PHYSICAL REVIEW D VOLUME 25, NUMBER 12 15 JUNE 1982

Further study of quantum superspaces

Ingemar Bengtsson
Institute of Theoretical Physics, 412 96 Göteborg, Sweden
(Received 27 May 1981)

The phase space of a supersymmetric point particle turns out to be constrained in certain cases, which lead to a new type of superspace. This report treats the cases where the Weyl condition is imposed on the spinors, as well as the case of a de Sitter supersymmetry.

I. INTRODUCTION

Supersymmetric field theories have attracted much attention in recent years. Of particular interest is the $N=4$ Yang-Mills theory in four dimensions of space-time, which is the maximal theory of its kind and can be constructed from a theory with simple supersymmetry in ten dimensions of space-time.[1] This theory has been shown to be finite up to the three-loop level[2] and offers the hope of being a finite quantum field theory. To be able to prove this, it would probably be advantageous to formulate the theory in a way which is supersymmetric also off the mass shell. So far, however, only a few supersymmetric field theories have been constructed off shell; in particular, it has been demonstrated that the $N=4$ Yang-Mills theory cannot be taken off shell in any simple way.[3] This problem motivated the investigation of the logical foundations of superspace carried out in Ref. 4.

II. QUANTUM SUPERSPACE

In Ref. 4 one considers a supersymmetrical point particle described by the trajectory $x^\alpha(\tau)$, $\theta^a(\tau)$ (where θ^a is a Majorana spinor). The action is

$$I=\int d\tau\{V^{-1}[(\dot{x}^\alpha-i\theta\gamma^\alpha\dot{\theta})^2+l\dot{\bar{\theta}}\dot{\theta}]-Vm^2\}, \tag{1}$$

where l is a dimensional constant (this action, without the l term, was first studied in Ref. 5). (The conventions are those of Ref. 4; in particular, we use a spacelike metric and $\{\gamma^\alpha,\gamma^\beta\}=-2\eta^{\alpha\beta}$.) When $l\neq 0$, the action is unconstrained and one can impose the canonical commutation relations

$$[p^\alpha,x^\beta]=-i\eta^{\alpha\beta}, \tag{2a}$$

$$\{p_\theta^a,\theta^b\}=-i(\gamma^0)^{ab}, \tag{2b}$$

all the other commutators being zero. When $l=0$, on the other hand, we have the second-class constraints

$$\phi^a\equiv p_\theta^a-i(\gamma\cdot p\theta)^a=0. \tag{3}$$

The terminology is that of Ref. 6. When one introduces the Dirac brackets compatible with the constraints and quantizes, one finds that the coordinates do not commute anymore.

Note that the Lorentz generator is

$$J^{\alpha\beta}=x^\alpha p^\beta-x^\beta p^\alpha+s^{\alpha\beta}=x^\alpha p^\beta-x^\beta p^\alpha-\tfrac{1}{2}\bar{\theta}\gamma^{\alpha\beta}p_\theta, \tag{4}$$

so that the constraints imply

$$p^\alpha s_{\alpha\beta}=0. \tag{5}$$

These constraints are encountered in the study of spinning particles. However, they are not all second class. In Ref. 7 some gauge conditions are added which result in the same Dirac brackets for the quantities x^α, p^α, and $s^{\alpha\beta}$ as in the present case. The present case is different, however, since all the second-class constraints are derivable from the Lagrangian and one does not have to impose any gauge conditions from the outside.

Noncommuting coordinates are clearly unsuitable for quantum mechanics. In the study of spinning particles one is led to the following gauge-invariant expressions for the position[8]:

$$q_\eta^\alpha(\tau)=\frac{1}{\eta\cdot p}(J^{\alpha\beta}\eta_\beta+\tau p^\alpha). \tag{6}$$

They are solutions to the gauge condition $\eta_\alpha x^\alpha-\tau=0$ imposed to solve for the first-class constraint $p^2-m^2=0$. They commute with each other when η is lightlike. This turns out to be true in the present case also; after introducing new spinorial coordinates as well one finds indeed that the noncovariant coordinates

$$q^\alpha=x^\alpha+\frac{ip_\beta}{2p^+}\bar{\theta}\gamma^{\alpha\beta+}\theta, \tag{7a}$$

$$S=\frac{1}{\sqrt{p^+}}(\gamma_-\gamma\cdot p\theta)^a, \tag{7b}$$

$$T^a=i\left(\frac{-p^2}{p^+}\right)^{1/2}(\gamma_-\theta)^a \tag{7c}$$

obey a remarkably simple algebra where the only nonzero brackets are

$$[p^\alpha,q^\beta]=-i\eta^{\alpha\beta}, \tag{8a}$$

$$\{S^a,S^b\}=\{T^a,T^b\}=-(\gamma_-\gamma^0)^{ab}. \tag{8b}$$

The space spanned by these coordinates is called "Clifford algebra superspace" or "quantum superspace." The suggestion in Ref. 4 is that certain supersymmetric field theories can be usefully written in terms of superfields which are functions of these coordinates.

A note may be added concerning the massless limit. When $m=0$, the constraint matrix

$$\{\phi^a,\phi^b\}=-2(\gamma\cdot p\gamma^0)^{ab} \tag{9}$$

used to get the Dirac brackets becomes singular (because of reparametrization invariance we have the first-class constraint $p^2-m^2=0$), so one has to add some further gauge conditions reducing the number of degrees of freedom. The final quantum superspace coordinates do not suffer from this defect, however, and it is assumed that this point does not cause any trouble. Note that the T^a's disappear when $p^2=0$.

In this report we will study the quantum superspace algebra in some additional cases, namely, when one imposes the Weyl condition on the spinors and in the case of a de Sitter symmetry.

III. THE HIGHER-DIMENSIONAL CASES

The formulas of Ref. 4 are valid whenever we have Majorana spinors (as in four-dimensional Yang-Mills theory). In the six-dimensional case, we cannot impose the Majorana condition since the charge-conjugation matrix C is symmetric. Instead, one imposes the Weyl condition

$$(1+\Gamma^7)_x{}^y\theta_y=0\,. \tag{10}$$

Using $\bar\theta^a\equiv(\theta^\dagger\Gamma^0)^a$ we have

$$\theta_x=-(\Gamma^7)_x{}^y\theta_y\,, \tag{11a}$$

$$\bar\theta^a=\bar\theta^b(\Gamma^7)_b{}^a\,. \tag{11b}$$

The index of $\bar\theta^a$ is said to be a Weyl index (denoted by $a,\ b,\ c,\ldots$ in the following), and the index of θ_x is said to be anti-Weyl (denoted by $x,\ y,\ z,\ldots$). Note that the indices of the charge-conjugation matrix C^{xa} have different Weyl properties, so that raising and lowering indices change their Weyl properties. Note also that contraction over indices with different Weyl properties yields zero. The term $l\dot{\bar\theta}\dot\theta$ does not survive. It is, however, still possible to give an unconstrained Lagrangian. In fact, it is

$$L=\tfrac{1}{2}V^{-1}\Big(\dot x^\alpha-\frac{i}{2}\dot{\bar\theta}\Gamma^\alpha\theta+\frac{i}{2}\bar\theta\Gamma^\alpha\dot\theta+l\dot{\bar\theta}\Gamma^\alpha\dot\theta\Big)^2+Vm^2\,, \tag{12}$$

where the superspace coordinates are $Z_A=(x^\alpha,\theta_x,\bar\theta^a)$ and the supersymmetry transformations, given by

$$\delta Z^A=i[\bar\epsilon Q+\bar Q\epsilon,Z^A]\,, \tag{13}$$

are

$$\delta x^\alpha=\frac{i}{2}\bar\theta\Gamma^\alpha\epsilon-\frac{i}{2}\bar\epsilon\Gamma^\alpha\theta\,, \tag{14a}$$

$$\delta\theta_x=\epsilon_x\,, \tag{14b}$$

$$\delta\bar\theta^a=\bar\epsilon^a\,. \tag{14c}$$

We can now impose the naive commutation relations, the nonzero ones of which are

$$[p^\alpha,x^\beta]=-i\eta^{\alpha\beta}\,, \tag{15a}$$

$$\{p_{\bar\theta}^x,\bar\theta^a\}=-iC^{xa}\,, \tag{15b}$$

$$\{p_\theta^x,\theta^a\}=-iC^{xa}\,. \tag{15c}$$

When $l=0$ the Lagrangian is constrained; we have

$$p_\theta^x=-\frac{i}{2}(\bar\theta\Gamma\cdot p)^x\,, \tag{16a}$$

$$p_{\bar\theta}^x=-\frac{i}{2}(\Gamma\cdot p\theta)^x\,. \tag{16b}$$

The Dirac brackets become

$$[x^\alpha,x^\beta]=\frac{1}{4p^2}\bar\theta\Gamma^{\alpha\beta}\Gamma\cdot p\theta+\frac{1}{4p^2}\bar\theta\Gamma\cdot p\Gamma^{\alpha\beta}\theta\,, \tag{17a}$$

$$[x^\alpha,\theta_x]=\frac{i}{2p^2}(\Gamma\cdot p\Gamma^\alpha\theta)_x\,, \tag{17b}$$

$$[x^\alpha,\bar\theta^a]=\frac{i}{2p^2}(\bar\theta\Gamma^\alpha\Gamma\cdot p)^a\,, \tag{17c}$$

$$\{\bar\theta^a,\theta_x\}=\frac{1}{p^2}(\Gamma\cdot p)^a{}_x\,, \tag{17d}$$

$$\{\theta_x,\theta_y\}=\{\bar\theta^a,\bar\theta^b\}=[p^\alpha,\theta_x]=[p^\alpha,\bar\theta^a]=0\,, \tag{17e}$$

$$[p^\alpha,x^\beta]=-i\eta^{\alpha\beta}\,. \tag{17f}$$

In the same way as in the Majorana case, we can introduce noncovariant coordinates having simple commutation relations as follows:

$$q^\alpha=x^\alpha+\frac{i}{4p^+}\bar\theta\Gamma^{\alpha+}\Gamma\cdot p\theta+\frac{i}{4p^+}\bar\theta\Gamma\cdot p\Gamma^{\alpha+}\theta\,, \tag{18a}$$

$$S_x=\frac{1}{(2p^+)^{1/2}}(\Gamma^+\Gamma\cdot p\theta)_x\,, \tag{18b}$$

$$\bar S^a=\frac{1}{(2p^+)^{1/2}}(\bar\theta\Gamma\cdot p\Gamma^+)^a\,, \tag{18c}$$

$$T_x=i\left(\frac{-p^2}{2p^+}\right)^{1/2}(\Gamma^+\theta)_x\,, \tag{18d}$$

$$\bar T^a=-i\left(\frac{-p^2}{2p^+}\right)^{1/2}(\bar\theta\Gamma^+)^a\,. \tag{18e}$$

These quantum superspace coordinates obey an algebra with the following nonzero brackets:

$$\{\bar S^a,S_x\}=\{\bar T^a,T_x\}=(\Gamma^+)^a{}_x\,, \tag{19a}$$

$$[p^\alpha,q^\beta]=-i\eta^{\alpha\beta}\,. \tag{19b}$$

The generators $Q_a,\bar Q^x$, and $J^{\alpha\beta}$ are

$$Q_a=p_{\theta a}-\frac{i}{2}(\Gamma\cdot p\theta)_a=-i(\Gamma\cdot p\theta)_a\,, \tag{20a}$$

$$\bar{Q}^x = -p^x_{\bar\theta} + \frac{i}{2}(\bar\theta\Gamma\cdot p)^x = i(\bar\theta\Gamma\cdot p)^x\,, \tag{20b}$$

$$\begin{aligned} J^{\alpha\beta} &= x^\alpha p^\beta - x^\beta p^\alpha - \tfrac{1}{2}\bar\theta\Gamma^{\alpha\beta}p_{\bar\theta} - \tfrac{1}{2}p_\theta\Gamma^{\alpha\beta}\theta \\ &= x^\alpha p^\beta - x^\beta p^\alpha + \frac{i}{4}\bar\theta\Gamma^{\alpha\beta}\Gamma\cdot p\theta + \frac{i}{4}\bar\theta\Gamma\cdot p\Gamma^{\alpha\beta}\theta\,. \end{aligned} \tag{21}$$

They become when expressed in the new coordinates

$$Q_a = \frac{1}{(2p^+)^{1/2}}[i\Gamma\cdot pS + (-p^2)^{1/2}T]\,, \tag{22a}$$

$$\bar{Q}^x = -\frac{1}{(2p^+)^{1/2}}[i\bar{S}\Gamma\cdot p - (-p^2)^{1/2}\bar{T}]\,, \tag{22b}$$

$$J^{\alpha\beta} = q^\alpha p^\beta - q^\beta p^\alpha + K^{\alpha\beta}\,, \tag{23a}$$

where

$$K^{+-} = K^{+i} = 0\,, \tag{23b}$$

$$K^{ij} = -\frac{i}{2}(\bar{S}\Gamma^{ij}\Gamma^- S + \bar{T}\Gamma^{ij}\Gamma^- T)\,, \tag{23c}$$

and

$$\begin{aligned} K^{i-} = &-\frac{ip^-}{4(p^+)^2}(\bar{S}\Gamma^{i+}\Gamma\cdot pS + \bar{T}\Gamma^{i+}\Gamma\cdot pT) + \frac{i}{8p^+}(\bar{S}\Gamma^{i-}\Gamma\cdot pS + \bar{T}\Gamma^{i-}\Gamma\cdot pT) \\ &+ \frac{p^-}{4(p^+)^2}(-p^2)^{1/2}(\bar{T}\Gamma^{i+}S - \bar{S}\Gamma^{i+}T) - \frac{1}{8p^+}(-p^2)^{1/2}(\bar{T}\Gamma^{i-}S - \bar{S}\Gamma^{i-}T) - (i \leftrightarrow -)\,. \end{aligned} \tag{23d}$$

In the ten-dimensional case, one imposes both the Majorana and the Weyl conditions on the spinors. In this case, the quantum superspace coordinates of Ref. 4 remain as they stand; a difficulty appears, however, in trying to give an unconstrained Lagrangian, since both $l\dot{\bar\theta}\dot\theta$ and $l\dot{\bar\theta}\Gamma^\alpha\dot\theta$ are zero. The only term of this kind available is $l\dot{\bar\theta}\Gamma^{\alpha\beta\gamma}\dot\theta$ and it cannot be used in the Lagrangian. In this case the quantum superspace seems to be the only available superspace. This might be the explanation why one cannot find the auxiliary fields needed for the off-shell representation of the ten-dimensional supersymmetric Yang-Mills theory.[3]

IV. THE de SITTER CASE

It is also of some interest to consider supersymmetric-field theories in a de Sitter space. A de Sitter space is defined as the four-dimensional manifold

$$y_A y^A = -(y^0)^2 + \vec{y}^2 - (y^4)^2 = -R^2 \tag{24}$$

imbedded in a five-dimensional flat space. R is a constant, the radius of curvature of the universe. The signature of the five-dimensional space is chosen as $(-,+,+,+,-)$, since this signature admits four-component Majorana spinors.

Now, when both m and l are set to zero, the action (1) possesses also a conformal symmetry. This case has been treated in Ref. 9. The conformal symmetry group in four space-time dimensions is SO(2, 4), and the corresponding graded group is SU(1; 2, 2). From this case it is easy to extract the subgroups which give us the de Sitter case. They are SO(2, 3), with the graded group Osp(1, 4).

For SU(1; 2, 2), the algebra is

$$[D, p^\alpha] = -ip^\alpha\,, \tag{25a}$$

$$[D, K^\alpha] = iK^\alpha\,, \tag{25b}$$

$$[J^{\alpha\beta}, p^\gamma] = i(\eta^{\alpha\gamma}p^\beta - \eta^{\beta\gamma}p^\alpha)\,, \tag{25c}$$

$$[J^{\alpha\beta}, K^\gamma] = i(\eta^{\alpha\gamma}K^\beta - \eta^{\beta\gamma}K^\alpha)\,, \tag{25d}$$

$$[J^{\alpha\beta}, J^{\gamma\delta}] = i(J^{\alpha\gamma}\eta^{\beta\delta} + J^{\beta\delta}\eta^{\alpha\gamma} - J^{\beta\gamma}\eta^{\alpha\delta} - J^{\alpha\delta}\eta^{\beta\gamma})\,, \tag{25e}$$

$$[p^\alpha, K^\beta] = 2i(\eta^{\alpha\beta}D + J^{\alpha\beta})\,, \tag{25f}$$

$$[D, Q^a] = -\frac{i}{2}Q^a\,, \tag{25g}$$

$$[D, Z^a] = \frac{i}{2}Z^a\,, \tag{25h}$$

$$[p^\alpha, Z^a] = -(\gamma^\alpha Q)^a\,, \tag{25i}$$

$$[Q^a, K^\alpha] = -(\gamma^\alpha Z)^a\,, \tag{25j}$$

$$\{Q^a, Q^b\} = -2(\gamma_\alpha\gamma^0)^{ab}p^\alpha\,, \tag{25k}$$

$$\{Z^a, Z^b\} = 2(\gamma_\alpha\gamma^0)^{ab}K^\alpha\,, \tag{25l}$$

$$\{Q^a, Z^b\} = i(\gamma_{\alpha\beta}\gamma^0)^{ab}J^{\alpha\beta} - 2i(\gamma^0)^{ab}D - 3(\gamma_5\gamma^0)^{ab}A\,, \tag{25m}$$

$$[Q^a, A] = (\gamma_5 Q)^a\,, \tag{25n}$$

$$[Z^a, A] = -(\gamma_5 Z)^a\,. \tag{25o}$$

The remaining brackets are zero.

From here, the Osp (1, 4) algebra in a de Sitter space with metric $(-,+,+,+,-)$ and radius of curvature R is easily obtained by defining

$$\pi^\alpha = \frac{1}{R}J^{\alpha 5} = p^\alpha - \frac{1}{4R^2}K^\alpha\,, \tag{26a}$$

$$G^a = Q^a + \frac{1}{2R}Z^a\,. \tag{26b}$$

We get the following subalgebra:

$$[J^{\alpha\beta}, \pi^\gamma] = i(\eta^{\alpha\gamma}\pi^\beta - \eta^{\beta\gamma}\pi^\alpha)\,, \tag{27a}$$

$$[\pi^\alpha, \pi^\beta] = -\frac{i}{R^2} J^{\alpha\beta}, \tag{27b}$$

$$\{G^a, G^b\} = -2(\gamma_\alpha\gamma^0)^{ab}\pi^\alpha + \frac{i}{R}(\gamma_{\alpha\beta}\gamma^0)^{ab}J^{\alpha\beta}, \tag{27c}$$

$$[\pi^\alpha, G^a] = -\frac{1}{2R}(\gamma^\alpha G)^a. \tag{27d}$$

This is the Osp (1, 4) algebra. Note that it contracts to the usual graded Poincaré algebra when $R \to \infty$.

Reference 9 contains the explicit expressions for the variations of the coordinates as defined by

$$\delta Z^A = i[a\cdot p + \bar\epsilon Q - \tfrac{1}{2}l_{\alpha\beta}J^{\alpha\beta} - \lambda D + \rho A - c\cdot K + \bar\xi\, ZZ^A]. \tag{28}$$

Putting $C^\alpha = (1/4R^2)a^\alpha$, $\xi^a = (1/2R)\epsilon^a$, and dropping D and A we obtain the appropriate expressions for the de Sitter case,

$$\delta Z^A = i[a\cdot\pi - \tfrac{1}{2}l_{\alpha\beta}J^{\alpha\beta} + \bar\epsilon G, Z^A], \tag{29}$$

$$\delta x^\alpha = a^\alpha + \frac{a^\alpha x^2}{4R^2} - x^\alpha\frac{a\cdot x}{2R^2} + \frac{1}{8R^2}a^\alpha\bar\theta\theta\bar\theta\theta + l^\alpha{}_\beta x^\beta + \frac{1}{2R}\bar\epsilon\gamma\cdot x\gamma^\alpha\theta + \frac{i}{2R}\bar\epsilon\gamma^\alpha\theta\bar\theta\theta + i\bar\epsilon\gamma^\alpha\theta, \tag{30a}$$

$$\begin{aligned}\delta\theta^a = {}& \frac{i}{4R^2}(\gamma\cdot x\gamma\cdot a\theta)^a + \frac{i}{4R^2}(\gamma\cdot a\theta\bar\theta\theta)^a - \tfrac{1}{4}l_{\alpha\beta}(\gamma^{\alpha\beta}\theta)^a + \epsilon^a + \frac{i}{2R}(\gamma\cdot x\epsilon)^a \\ & + \frac{i}{4R}(\gamma_5\gamma_\alpha\epsilon\bar\theta\gamma_5\gamma^\alpha\theta)^a - \frac{1}{2R}\epsilon^a\bar\theta\theta - \frac{1}{2R}(\gamma_5\epsilon\bar\theta\gamma_5\theta)^a.\end{aligned} \tag{30b}$$

We also have

$$\delta V = -\left(\frac{1}{R^2}a\cdot x + \frac{2}{R}\bar\epsilon\theta\right)V. \tag{30c}$$

This algebra closes. By similarly comparing to Ref. 9, we obtain the quantum superspace representation of the generators:

$$J^{\alpha\beta} = x^\alpha p^\beta - x^\beta p^\alpha - \frac{i}{2}p_\gamma\bar\theta\gamma^{\alpha\beta\gamma}\theta = q^\alpha p^\beta - q^\beta p^\alpha + K^{\alpha\beta}, \tag{31a}$$

where $K^{+-} = K^{+1} = 0$ and

$$K^{12} = \tfrac{1}{8}(\bar S\gamma_+\gamma_5 S + \bar T\gamma_+\gamma_5 T), \tag{31b}$$

$$K^{i-} = K^{ij}\frac{p^j}{p^+} + \frac{(-p^2)^{1/2}}{4p^+}\bar S\gamma^i\gamma_+ T, \tag{31c}$$

$$\begin{aligned}\pi^\alpha = {}& p^\alpha - \frac{1}{4R^2}(2x^\alpha x\cdot p - x\cdot xp^\alpha - 4ix^\alpha - i\bar\theta\gamma^{\alpha\beta\gamma}\theta x_\beta p_\gamma - \tfrac{1}{4}\bar\theta\gamma_5\gamma_\beta\theta\bar\theta\gamma_5\gamma^\beta\theta p^\alpha + \bar\theta\gamma_5\gamma^\alpha\theta\bar\theta\gamma_5\gamma\cdot p\theta) \\ = {}& p^\alpha - \frac{1}{4R^2}\Big\{2q^\alpha q\cdot p - q\cdot qp^\alpha - q_\beta\frac{(-p^2)^{1/2}}{2p^+}\bar S\gamma^{\alpha\beta}T + \frac{p^\alpha}{16p^2}\bar T\gamma_5\gamma_+ T(2\bar T\gamma_5\gamma_+ T - \bar S\gamma_5\gamma_+ S) \\ & + \frac{i}{16p^+(-p^2)^{1/2}}(\bar S\gamma_5\gamma_+ S - 3\bar T\gamma_5\gamma_+ T)\bar S\gamma_5\gamma\cdot p\gamma^\alpha T \\ & + \delta^\alpha_-\frac{1}{16p^+}[(\bar T\gamma_5\gamma_+ T)^2 - (\bar S\gamma_5\gamma_+ S)^2 + 2\bar S\gamma_5\gamma^i\gamma_+ T\bar S\gamma_5\gamma_i\gamma_+ T] - 4iq^\alpha - \frac{3ip_\beta}{16p^+(-p^2)^{1/2}}\bar S\gamma^{\alpha\beta}T\Big\},\end{aligned} \tag{32}$$

$$\begin{aligned}G^a = {}& 2i(\gamma\cdot p\theta)^a + \frac{1}{2R}[2(\gamma\cdot x\gamma\cdot p\theta)^a + i(\gamma_5\gamma_\alpha\gamma\cdot p\theta)^a\bar\theta\gamma_5\gamma^\alpha\theta + 4i\theta^a] \\ = {}& \frac{1}{(p^+)^{1/2}}[i\gamma\cdot pS + (-p^2)^{1/2}T]^a + \frac{1}{2R}\Big\{-i(\gamma\cdot q)Q^a + \frac{i}{2(p^+)^{1/2}}(\gamma_5\gamma^i T)^a\bar T\gamma_5\gamma_i\gamma_+ S \\ & - \frac{i}{4(p^+)^{1/2}}(\gamma_5 S)^a\bar S\gamma_5\gamma_+ S + \frac{5i}{2(p^+)^{1/2}}S^a \\ & + \tfrac{1}{2}(-p^2p^+)^{-1/2}[\gamma_5\gamma\cdot pT)^a\bar T\gamma_5\gamma_+ T + 3(\gamma\cdot pT)^a]\Big\}.\end{aligned} \tag{33}$$

Ordering problems have here been dealt with by requiring hermiticity.

It is intriguing to ask whether one could construct an unconstrained Lagrangian using the dimensional constant R now available. The problem does not seem to have a simple solution, however.

V. CONCLUSION

The fact that one cannot describe the supersymmetric point particle within the ordinary superspace formulation certainly casts some doubts on the latter. Whether the "quantum superspace" coordinates will ever turn out to be useful remains, of course, highly speculative. It is interesting to notice, however, that the introduction of a Clifford algebra for the fermionic coordinates suggests the possibility of obtaining the space-time coordinates as "composites" of the fermionic ones.[10]

ACKNOWLEDGMENTS

The author wishes to thank Lars Brink for suggesting this problem and Lars Brink, Robert Marnelius, and Bengt Nilsson for helpful discussions.

---

[1]F. Gliozzi, J. Scherk, and D. Olive, Nucl. Phys. B122, 256 (1977); L. Brink, J. H. Schwarz, and J. Scherk, *ibid.* B121, 77 (1977).

[2]W. Caswell and D. Zanon, Nucl. Phys. B182, 125 (1981); M. Grisaru, M. Roček, and W. Siegel, Phys. Rev. Lett. 45, 1063 (1980); Nucl. Phys. B183, 141 (1981).

[3]W. Siegel and M. Roček, Caltech Report No. 68–818, 1981 (unpublished); B. E. W. Nilsson, Institute of Theoretical Physics (Göteborg) Report No. 81–6, 1981 (unpublished).

[4]L. Brink and J. H. Schwarz, Phys. Lett. 100B, 310 (1981).

[5]R. Casalbuoni, Nuovo Cimento 33A, 389 (1976).

[6]P. A. M. Dirac, *Lectures on Quantum Mechanics* (Belfer Graduate School of Science, Yeshiva University, New York, 1964).

[7]A. J. Hanson and T. Regge, Ann. Phys. (N.Y.) 87, 498 (1974).

[8]R. Marnelius and B. Nilsson, Institute of Theoretical Physics, Göteborg, Report No. 79-52, 1979 (unpublished).

[9]J. H. Schwarz, Caltech Report No. 68–817, 1981 (unPublished).

[10]J. H. Schwarz and P. van Nieuwenhuizen, Caltech Report No. 68–822, 1981 (unpublished).