after the measurements) with the following state ket:

$$|\Psi\rangle = \sum_{s_1,s_2,\cdots,s_N} C_{s_1}C_{s_2}\cdots |s_1\rangle |s_2\rangle \cdots \times |\Phi(s_1\cdots s_N)\rangle . \quad (1.1)$$

In this equation, s and $|s\rangle$ are an eigenvalue and eigenket for one of the N systems belonging to the operator corresponding to the measurement. The measuring apparatus that is applied once to each of these systems reaches a state represented by $|\Phi(s_1\cdots s_N)\rangle$ when its sequence of measurement results was the numbers $s_1\cdots s_N$. These authors consider $\langle\Psi|\Psi\rangle$ and show that those components of $|\Psi\rangle$, whose measurement sequences $s_1\cdots s_N$ are consistent with the statement that they were selected with probabilities $C_s^*C_s$ make almost the entire contribution as N approaches infinity. Stated differently, those components of $|\Psi\rangle$ in Eq. (1.1) that are *not* consistent with a selection probability of $C_s^*C_s$ decrease as N^{-1} as the number of measured systems increases without limit.

There is no restriction on the coefficients C_s in Eq. (1.1) that requires that they vanish for a given set of sequences. Thus the measuring device state $|\Phi(s_1\cdots s_N)\rangle$ can include *all* possible sequences without regard to any preferred probability of selection given by $C_s^*C_s$. The statement that only a limited set contributes to the norm $\langle\Psi|\Psi\rangle$ is a statement about the *magnitudes* of the products $C_{s_1}C_{s_2}\cdots$. There is no proof given, however, that the magnitude of any component in Eq. (1.1) has any significance. Although it is usually postulated in quantum mechanics that the probability of occurrence is proportional to the square of the magnitude of the coefficients, we are not allowed to use that postulate here because we are attempting to derive it.

We need, instead, to show that sequences inconsistent with a probability given by C^*C simply do not occur in Eq. (1.1). Then all observers (or measuring devices) will conclude that C^*C does,

indeed, give the proper probability. Each component corresponds to the observer's memory recording a given sequence of measurements, but his memory does not record the coefficient in an expansion like Eq. (1.1). This will require us to find some restriction on $|\Psi\rangle$ itself. This restriction must arise from the requirement that a *proper* sequence of measurements has been made. This is what we shall do in the following pages.

In order to make use of the properties of the Schrödinger equation to restrict the wave function corresponding to the state ket in Eq. (1.1), we shall make repeated identical measurements on one system rather than one measurement on each member of an ensemble of systems. The measurement that we shall choose to make will be for coordinates, and we shall use the apparatus described in I. This apparatus is shown in Fig. 1. We shall use a number of copies of such devices in order to obtain repeated measurements.

More specifically, we will compare two measurements that give a "yes/no" answer to the question "Is a quantum particle near one of two points?" We will count only those measurements that give a yes answer at one of the two points and will observe that the ratio of the number of yes answers at the first point to the number of yes answers at the second approaches the ratio of $\psi^*(\bar{x})\psi(\bar{x})$ at the two points.

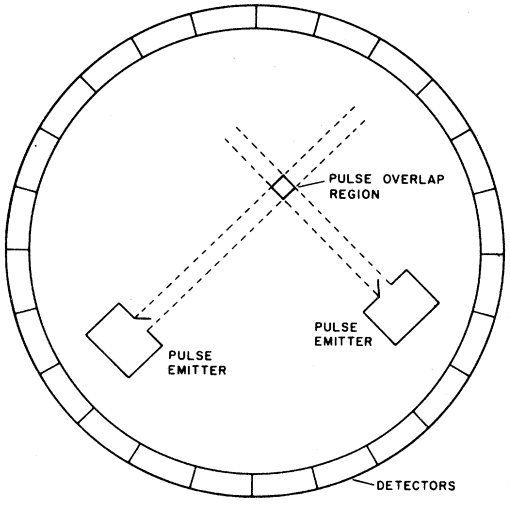

FIG. 1. Space-time event coordinate-measuring device. Two light pulses are emitted so that they overlap at the point in space and at the time where a charged particle is to be detected. If the particle absorbs a photon from each pulse, it can emit a photon that can activate one of the detectors surrounding the overlap region.

It will be necessary for us to include the observer in the overall wave function. This will be done in the usual manner of measurement theory as described by Wigner and London and Bauer and originally credited to von Neumann.[6]

We shall see that the requirement that the system wave function be normalized properly before each of the repeated measurements imposes a condition that leads to a final wave function with the proper characteristics. Each state of the observer in this final wave function will be identified with a sequence of measured values that would result if the probability of the result of a given measurement were proportional to $\psi^*(\bar{x})\psi(\bar{x})$.

II. A COORDINATE-MEASURING EXPERIMENT

The apparatus for coordinate measurements discussed in I consisted of two light sources capable of emitting two sharply defined pulses timed to coincide at a prescribed point in space at a given time (see Fig. 1). If we were dealing with a classical system and a classical charged particle were present in this region, it might absorb energy from each pulse and reemit a third electromagnetic wave. This third wave could then propagate to a battery of detectors that could determine its energy and direction. If this detected light wave had the proper direction to originate in the pulse overlap region, and if its energy were a proper value to correspond to the absorption from the two pulses, then the charged particle must have been present in the pulse overlap region. We would then assign the space-time coordinates of a point in the overlap region to the classical charged particle.

According to quantum mechanics, however, we do not measure the coordinates of a classical particle. Instead, the initial wave function for a charged particle ψ_i is split into the part in the overlap region ψ_x and the part outside this region ψ_0. This photon pulses produce a transition of ψ_x into a highly excited state extending roughly over the pulse overlap region and lasting approximately the time of overlap.[1] This excited state emits a photon wave that propagates toward the detectors where it is absorbed.

If the initial wave function ψ_i consists entirely of the component ψ_x that lies in the space-time volume of the pulse overlap region, and if the detectors coupled with an observer are essentially 100% efficient, then the observer will recognize the detection and will go into a state such that his memory records the presence of the wave in the

pulse overlap region. Such a recording is interpreted by a human brain as the presence of the charged "particle" in the overlap region.

If, on the other hand, ψ_x vanishes and ψ_i consists only of the part outside the overlap region, a human brain would interpret the recorded memory state as one showing no "particle" in the overlap region.

In general, ψ_i will be a superposition of ψ_x and ψ_0. Each of the two terms must develop independently according to the Schrödinger equation to produce states of particle detection and particle nondetection, respectively. Thus the final state of the entire system including the observer will consist of a sum of two terms, one containing the particle state into which ψ_x evolves together with an observer wave function corresponding to particle detection, the other containing the state into which ψ_0 evolves together with the nondetection state of the apparatus and observer.

We can describe this in terms of wave functions as follows.[6] Initially, just before the first transition, the wave function for the overall system of charged particle + apparatus + observer is in the form

$$\Psi_i=\psi_i^{(1)}(\bar{x})\phi_i(y)\chi_i(z)$$
$$=\psi_x^{(1)}(\bar{x})\phi_i(y)\chi_i(z)+\psi_0^{(1)}(\bar{x})\phi_i(y)\chi_i(z)\ , \qquad (2.1)$$

where $\phi_i(y)$ describes the apparatus and $\chi_i(z)$ the observer's brain or a mechanical device. This initial wave function is normalized to unity. After the measurement, the state is

$$\Psi_f^{(1)}=\psi_x^{(1)'}(\bar{x})\phi_x(y)\chi_x(z)+\psi_0^{(1)}(\bar{x})\phi_0(y)\chi_0(z)\ , \qquad (2.2)$$

where $\psi_x^{(1)'}$ is the final state of the charged-particle wave function produced by the decay of the excited state with the emission of the final photon. In addition, $\phi_x(y)$ is the state of the apparatus after the photon wave has been emitted and the detectors have operated. On the other hand, $\phi_0(y)$ is the state of the apparatus when no photon detection has been made. Finally, χ_x is the observer's wave function when the detection has been recognized, and χ_0 is his function when no detection has been observed.

We assume that a generator is available to produce the same charged-particle function ψ_i as often as we like. For example, we might consider ψ_i to be the wave function of a pion generated in an accelerator by the collision of a proton pulse with a target. If the pion packet ψ_i is required to first pass through a wire loop (See Fig. 2), we can correlate the subsequent coordinate measurements with a current pulse induced in the wire by the passage of a single pion. Then by repeatedly reproducing all the physical conditions associated with the generation of ψ_i, we can make as many measurements as we like to determine the probability distribution produced by it.

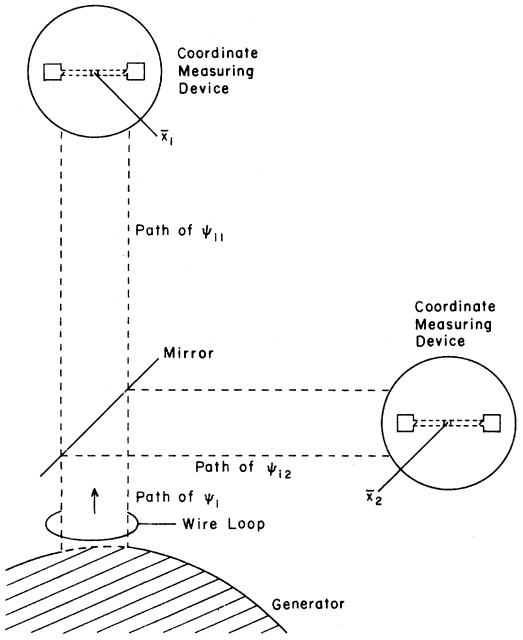

FIG. 2. Apparatus to determine if coordinates relative to the center are $\bar{x}_1$ or $\bar{x}_2$. The charged-particle packet from the generator ψ_i is split into two equal parts ψ_{i1} and ψ_{i2} by a half-silvered mirror. Each part passes into a coordinate-measuring device. One device has its pulse-overlap region set at the coordinates $\bar{x}_1$ relative to the center of ψ_{i1} while the other is set for $\bar{x}_2$ relative to ψ_{i2}. The wire loop detects the passage of the charged "particle" for correlation with the coordinate measurements.

If we repeat the coordinate measurement once, we bring in the wave function of a second charged particle $\psi_i^{(2)}$. Of course, $\psi_i^{(2)}$ is identical to $\psi_i^{(1)}$ except that it is generated at a later time. The initial wave function for this second measurement will then be

$$\Psi_i^{(2)}=\psi_i^{(2)}(\bar{x}_2)\Psi_f^{(1)}\ . \qquad (2.3)$$

The second measurement splits each term in Eq. (2.2) into two terms again so that

$$\Psi_f^{(2)}=\psi_x^{(2)'}\psi_x^{(1)'}\phi_{xx}\chi_{xx}+\psi_0^{(2)'}\psi_x^{(1)'}\phi_{0x}\chi_{0x}$$
$$+\psi_x^{(2)'}\psi_0^{(1)'}\phi_{x0}\chi_{x0}+\psi_0^{(2)'}\psi_0^{(1)'}\phi_{00}\chi_{00}\ . \qquad (2.4)$$

The first term on the right represents the component involving two final photon waves and two positive measurement results recorded in the observer's brain. The second term represents a positive reading for the first measurement but a negative one for the second.

We may ask if the second measurement leading to $\Psi_f^{(2)}$ is properly done. That is, does it represent a measurement with the same initial conditions as the first one to the extent that further repetition would give a probability distribution to be compared with $\psi^*(\bar{x})\psi(\bar{x})$? We immediately note that each branch of $\Psi_f^{(1)}$ in Eq. (2.2) serves as an initial state for the second measurement. Each branch should, therefore, be normalized to the same value. However the first measurement produces branches of different magnitudes. This "weights" the second measurement results unequally so that the four terms in $\Psi_f^{(2)}$ in Eq. (2.4) do not have the proper magnitudes to represent subsequent "identical" measurements. Each term in Eq. (2.2) needs to be normalized, but this would prevent us from using the Schrödinger equation to determine the properties of the final wave function. We shall correct this defect in the next section.

III. AN EQUAL-PROBABILITY MEASURING DEVICE

Let us alter the experimental apparatus so that we make a number of *identical* measurements for comparison. In order to determine the relative probabilities for measuring different coordinates, we shall determine the ratio of the probability for obtaining $\bar{x}_1$ to that for $\bar{x}_2$. For this purpose, we shall arrange to have a "partially silvered mirror" placed in the pathway of the charged-particle wave function as it moves from the generator to the measuring apparatus shown in Fig. 1. This mirror will serve to split the wave function into two equal parts traveling in different directions as shown in Fig. 2. A rectangular barrier serves as a mirror for the nonrelativistic Schrödinger equation. A measuring apparatus can then be arranged to determine if the coordinates are at $\bar{x}_1$ for one of the components ψ_{il} while another setup can determine if the coordinates are at $\bar{x}_2$ for the other component ψ_{i2}. By "equal parts" we mean that if we displace ψ_{il} so that its center coincides with that of ψ_{i2}, the two functions will have the same numerical value for their magnitudes at every point.

In order to be able to compare identical measurements, we shall arrange to have the coordinate measuring apparatus in Fig. 1 set to measure $\bar{x}_1$ in exactly the same state ϕ_{il} before each measurement. Similarly, the other apparatus will be in state ϕ_{i2}. This would require the pulse emitters to have the same orientations relative to the detectors, etc., and the shutters to be open to form pulses at the same times relative to ψ_i generation prior to each measurement. In addition, we will provide for all pulse overlap regions to have the same infinitesimal volume so that the charged-particle state generated in each region will have an amplitude proportional to the value of the initial wave function there.

It is convenient to introduce the spatial coordinates $\vec{x}^{\,c}$ for the center of each component of ψ_i at the time x^0. Then we can measure $\vec{x}$ relative to its center. Thus we write $\psi_{il}(\vec{x}^{\,c},\bar{x})$ and $\psi_{i2}(\vec{x}^{\,c},\bar{x})$ with a different value of $\vec{x}^{\,c}$ for each of the two components after they have been split into two parts. The coordinate measurement for one component is made for the relative coordinate $\vec{x}=\vec{x}_1$ and for the other with $\vec{x}=\vec{x}_2$.

After one measurement of both coordinates, we will obtain a wave function with three components, two like the first one on the right-hand side of Eq. (2.2) and the third one like the last term in Eq. (2.2). Thus the final-state wave function looks like

$$\Psi_f=\psi_{x1}^{(1)\prime}(\bar{x})\phi_{x1}(y)\chi_{x1}(z)+\psi_{x2}^{(1)\prime}(\bar{x})\phi_{x2}(y)\chi_{x2}(z)+\psi_0^{(1)}(\bar{x})\phi_0(y)\chi_0(z)\,. \tag{3.1}$$

The two coordinate-measuring devices have split the charged-particle wave function into three parts, $\psi_{x1}^{(1)\prime}$, $\psi_{x2}^{(1)\prime}$, and $\psi_0^{(1)}$ where the first component is left from the photon emission from the pulse overlap region at relative coordinate $\bar{x}_1$, the second from $\bar{x}_2$, and the third from the parts of $\psi_i^{(1)}$ outside the two pulse overlap regions. Each component is correlated with the corresponding states of the apparatus and of the observer.

We must repeat the measurement a large number of times and ask if the observer's state corresponds to a frequency proportional to $\psi^*(\bar{x})\psi(\bar{x})$. Since we will actually determine only the ratio of the probability of a coordinate measurement at $\bar{x}_1$ to that at $\bar{x}_2$, we shall only be interested in those sequences where all measurements give either $\bar{x}_1$ or $\bar{x}_2$. We shall, therefore, not be concerned with observer states that include measurements outside the pulse overlap regions centered on $\bar{x}_1$ or $\bar{x}_2$. The third term in Eq. (3.1) will lead to observer states of this latter type and will, therefore, not be of in-

terest to us.

The first two terms in Eq. (3.1) will be proportional to ψ_i at $\bar{x}_1$ and $\bar{x}_2$, respectively. In order to make these values of ψ_i the same, we shall redesign the mirror already there and place additional partially silvered mirrors in the path of the charged-particle wave function arranged to reflect fractions of ψ_i at right angles as shown in Fig. 3. These mirrors will further split ψ_{i1} into n_1 equal parts and ψ_{i2} into n_2 equal parts ψ_i', respectively, in such a way that

$$|\psi_{i1}'(\vec{x}_1^c,\bar{x}_1)| = |\psi_{i2}'(\vec{x}_2^c,\bar{x}_2)| \ . \tag{3.2}$$

Here, $\vec{x}_1^c$ is the center of any one of the n_1 components of ψ_{i1}, and $\vec{x}_2^c$ is the center of any one of the n_2 components of ψ_{i2}. We are splitting the original component that was to contribute the measurement at $\bar{x}_1$, ψ_{i1}, into n_1 equal parts and the component with measurements at $\bar{x}_2$, ψ_{i2}, into n_2 equal parts with n_1 and n_2 determined so as to satisfy the last equation. Of course, this cannot be done exactly with discrete n_1 and n_2, but it can be done within a prescribed accuracy. The above equation states that the magnitudes at relative coordinate $\bar{x}_1$ of the components derived from ψ_{i1} will have the same value as any one of the components derived from ψ_{i2} at the point $\bar{x}_2$.

We now arrange n_1 measuring devices to make the measurement for the n_1 components of ψ_{i1} at $\bar{x}_1$ and another n_2 devices to make the n_2 measurements for the components of ψ_{i2} at $\bar{x}_2$. The last equation guarantees that the initial wave functions arising from pulse-overlap regions centered at $\bar{x}_1$ or $\bar{x}_2$ all have the same magnitudes. Each of these wave functions can then be used as initial wave functions to repeat the measurement. As we have already noted, components of the total wave functions that lie outside the pulse-overlap regions contribute to observer states corresponding to measurements at points other than $\bar{x}_1$ and $\bar{x}_2$. They will not, therefore, be considered in determining the relative probabilities at $\bar{x}_1$ and $\bar{x}_2$.

The observations for all of these different coordinate measurements will be recorded in the observer's memory. This means that the state of the whole system (including the observer) will be a superposition of n_1+n_2+1 terms with the observer's states correlated with those of the charged particle and the apparatus. (The 1 counts the term resulting from $\psi_i(\bar{x})$ outside the pulse-overlap regions.) Since we have adjusted the wave functions in *all* the pulse-overlap regions to have the same values, all of the n_1+n_2 components of Ψ_f like the first two in Eq. (3.1) have the same amplitudes.

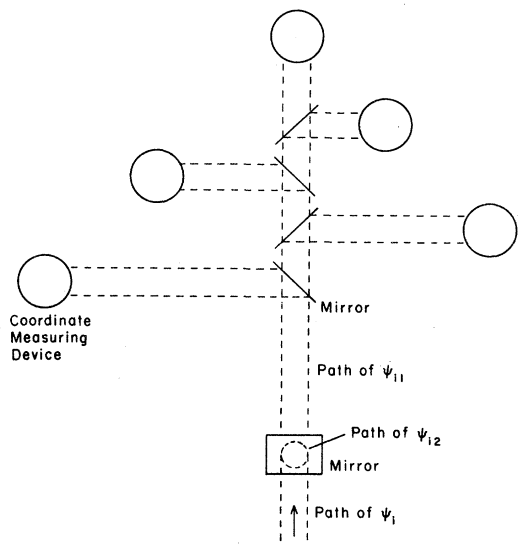

FIG. 3. Apparatus designed to give equal magnitude to subsequent measurements of coordinates. The apparatus of Fig. 1 is altered by adding partially silvered mirrors and additional coordinate-measuring devices. The silvering of all the mirrors is adjusted to satisfy Eq. (3.2). The coordinate-measuring devices attached to the path of ψ_{i1} have their overlap regions located at $\bar{x}_1$ relative to the packet center while those attached to the path of ψ_{i2} have their overlap regions located at $\bar{x}_2$. Only those devices attached to ψ_{i1} are shown. This figure is oriented at right angles around the path of ψ_i relative to Fig. 2, and ψ_{i2} is shown perpendicular to the plane of the paper.

IV. REPEATED MEASUREMENTS

We can repeat the measurements we have just described a large number of times N. Then, in place of the one in Eq. (3.1), we will have a final state for the entire system including charged particles, measuring apparatus, and observer of the form

$$\Psi_f = \cdots + \psi_{x12}^{(1)\prime}\psi_{x15}^{(2)\prime}\cdots\psi_{x23}^{(N)\prime}\phi_{x12,15,\ldots,23}\chi_{x12,15,\ldots,23} + \cdots \ . \tag{4.1}$$

The first number in each pair in the subscripts is 1 or 2 and distinguishes coordinate measurements made at $\bar{x}_1$ from those made at $\bar{x}_2$. The second number in the subscript identifies the particular coordinate-measuring device (see Fig. 3).

The state function $\chi_{x\,12,15,\ldots,23}$ represents a state of the observer's memory that records a sequence of N measurements of the relative coordinates $\bar{x}_1, \bar{x}_1, \ldots, \bar{x}_2$. By dividing the components of ψ^i by using partially silvered mirrors, we have arranged for each term to have the same magnitude. These sequences, after N has become very large, will be those that would be predicted under Laplace's definition of probability where there are n_1 equal opportunities for obtaining $\bar{x}_1$ at each measurement and n_2 equal opportunities for $\bar{x}_2$. Thus the observer in each of his states would conclude that the probability $P(\bar{x}_1)$ of obtaining $\bar{x}_1$ at a given measurement is proportional to n_1 and for obtaining $\bar{x}_2$, $P(\bar{x}_2)$, is proportional to n_2. That is,

$$P(\bar{x}_2)/P(\bar{x}_1)=n_2/n_1 \,. \tag{4.2}$$

We may now investigate the relation of n_2/n_1 to ψ_i. According to Eq. (3.1), n_2/n_1 was chosen so that the parts of the various components of ψ_i lying in the pulse-overlap regions are of equal numerical magnitudes. As we have already noted, since n_1 and n_2 are integers, the equality in Eq. (3.1) can only be approximately true. To decrease the error, we must increase n_1 and n_2. We can always increase them enough to reduce the difference of the two sides of Eq. (3.1) to a value less than a prescribed number. In order to accomplish this, we must increase the number of branches shown in Fig. 3. These branches are designed to give equal path lengths, from the bottom mirror shown, to the center of each coordinate-measuring device. This is so that every component of ψ_i will have spread by the same amount as every other component so that it will produce a wave function of size and shape identical to every other one relative to the center of the device at the time a measurement is made. Increasing the number of branches will require more volume for the apparatus and, therefore, paths of longer length. This will, of course, increase the spreading of the wave packets before the measurements take place. It will not, however, alter the fact that they will have identical sizes when they reach their respective measurement devices. If the spreading is too great, each path will need to be surrounded by an absorbing tube to prevent interference between packets on different branches. Furthermore, the mirrors will need to be tilted so as to send each packet in a direction that will place the part containing the relative point at $\bar{x}_1$ (or $\bar{x}_2$) in the pulse overlap region of the coordinate-measuring device.

Since $\int \psi^*(\bar{x})\psi(\bar{x})d^3x$ is conserved by the Schrödinger equation, the splitting of ψ_i into n_1+n_2 parts by the mirrors will be such that

$$n_1 \int |\psi'_{i1}(\vec{x}^c_1,\bar{x})|^2 d^3x + n_2 \int |\psi'_{i2}(\vec{x}^c_2,\bar{x})|^2 d^3x = \int |\psi_i(\bar{x})|^2 d^3x \,, \tag{4.3}$$

where $\psi'_{i1}(\vec{x}^c_1,\bar{x})$ is any one of the components produced by the mirrors on which the measurement at relative coordinate $\bar{x}_1$ is made, $\psi'_{i2}(\vec{x}^c_2,\bar{x})$ is any one of the components on which the measurement at $\bar{x}_2$ is made, and ψ_i is the original charged-particle wave function. Since the mirrors produce components of ψ all with the same shape as ψ_i, Eqs. (3.2) and (4.3) show that

$$n_1^{-1}|\psi_i(\bar{x}_1)|^2 = n_2^{-1}|\psi_i(\bar{x}_2)|^2 \,, \qquad n_1+n_2\to\infty \,. \tag{4.4}$$

From this equation and (4.2),

$$P(\bar{x}_2)/P(\bar{x}_1) = |\psi_i(\bar{x}_2)|^2/|\psi_i(\bar{x}_1)|^2 \,, \qquad n_1+n_2\to\infty \,. \tag{4.5}$$

Thus we find that the observer, in each of his states, will conclude that the measurement sequence that he observes implies the same probability ratio for the two points as the postulate normally made for quantum mechanics. Since this analysis applies to *any* pair of points,

$$P(\bar{x}) \propto |\psi_i(\bar{x})|^2 \,. \tag{4.6}$$

V. DISCUSSION

We have seen that, in order to have the same amplitudes for identical repeated experiments, we must break up the wave function of the charged particle into components of the proper size. The fact that the quantity conserved by the Schrödinger equation in this breakup has the form $\int \psi^*(\bar{x})\psi(\bar{x})d^3x$ determines these components to such that the observer concludes that the coordinate probability distribution is proportional to

$\psi^*(\bar{x})\psi(\bar{x})$.

This author believes that the human brain is a purely physical object and, as such, is describable by a wave function. In this case, the "observer" may be a human being. However, for those who do not accept this point of view, the results of the measurements that we have described can be recorded by a mechanical device. This device then becomes the "observer" and the arguments we have seen are in no way altered.

If we consider the human brain to be purely physical and, therefore, describable by a wave function, a human observer's state after the measurement will be described by a wave function of the type shown in Eq. (4.1). Each term in the superposition will correspond to an observer's state such that his memory records a given sequence of coordinate measurements. The observer will interpret his condition in terms of a sequence of events at the two points whose (relative) coordinates were those of a sequence like $\bar{x}_1, \bar{x}_1, \ldots, \bar{x}_2$. To him, then, events occured at each of these points. Actually, however, his state is represented by the superposition of terms in Eq. (4.1), each term representing an observer state corresponding to his recognition of a different sequence of events. Thus a human being would describe a charged particle as an event whereas, in actuality, the only thing that exists is a total wave function of the system.

There are many physicists who reject the description of reality as represented by the wave function in Eq. (4.1) because of the lack of uniqueness of the state of a human observer for which it calls. They look for a process by which one of the terms in Eq. (4.1) is selected over all the others. The wave function "collapses" to one of its components. No satisfactory equation of motion has been proposed for this conversion of the superposition to one of its components. In particular, this author is unaware of any suggestion of what mechanism selects one of the components in preference to all of the others. Zurek[7] proposes that this collapse is "forced." Nothing in the derivation of Eq. (4.1) provides any such forcing.

This collapse is avoided entirely by the Everett proposal.[3,4]

VI. ACKNOWLEDGMENT

The author is indebted to Charles B. Thorn for helpful discussions.

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