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Effects of nucleon-nucleon interactions on scattering of neutrinos in neutron matter

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We calculate neutrino mean free paths from neutrino-neutron scattering in neutron matter. We formulate the problem such that the correlation effects of the neutron system can be taken into account in terms of Landau parameters. It is shown that the neutrinos are scattered not only by single-pair excitations (in the density and spin-density channels) but also by the collective mode of the system (spin-zero sound). We find that for degenerate neutrinos the effects of nucleon-nucleon interactions reduce the neutrino scattering rate by a factor 2–3, where the contribution from the collective mode is negligible because its temperature dependence is different from that of the single-pair excitations.

I. INTRODUCTION

Neutrino processes in dense matter play an important role in the later stages of stellar evolution. In this paper we consider neutrino scattering processes in neutron matter. Interaction effects are large in neutron matter, which is a strongly interacting quantum system, and it is therefore important to determine how the neutron-neutron interaction affects neutrino scattering rates. The most convenient way of discussing the problem is to consider scattering from fluctuations in the system, as one does when studying scattering of particles and radiation from condensed matter in the laboratory. The scattering of neutrinos by density fluctuations in neutron-star matter was first discussed by Sawyer,[1] who estimated the neutrino mean free path. He used a thermodynamic relation to express the static density fluctuation spectrum in the long-wavelength limit in terms of the compressibility, which in turn he obtained from the equation of state derived from the Reid nucleon-nucleon potential. His treatment was static; no dynamic properties were incorporated. In addition, only the vector part of the weak neutral current, which couples to density fluctuations, was taken into account and the axial-vector part, which couples to spin-density fluctuations, was neglected.

Quantum liquids have other elementary excitations in addition to the single-pair excitations which exist in noninteracting systems. Among these are collective modes (such as phonons) and multipair excitations. Sutherland and Flowers[2] considered scattering of neutrinos by phonons in neutron-star matter. They treated only density fluctuations and assumed that the collective mode exhausts the f-sum rule. Their conclusion was that scattering of neutrinos by phonons could have a significant effect on the mean free path.

The purpose of this paper is to treat the problem in a consistent fashion: we incorporate the nucleon-nucleon interaction effects within the framework of Landau-Fermi liquid theory. Using the Landau parameters calculated for pure neutron matter,[3] we consider the scattering of neutrinos by both density and spin-density fluctuations. This formalism enables us to treat both single-pair excitations and collective modes on an equal footing. Compared with a noninteracting neutron gas, we find the following major differences. (1) The scattering by single-pair excitations is modified. In particular, the Gamow-Teller transitions (the axial-vector ones) are suppressed appreciably due to the large value of the Landau parameter F_0^a (~ 0.97 at $\rho=\rho_0$, where $\rho_0=2.8\times 10^{14}$ g cm^{-3} is nuclear matter density). (2) A collective mode (spin-zero sound) appears, and gives a contribution to the neutrino mean free path which has a qualitatively different temperature dependence compared with the contribution from single-pair excitations. At nuclear matter density zero sound involving density fluctuations is not expected to exist in neutron matter, since F_0^s is not sufficiently repulsive. (It is estimated that zero sound begins to appear only at densities above $\sim 2.3\rho_0$.[4,5])

The organization of the paper is as follows. In

Sec. II, the probability of neutrino-neutron scattering is expressed in terms of dynamic form factors. In Sec. III, we calculate the linear response functions by solving the quantum kinetic equation, and derive expressions for the dynamic form factors for density and spin-density fluctuations. We then discuss kinematics in Sec. IV, and neutrino mean free paths are calculated for both degenerate and nondegenerate neutrinos in Sec. V. A summary of the results and discussions are given in Sec. VI.

II. SCATTERING RATE

In this section we calculate the rate of neutrino-neutron scattering in terms of the dynamic form factors of the neutron system.[6]

Neutrinos scatter from neutrons via the weak neutral current. According to the Weinberg-Salam model, the interaction Lagrangian density, in the low-momentum-transfer region, has the form[7]

$$\mathscr{L}_I(x)=\frac{G}{\sqrt{2}}l_\mu(x)j_z^\mu(x)\ , \tag{2.1}$$

where the weak coupling constant is

$$G\approx 1.4358\times 10^{-49}\ \mathrm{erg\,cm}^3,$$

$$l_\mu(x)=\bar{\psi}_\nu\gamma_\mu(1-\gamma_5)\psi_\nu \tag{2.2}$$

is the lepton weak neutral current, and

$$j_z^\mu(x)=\tfrac{1}{2}\bar{\psi}_n\gamma^\mu(1-C_A\gamma_5)\psi_n \tag{2.3}$$

is the third component of the isospin current with $C_A\approx 1.25$.[8] Since the neutrons are nonrelativistic in neutron matter we may use the approximation

$$\bar{\psi}_n\gamma^\mu(1-C_A\gamma_5)\psi_n\rightarrow\psi_n^\dagger\psi_n\delta_0^\mu-C_A\psi_n^\dagger\sigma_i\psi_n\delta_i^\mu\ . \tag{2.4}$$

The first term on the right-hand side of Eq. (2.4) represents a density fluctuation and the second term a spin-density fluctuation. We denote the initial and the final four-momenta of the neutrino by $q^\mu=(q^0,\vec{q})$ and $q^{\mu\prime}=(q^{0\prime},\vec{q}\,')$, and the four-momentum transfer by $k^\mu=(\omega,\vec{k})=(q^0-q^{0\prime},\vec{q}-\vec{q}\,')$. The rate at which a neutrino is scattered by a system of neutrons is calculated from Eqs. (2.1)–(2.4) by using Fermi's golden rule, and taking a statistical average over initial neutron states and summing over final neutron states:

$$W_{fi}=\frac{G^2n}{4V}\{(1+\cos\theta)S(\vec{k},\omega)+C_A{}^2[(1-\cos\theta)\delta_{ij}+\hat{q}_i'\hat{q}_j+\hat{q}_i\hat{q}_j']\ \mathscr{S}_{ij}^s(\vec{k},\omega)+iC_A{}^2e_{ijk}(\hat{q}_k-\hat{q}_k')\mathscr{S}_{ij}^a(\vec{k},\omega)\}\ . \tag{2.5}$$

Here n is the neutron number density, $\cos\theta=\hat{q}\cdot\hat{q}'$, and e_{ijk} is the antisymmetric tensor of rank three. The dynamic form factors are defined as

$$S(\vec{k},\omega)\equiv\frac{1}{nV}\int_{-\infty}^{\infty}dt\,e^{i\omega t}\langle\rho(\vec{k},t)\rho(-\vec{k},0)\rangle \tag{2.6}$$

for density fluctuations, and

$$\mathscr{S}_{ij}(\vec{k},\omega)\equiv\frac{1}{nV}\int_{-\infty}^{\infty}dt\,e^{i\omega t}\langle\sigma_i(\vec{k},t)\sigma_j(-\vec{k},0)\rangle \tag{2.7}$$

for spin-density fluctuations with symmetric and antisymmetric components,

$$\mathscr{S}_{ij}^{s(a)}(\vec{k},\omega)=\tfrac{1}{2}[\mathscr{S}_{ij}(\vec{k},\omega)+(-)\mathscr{S}_{ji}(\vec{k},\omega)]\ , \tag{2.8}$$

where $\rho(\vec{k},\omega)$ and $\sigma_i(\vec{k},\omega)$ are the Fourier components of the density operator $(\psi_n^\dagger\psi_n)$ and the spin-density operator $(\psi_n^\dagger\sigma_i\psi_n)$, respectively; $\langle\ \rangle$ denotes the statistical average, and V is the normalization volume. In the present case the dynamic form factor for spin-density fluctuations is symmetric $[\mathscr{S}_{ij}(\vec{k},\omega)\equiv\delta_{ij}\mathscr{S}(\vec{k},\omega)]$, so that Eq. (2.5) may be written in a compact form as

$$W_{fi}=\frac{G^2n}{4V\hbar^2}[(1+\cos\theta)S(\vec{k},\omega)+C_A^2(3-\cos\theta)\mathscr{S}(\vec{k},\omega)]\ . \tag{2.9}$$

The collision term in the Boltzmann equation is given in terms of W_{fi} by the equation

$$\frac{\partial n(\vec{q})}{\partial t}=-V\int\frac{d^3\vec{q}\,'}{(2\pi)^3}W_{fi}(\vec{q}-\vec{q}\,',q^0-q^{0\prime})n(\vec{q})[1-n(\vec{q}\,')]+V\int\frac{d^3\vec{q}\,'}{(2\pi)^3}W_{fi}(\vec{q}\,'-\vec{q},q^{0\prime}-q^0)n(\vec{q}\,')[1-n(\vec{q})]\ . \tag{2.10}$$

Here, $n(\vec{q})$ is the neutrino distribution function. The first term in Eq. (2.10) represents the rate at which neutrinos are lost by scattering out of the state $\vec{q}$, while the second represents scattering into the state $\vec{q}$. The n and $1-n$ factors in (2.10) physically correspond to the fact that the process can occur only if the initial state is occupied and the final one is unoccupied. The relaxation time $\tau(\vec{q})$ is calculated by linearizing (2.10) with respect to the deviation $\delta n(\vec{q})$ from the equilibrium (Fermi) function,

$$n^0(\vec{q})=\{\exp[(q^0-\mu_\nu)/k_BT]+1\}^{-1}\ , \tag{2.11}$$

where μ_ν is the neutrino chemical potential. One finds

$$\frac{\partial n(\vec{q})}{\partial t}=-\delta n(\vec{q})/\tau(\vec{q})\ , \tag{2.12}$$

where

$$\frac{1}{\tau(\vec{q})}=V\int\frac{d^3\vec{q}\,'}{(2\pi)^3}\{W_{fi}(\vec{q}-\vec{q}\,',q^0-q^{0\prime})[1-n(\vec{q}\,')]+W_{fi}(\vec{q}\,'-\vec{q},q^{0\prime}-q^0)n(\vec{q}\,')\}\ . \tag{2.13}$$

With the help of the principle of detailed balancing, which for a system invariant under spatial reflections is

$$W_{fi}(\vec{k},\omega)=e^{\beta\omega}W_{fi}(\vec{k},-\omega)\ , \tag{2.14}$$

where $\beta\equiv 1/k_BT$, Eq. (2.13) may be rewritten as

$$\frac{1}{\tau(\vec{q})}=[1+e^{-(q^0-\mu_\nu)/k_BT}]V\int\frac{d^3\vec{q}\,'}{(2\pi)^3}W_{fi}(\vec{q}-\vec{q}\,',q^0-q^{0\prime})[1-n(\vec{q}\,')]\ . \tag{2.15}$$

This result contains the effects of neutron-neutron interactions through the dynamic form factors that occur in W_{fi}. To evaluate the expression we need the form factors of the neutron system, and we now calculate these.

III. DYNAMIC FORM FACTORS OF THE NEUTRON LIQUID

In this section we derive expressions for the density-density and spin-density—spin-density response functions in the neutron liquid, and evaluate the dynamic form factors from them by using the fluctuation-dissipation theorem. The neutrino momenta and the momentum transfers in many applications are small compared with the neutron Fermi momentum, and energy transfers and the temperature are small compared with the neutron Fermi energy. Consequently, we may describe the properties of the neutron system using Landau-Fermi liquid theory.

We begin with the linearized form of the Landau transport equation in the presence of an external force $\vec{F}_{\vec{p}\sigma}(k,\omega)$,[9–11]

$$(\omega-\vec{k}\cdot\vec{v}_{\vec{p}})\delta n_{\vec{p}\sigma}(\vec{k},\omega)+\vec{k}\cdot\vec{v}_{\vec{p}}\frac{\partial n^0}{\partial\epsilon_p}\sum_{\vec{p}\,',\sigma'}f_{\vec{p}\sigma\vec{p}\,'\sigma'}\delta n_{\vec{p}\,'\sigma'}(\vec{k},\omega)+i\vec{F}_{\vec{p}\sigma}(\vec{k},\omega)\cdot\vec{v}_{\vec{p}}\frac{\partial n^0}{\partial\epsilon_p}=0\ , \tag{3.1}$$

where $\delta n_{\vec{p}\sigma}=n_{\vec{p}\sigma}-n^0_{\vec{p}\sigma}$ is the departure of the quasiparticle distribution from the ground-state distribution, $\epsilon_p=\epsilon^0_{\vec{p}\sigma}$ is the ground-state quasiparticle energy, and $f_{\vec{p}\sigma\vec{p}\,'\sigma'}$ is the interaction energy of the excited quasiparticles $\vec{p}\sigma$ and $\vec{p}\,'\sigma'$. The quasiparticle group velocity is given by $\vec{v}_{\vec{p}}=\vec{\nabla}_{\vec{p}}\epsilon_{\vec{p}\sigma}$, where

$$\epsilon_{\vec{p}\sigma}=\epsilon^0_{\vec{p}\sigma}+\sum_{\vec{p}\,'\sigma'}f_{\vec{p}\sigma\vec{p}\,'\sigma'}\delta n_{\vec{p}\,'\sigma'} \tag{3.2}$$

is the local excitation energy of a quasiparticle.

The density fluctuation

$$\delta n^s_{\vec{p}}(\vec{k},\omega)=\delta n_{\vec{p}\uparrow}(\vec{k},\omega)+\delta n_{\vec{p}\downarrow}(\vec{k},\omega) \tag{3.3}$$

couples to the spin-independent external force

$$\vec{F}_{\vec{p}\sigma}(\vec{k},\omega)=i\vec{q}\phi(\vec{k},\omega)\ , \tag{3.4}$$

while the spin-density fluctuation

$$\delta n^a_{\vec{p}}(\vec{k},\omega)=\delta n_{\vec{p}\uparrow}(\vec{k},\omega)-\delta n_{\vec{p}\downarrow}(\vec{k},\omega) \tag{3.5}$$

couples to the spin-dependent external force

$$\vec{F}_{\vec{p}\sigma}(\vec{k},\omega)=i\vec{k}g\mu_B\sigma H(\vec{k},\omega)\ , \tag{3.6}$$

where the perturbing magnetic field is chosen in the z direction, g is the Landé g factor, and μ_B is the Bohr magneton. Since this is the first detailed investigation of how neutron-neutron interactions affect scattering of neutrinos, we shall neglect the tensor contributions to the interaction between quasiparticles. The quasiparticle interaction is then a function which is invariant under rotation of the quasiparticle spins. In an isotropic system, introducing

$$f^{s(a)}_{\vec{p}\vec{p}'}=\tfrac{1}{2}[f_{\vec{p}\uparrow\vec{p}'\uparrow}+(-)f_{\vec{p}\uparrow\vec{p}'\downarrow}]\ , \tag{3.7}$$

one can decompose the transport equation (3.1) into a spin-antisymmetric part,

$$(\omega-\vec{k}\cdot\vec{v}_{\vec{p}})\delta n^a_{\vec{p}}(\vec{k},\omega)+\vec{k}\cdot\vec{v}_{\vec{p}}\frac{\partial n^0}{\partial\epsilon_p}\sum_{\vec{p}'}2f^a_{\vec{p}\vec{p}'}\delta n^a_{\vec{p}'}(\vec{k},\omega)-2\vec{k}\cdot\vec{v}_{\vec{p}}g\mu_B\,|\,\sigma\,|\,H(\vec{k},\omega)\frac{\partial n^0}{\partial\epsilon_p}=0\ , \tag{3.8}$$

and a spin-symmetric part, which has a form similar to Eq. (3.8). In (3.8) the external force is chosen explicitly as (3.6). Let us calculate the spin-density—spin-density response function, which is defined as

$$\chi_\sigma(\vec{k},\omega)\equiv\frac{1}{V}\sum_{\vec{p}}g\mu_B\,|\,\sigma\,|\,\delta n^a_{\vec{p}}(\vec{k},\omega)/(g\mu_B\sigma)^2H(\vec{k},\omega)\ . \tag{3.9}$$

Expecting a solution to Eq. (3.8) of the form

$$\delta n^a_{\vec{p}}(\vec{k},\omega)=\frac{\vec{k}\cdot\vec{v}_{\vec{p}}}{\omega-\vec{k}\cdot\vec{v}_{\vec{p}}}\frac{\partial n^0}{\partial\epsilon_p}x_{\vec{p}}\ , \tag{3.10}$$

one expands $x_{\vec{p}}$ and $f^a_{\vec{p}\vec{p}'}$ in Legendre polynomials

$$x_{\vec{p}}=\sum_{l=0}^{\infty}x_lP_l(\cos\tilde{\theta})\ , \tag{3.11}$$

$$f^a_{\vec{p}\vec{p}'}=\sum_{l=0}^{\infty}f^a_lP_l(\cos\Theta)\ , \tag{3.12}$$

where $\cos\tilde{\theta}=\hat{p}\cdot\hat{k}$ and $\cos\Theta=\hat{p}\cdot\hat{p}'$. Retaining only the $l=0$ and $l=1$ components in Eq. (3.8), one obtains

$$\chi_\sigma(\vec{k},\omega)=\frac{N(0)}{V}\frac{g(\lambda)}{1+[F^a_0+\lambda^2F^a_1/(1+\frac{1}{3}F^a_1)]g(\lambda)}\ , \tag{3.13}$$

in terms of the reduced interactions $F^{s(a)}_l=N(0)f^{s(a)}_l$, where $N(0)=Vm^*p_F(n)/\pi^2\hbar^3$ is the density of states for both spins, $\lambda\equiv\omega/kv_F(n)$ [$p_F(n)$ and $v_F(n)$ being the neutron Fermi momentum and Fermi velocity, respectively], and

$$g(\lambda)\equiv1-\frac{\lambda}{2}\ln\left[\frac{\lambda+1}{\lambda-1}\right]=1-\frac{\lambda}{2}\ln\left|\frac{\lambda+1}{\lambda-1}\right|+\frac{i\pi}{2}\lambda\theta(1-|\lambda|)\ . \tag{3.14}$$

In general, the spin-density—spin-density response function has a tensor form, $\chi^\sigma_{ij}(\vec{k},\omega)$, whose nonzero components are $\chi^\sigma_{xx}=\chi^\sigma_{yy}=\chi^\sigma_{zz}=\chi_\sigma(\vec{k},\omega)$ in an isotropic system. The density-density response function, $\chi_\rho(\vec{k},\omega)$, can be calculated similarly and has the same form as (3.13) except that the F^a_l's are replaced by F^s_l's. The dynamic form factors of density and spin-density fluctuations, $S(\vec{k},\omega)$ and $\mathscr{S}_{ij}(\vec{k},\omega)$ may be obtained from the corresponding response functions via the fluctuation-dissipation theorem. For spin-density fluctuations, for example,

$$\mathscr{S}_{ij}(\vec{k},\omega)=\frac{2\hbar}{n}\frac{1}{1-e^{-\beta\hbar\omega}}\mathrm{Im}\chi^{\sigma}_{ij}(\vec{k},\omega)\ , \quad (3.15)$$

where n is the neutron number density.

Let us now examine the density and spin-density fluctuation spectra by using the Landau parameters calculated for pure neutron matter.[3] In Table I the parameters at nuclear matter density (ρ_0), which corresponds to the neutron Fermi momentum $p_F(n)\simeq 1.70\hbar\ \mathrm{fm}^{-1}$, are listed. From these parameters one immediately notices that zero sound does not exist as a well-defined collective mode in neutron matter at this density; it suffers strong Landau damping since the repulsive force is too weak, $F_0^s\approx 0$.[12] In fact, the Landau parameters of Ref. 3 indicate that zero sound appears as an undamped collective mode only at densities above $\sim 2.27\ \rho_0$ [$p_F(n)\simeq 2.24\hbar\ \mathrm{fm}^{-1}$, $F_0^s\simeq 0.73$, $F_1^s\simeq -0.59$].[5] At such densities, the conditions[13]

$$F_0^s>0,\ F_1^s<0,\ \text{and}\ F_0^s>|F_1^s/(1+\tfrac{1}{3}F_1^s)| \quad (3.16)$$

are satisfied so that $\chi(\vec{k},\omega)$ has a pole within the same approximation used to derive Eq. (3.13). Therefore, only the single-pair excitations exist in the density fluctuation spectrum at $\rho=\rho_0$. On the other hand $F_0^a=0.97$ is enough to sustain a well-defined spin-zero-sound mode.[14] From Eqs. (3.13) and (3.15) one can obtain the spin-zero-sound-pole contributions to the dynamic form factor,

$$\mathscr{S}_{ij}(\vec{k},\omega)=\delta_{ij}\frac{N(0)\hbar k v_F(n)}{nVB}\frac{1}{1-e^{-\beta\hbar\omega}}\times[\delta(\omega-c_s k)-\delta(\omega+c_s k)]\ , \quad (3.17)$$

where

$$B=\{F_0^a+[F_1^a/(1+\tfrac{1}{3}F_1^a)]\lambda_0^2\}A/2\pi\ , \quad (3.18)$$

$$A=2F_1^a\lambda_0 g(\lambda_0)/(1+\tfrac{1}{3}F_1^a)+[F_0^a+F_1^a\lambda_0^2/(1+\tfrac{1}{3}F_1^a)]g'(\lambda_0)\ , \quad (3.19)$$

and $\lambda_0\equiv c_s/v_F(n)$ is the solution to the equation

$$1+[F_0^a+F_1^a\lambda_0^2/(1+\tfrac{1}{3}F_1^a)]g(\lambda_0)=0\ . \quad (3.20)$$

From the Landau parameters in Table I we have $B\simeq 1.14$ and $\lambda_0\simeq 1.10$. From the same set of Eqs. (3.13) and (3.15) the single-pair excitation spectrum can be obtained, and its leading term in the low-frequency limit ($\lambda<<1$) has the simple form

$$S(\vec{k},\omega)\simeq\frac{\pi\hbar N(0)}{nV(1+F_0^s)^2}\frac{\lambda}{1-e^{-\beta\hbar\omega}}\quad(\lambda<<1)\ , \quad (3.21)$$

for density fluctuations, and

$$\mathscr{S}_{ij}(\vec{k},\omega)\simeq\delta_{ij}\frac{\pi\hbar N(0)}{nV(1+F_0^a)^2}\frac{\lambda}{1-e^{-\beta\hbar\omega}}\quad(\lambda<<1)\ , \quad (3.22)$$

for spin-density fluctuations. In this limit, the Fermi-liquid effects are included in the neutron effective mass $m^*=(1+\frac{1}{3}F_1^s)m$ through $N(0)$ and by factors $(1+F_0^s)^{-2}$ or $(1+F_0^a)^{-2}$. In Figs. 1 and 2, we compare calculations of a number of expressions for the dynamic form factors for density and spin-density fluctuations:

(1) the result for a free Fermi gas,

(2) the result for a Fermi gas with an effective mass,

(3) the low-frequency results (3.21) and (3.22), and

(4) the full expressions from Eqs. (3.13)–(3.20), including the effects of the Landau parameters F_0^s, F_0^a, F_1^s, and F_1^a.

From these figures one sees the following. In the case of density fluctuations, the single-pair excitation spectrum $S(\vec{k},\omega)$ for a free Fermi gas is modified by the effective-mass correction (F_1^s), which is further modified a little by F_0^s. In contrast to this, for the case of spin-density fluctuations, the reduction of the single-pair excitation part of $\mathscr{S}_{ij}(\vec{k},\omega)$ due to a factor $(1+F_0^a)^{-2}$ is marked, and the collective mode appears. In both cases the low-frequency forms of $S(\vec{k},\omega)$ and $\mathscr{S}_{ij}(\vec{k},\omega)$ give a reasonable approximation to the full expressions. Therefore, we adopt those forms, which enable us to make analytic calculations in the following sections.

TABLE I. The Landau parameters for pure neutron matter at nuclear matter density calculated by Bäckman, Källman, and Sjöberg (Ref. 3).

F_0^s	F_0^a	F_1^s	F_1^a
0.07	0.97	−0.43	0.51

IV. KINEMATICS

In this section we discuss the allowed momentum and energy transfers for the various cases of interest. The limits on ω and $k(\equiv|\vec{k}|)$ due to the neutron system may easily be obtained by inspect-

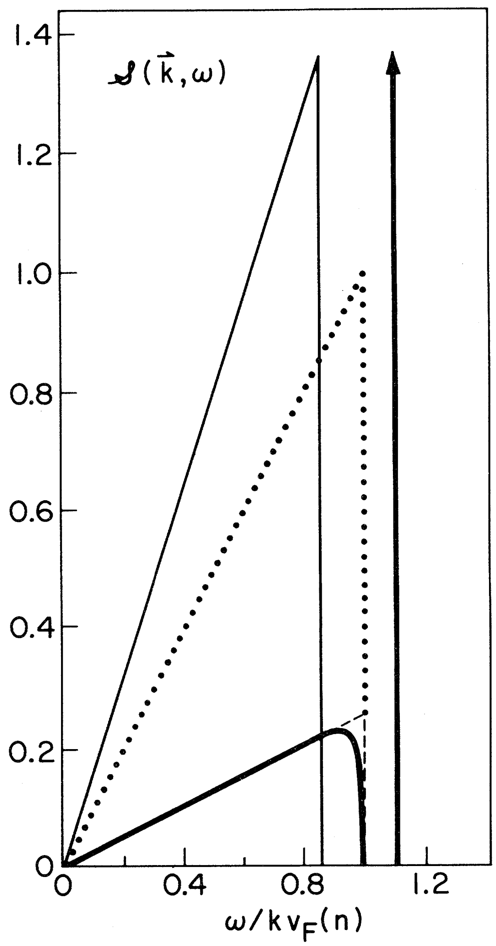

FIG. 1. Dynamic form factor for spin-density fluctuations in neutron matter. The fluctuation spectra are shown for (1) a free Fermi gas (thin line); (2) a Fermi gas with an effective mass $m^*=(1+\frac{1}{3}F_1^s)m$ (dotted line); (3) a Fermi liquid in the low-frequency approximation, Eqs. (3.17) and (3.22) (dashed line); and (4) a Fermi liquid, including the effects of the Landau parameters F_0^a, F_1^s, and F_1^a (bold line). This is obtained from Eqs. (3.13)–(3.20). Fermi-liquid effects reduce the contribution from the single-pair excitations, while they give rise to a spin-zero-sound pole at $\lambda\equiv\omega/kv_F(n)\simeq 1.10$. The spectra are normalized so that the maximum of curve (2) is unity. The Landau parameters are due to Bäckman, Källman, and Sjöberg (Ref. 3) at nuclear matter density.

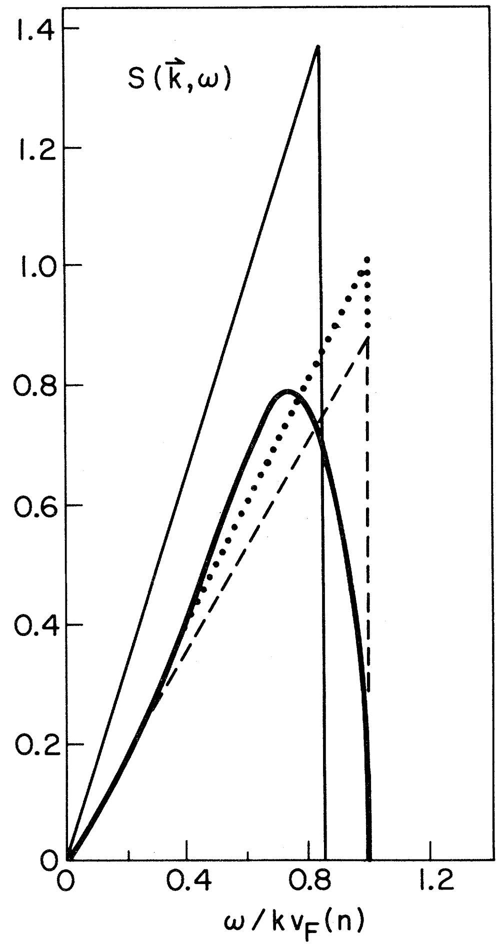

FIG. 2. Dynamic form factor for density fluctuations in neutron matter. The fluctuation spectra are shown for (1) a free Fermi gas (thin line); (2) a Fermi gas with an effective mass $m^*=(1+\frac{1}{3}F_1^s)m$ (dotted line); (3) a Fermi liquid in the low-frequency approximation, Eq. (3.21) (dashed line); and (4) a Fermi liquid, including the effects of the Landau parameters F_0^s and F_1^s (bold line). This is obtained from a formula for $S(\vec{k},\omega)$ similar to Eq. (3.13). Fermi-liquid effects modify the single-pair excitation spectrum and no collective mode exists at nuclear matter density.

ing the dynamic form factors. According to Eqs. (3.13) and (3.14), these are nonzero in the range

$$-kv_F(n)\le\omega\le kv_F(n)\ . \tag{4.1}$$

corresponding to the continuum of single-pair excitations, and at the energies

$$\omega=\pm c_s k\ , \tag{4.2}$$

corresponding to the collective mode.[15] The range of the single-pair continuum may easily be understood by considering scattering of a neutrino by a free nonrelativistic particle of mass m^*. The energy and momentum conservation laws are

$$\frac{\vec{p}^{\,2}}{2m^*}+\hbar cq=\frac{\vec{p}^{\,\prime 2}}{2m^*}+\hbar cq'\ , \tag{4.3}$$

$$\vec{p}+\hbar\vec{q}=\vec{p}\,'+\hbar\vec{q}\,'\ , \tag{4.4}$$

where $\vec{p}$ and $\hbar\vec{q}$ are the neutron and neutrino initial momenta, respectively, $q \equiv |\vec{q}|$, and the prime denotes the final-state quantities. The energy and momentum transferred from the neutrino to the neutron are defined by $\hbar\omega = \hbar c(q - q')$ and $\hbar\vec{k} = \hbar(\vec{q} - \vec{q}\,')$, respectively. Expressing the relation between ω and k in terms of the initial neutron momentum, one has, from Eqs. (4.3) and (4.4),

$$-kv + \frac{\hbar k^2}{2m^*} \le \omega \le kv + \frac{\hbar k^2}{2m^*} , \tag{4.5}$$

where $v = |\vec{p}\,|/m^*$. This is a familiar excitation domain for nonrelativistic particles. For a degenerate Fermi system the maximum velocity of an initial neutron is $v_F(n)$, the value which gives maximum possible range for ω. Curves corresponding to the condition (4.5) for this case are shown in Fig. 3, where lines $\omega = \pm c_s k$ (4.2) corresponding to phonon emission and absorption are also shown.[16]

In actual situations, since the typical neutrino momentum is much smaller than the neutron Fermi momentum $[\hbar q << p_F(n)]$, and since the momentum transfer is on the order of the larger of the two momenta, $\hbar q$ and $k_B T/c$, the recoil term, $\hbar k^2/2m^*$ is negligible compared with the resonant term $kv_F(n)$ in Eq. (4.5). The condition (4.5) then reduces to (4.1), obtained earlier from the dynamic form factor calculated in Landau theory. Similarly, in terms of the initial neutrino momentum one has

$$|\omega| \le ck \le |\omega - 2cq| . \tag{4.6}$$

The boundaries of the region described by (4.6) are

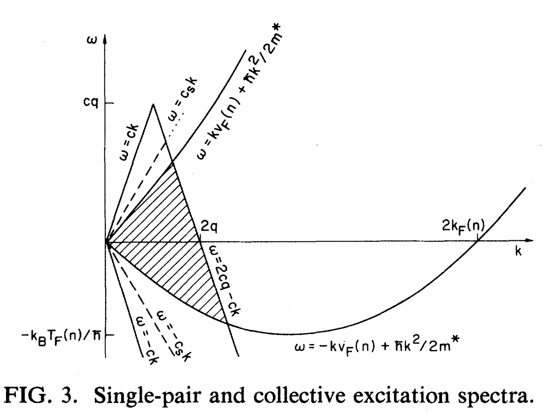

FIG. 3. Single-pair and collective excitation spectra. The hatched region represents the continuum of single-pair excitations and the dashed lines correspond to the collective-mode branch which are kinematically allowed in the scattering of a neutrino of energy $\hbar cq$ by a system of degenerate neutrons with a Fermi energy $k_B T_F(n) = \hbar^2 k_F(n)^2/2m^*$.

also shown in Fig. 3. The condition (4.6) is nothing but the relativistic counterpart of the condition (4.5) with the sign of ω reversed.

Thus, energy and momentum conservation requires that ω and k must lie in the region which satisfies both (4.5) and (4.6). This condition applies to single-pair excitations. The condition (4.6) restricts the phonon momenta to lie in the range

$$0 \le k \le 2q/[1 \pm (c_s/c)] . \tag{4.7}$$

The plus (minus) sign in Eqs. (4.2) and (4.7) corresponds to phonon emission (absorption).

In the ω-k plane, the region which satisfies both (4.5) and (4.6) is indicated by the hatched area in Fig. 3. This region corresponds to single-pair excitations. In addition, the collective-mode branch which satisfies both (4.2) and (4.7) is also shown in Fig. 3 by dashed lines. These two regions in the ω-k plane are kinematically allowed by the energy and momentum conservation laws.

Neutron degeneracy may further restrict the allowed domain in the ω-k plane. At low temperatures the momentum states of the neutron system with $p < p_F(n)$ are occupied. Therefore, although the neutron system can be excited ($\omega > 0$) to the extent allowed by Eqs. (4.1) and (4.6), the deexcitation of the system by an amount larger than $\sim k_B T$ is strongly suppressed by the Pauli principle. This means that the (negative) energy transfer is bounded from below,

$$-k_B T/\hbar \lesssim \omega < 0 . \tag{4.8}$$

Whether this restriction affects the original domain of the single-pair excitation spectrum (the hatched region in Fig. 3) or not depends upon the relative magnitude of the temperature, $k_B T$, and the maximum energy transfer $\sim \hbar q v_F(n)$. When $k_B T >> \hbar q v_F(n)$, which we shall designate as the high-temperature case, the condition (4.8) is less restrictive than (4.1) and (4.6).[17] Therefore, neutron degeneracy does not modify the original domain of possible energy and momentum transfers [see Fig. 4(a)]. On the other hand, when $k_B T << \hbar q v_F(n)$ (which we shall designate as the low-temperature case) the lower part ($\omega < 0$) of the original domain is cut down as shown in Fig. 4(b). This is a direct consequence of the neutron degeneracy. When the neutrinos are also degenerate and when the temperature is low $[k_B T << \hbar q v_F(n)]$, the kinematical region of excitation spectrum is bounded from above

$$0 < \omega \lesssim k_B T/\hbar , \tag{4.9}$$

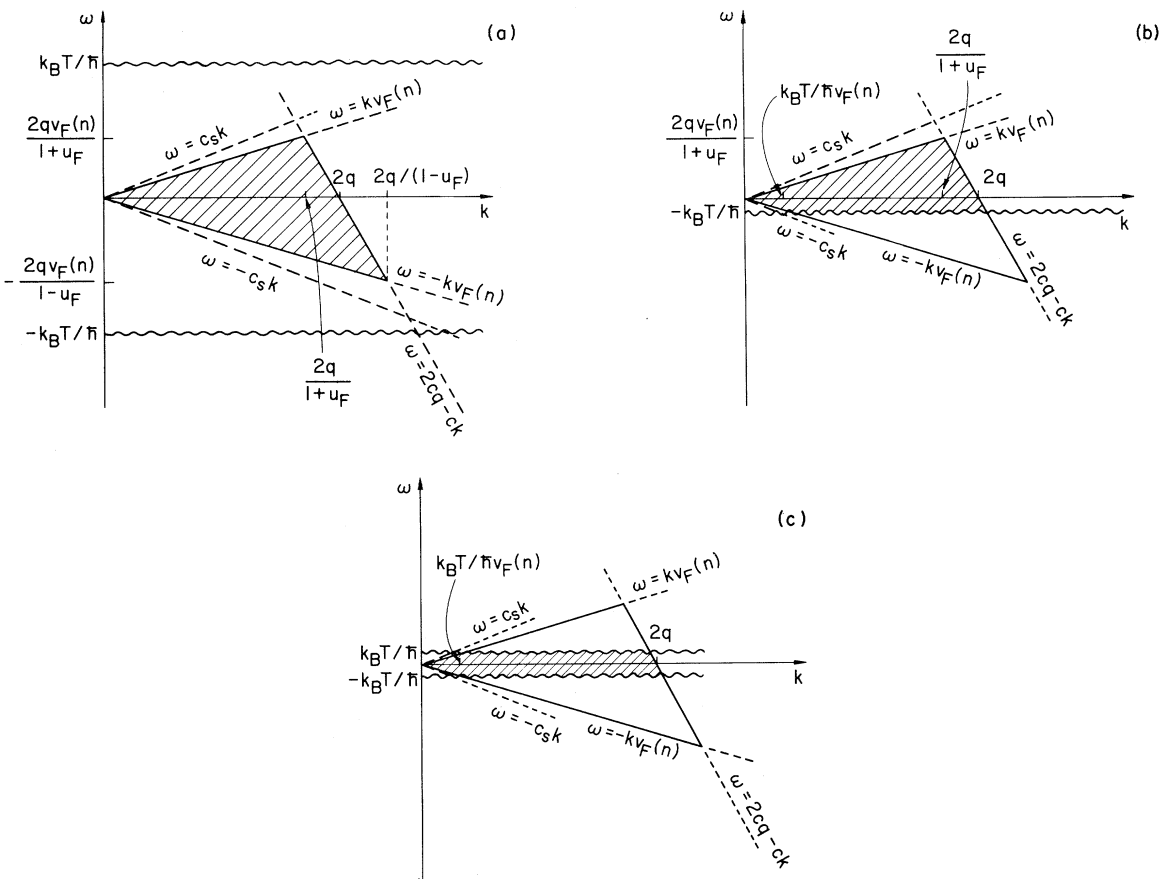

FIG. 4. (a) Single-pair and collective excitation spectra at high temperatures $[k_BT >> \hbar q v_F(n)]$. The hatched region of the single-pair excitation continuum and the part of the collective-mode branch indicated by dashed lines are kinematically allowed. The neutrinos may be either degenerate or nondegenerate. (b) Single-pair and collective excitation spectra at low temperatures $[k_BT << \hbar q v_F(n)]$ for nondegenerate neutrinos. The notation is the same as Fig. 4(a). At these temperatures, the energy transfer is bounded from below ($\hbar\omega \gtrsim -k_BT$) due to the neutron degeneracy. (c) Single-pair and collective excitation spectra at low temperatures $[k_BT << \hbar q v_F(n)]$ for degenerate neutrinos. The notation is the same as (a). Neutrino degeneracy further restricts the energy transfer from above ($\hbar\omega \lesssim k_BT$).

for the same reason [see Fig. 4(c)]. Neutrino degeneracy in the high-temperature case $[k_BT >> \hbar q v_F(n)]$ is possible only when the neutron Fermi velocity $v_F(n)$, is much smaller than the velocity of light c. However, since $v_F(n)$ is of the same order of magnitude as c in neutron matter, the high-temperature condition $[k_BT >> \hbar q v_F(n)]$ and the neutrino degeneracy condition ($k_BT << \mu_\nu \sim \hbar c q$) are actually incompatible. Nevertheless, we also include this case in our later calculations for the sake of comparison with the low-temperature case for the collective-mode contribution.

In the case of the collective excitation spectrum, the effect of neutron degeneracy (or neutrino degeneracy) on the kinematical region is similar. Those regions which are allowed for collective excitations are shown by the dashed lines in Figs. 4(a)–4(c).

V. NEUTRINO MEAN FREE PATHS

We are now ready to calculate the neutrino mean free path. Within the framework of Landau theory there are three contributions to the mean free path in neutron matter due to the following.

(1) Coupling to single-pair excitations by the Fermi transition. Neutrinos are scattered by density fluctuations, either creating or annihilating a

neutron quasiparticle-quasihole pair through the vector coupling. This is the first term in the square brackets of Eq. (2.9), with $S(\vec{k},\omega)$ approximately given by Eq. (3.21).

(2) Coupling to single-pair excitations by the Gamow-Teller transition. This is the axial-vector counterpart of (1), in which neutrinos are scattered by spin-density fluctuations at a rate given by the second term of Eq. (2.9) with the form factor given by Eq. (3.22).

(3) Coupling to collective excitations by the Gamow-Teller transition, which comes from the axial-vector coupling of the neutrinos to the collective mode (spin-zero sound). The scattering rate is also given by the second term of Eq. (2.9) with (3.17).

We estimate these contributions separately in both the degenerate and nondegenerate neutrino cases.

A. Degenerate neutrino case

1. Mean free path

We assume the neutrinos to be degenerate with a Fermi distribution $n(\vec{q})$ and chemical potential μ_ν. Then, the neutrino mean free path $[l_D = c\tau(\vec{q})]$ is

$$1/l_D = \frac{V}{c}\int \frac{d^3\vec{k}}{(2\pi)^3} W_{fi}[1-n(\vec{q}\,')] \times\{1+\exp[(\mu_\nu - E_\nu)/k_B T]\} . \quad (5.1)$$

Let us first evaluate the contribution from single-pair excitations. The integrand in Eq. (5.1) contains $\cos\theta$ [Eq. (2.9)], which from Eqs. (4.3) and (4.4) may be expressed as

$$\cos\theta \equiv \hat{q}\cdot\hat{q}' = \frac{1-(\omega/cq)+\frac{1}{2}(\omega/cq)^2-\frac{1}{2}(k/q)^2}{1-(\omega/cq)} . \quad (5.2)$$

At sufficiently low temperatures, most of the region in the ω-k plane allowed for single-pair excitations [the hatched region in Fig. 4(c)] satisfies $\omega/cq << 1$. Therefore, one may approximate

$$\cos\theta \simeq 1-\tfrac{1}{2}(k/q)^2 . \quad (5.3)$$

From Eqs. (4.3) and (4.4) one also has

$$\cos\theta_{qk} \equiv \hat{q}\cdot\hat{k} = \tfrac{1}{2}(k/q) + \omega/ck - \tfrac{1}{2}(\omega/ck)(\omega/cq) . \quad (5.4)$$

Thus

$$d(\cos\theta_{qk}) = [1-(\omega/cq)](d\omega/ck) \quad (5.5)$$

$$\simeq d\omega/ck . \quad (5.6)$$

The angular integral in (5.1) can be changed into an ω integral, by using (5.6), where the region of integration may be extended to $-\infty < \omega < \infty$. Since only the region $|\omega| \lesssim k_B T/\hbar$ contributes to the ω integral, this extension does not cause any serious error at low temperatures. First, the k integration may be done in the range $0 \le k \le 2q$ [cf. Fig. 4(c)]. Then, the remaining ω integral is standard,[18] and one has

$$(1/l_D)^{\mathrm{F}}_{\text{single pair}} = (1/l)_0 \frac{1}{32}\frac{1}{(1+F_0^s)^2}\left[\frac{k_B T}{\hbar q v_F(n)}\right]\left[\frac{T}{T_F(n)}\right]\{\pi^2+[(E_\nu-\mu_\nu)/k_B T]^2\} \quad (5.7)$$

for the Fermi part of the single-pair excitations, and

$$(1/l_D)^{\mathrm{GT}}_{\text{single pair}} = (1/l)_0 \frac{C_A{}^2}{16}\frac{1}{(1+F_0^a)^2}\left[\frac{k_B T}{\hbar q v_F(n)}\right]\left[\frac{T}{T_F(n)}\right]\{\pi^2+[(E_\nu-\mu_\nu)/k_B T]^2\} \quad (5.8)$$

for the Gamow-Teller part of the single-pair excitations. In Eqs. (5.7) and (5.8), $E_\nu = \hbar c q$,

$$(1/l)_0 \equiv n\sigma_0 (E_\nu/m_e c^2)^2 , \quad (5.9)$$

where

$$\sigma_0 \equiv 4G^2 m_e{}^2/\pi\hbar^4 \simeq 1.76\times 10^{-44}\ \mathrm{cm}^2 \quad (5.10)$$

and m_e is the electron mass. The difference between (5.7) and (5.8) arises from the axial-vector coupling constant (C_A), the angular factors in Eq.

(2.9), and from the Fermi-liquid correction factors (F_0^s and F_0^a).

Let us now turn to the contributions from the collective mode. In this case the dynamic form factor [Eq. (3.17)] is used in the axial-vector part of the transition probability [Eq. (2.9)] to calculate the mean-free-path expression (5.1). At finite temperatures the dynamic form factor contains two pole contributions from the collective mode, which correspond to phonon emission and absorption. The angular part of the $\vec{k}$ integration in (5.1) may be done by choosing the polar axis in the direction of $\vec{q}$. Then from Eqs. (5.2) and (5.5)

$$\int_{-1}^{1} d(\cos\theta_{qk})\delta(\omega \mp c_s k) = \frac{1 \mp (c_s/c)(k/q)}{ck} , \quad (5.11)$$

where the upper (lower) sign is for phonon emission (absorption) by a neutrino. Since, as we shall see, the phase space is different depending on whether a phonon is emitted or absorbed, it is convenient to treat these processes separately.

For phonon emission, (5.1) becomes

$$(1/l_D)_{\text{coll}}^{\text{em}} = \frac{G^2 C_A{}^2 N(0) v_F(n) q^3}{8\pi^2 V \hbar B c^2} I_E \times (1+e^{-x+\beta\mu_\nu}) , \quad (5.12)$$

where

$$I_E = \int_0^{2/(1+u)} y^2 dy \frac{1}{1+e^{-x+uxy+\beta\mu_\nu}} \frac{1}{1-e^{-uxy}} \times [1-uy+\tfrac{1}{4}(1-u^2)y^2] \quad (5.13)$$

and we have introduced dimensionless variables

$$u \equiv c_s/c, \quad x \equiv \hbar c q/k_B T \quad \text{and } y \equiv k/q \quad (5.14)$$

so that $uxy = \hbar c_s k/k_B T$. The first exponential factor in the integral of (5.13) comes from $1-n(\vec{q}\,')$ in (5.1); the second factor from (3.17); and the last factor from $3-\cos\theta$ in (2.9) expressed using (5.2) for $\cos\theta$. The region of the integration for the y integral in (5.13) comes from (4.7).

In some limiting cases we can calculate the integral I_E analytically (Appendix A). At low temperatures ($k_B T << \hbar c_s q$) and for a neutrino on the Fermi surface, $|E_\nu - \mu_\nu| << k_B T$, we have

$$(1/l_D)_{\text{coll}}^{\text{low}\,T} \approx (1/l)_0 \frac{9\zeta(3)}{128\pi B} \left[\frac{T}{T_F(n)}\right] \left[\frac{k_B T}{\hbar c_s q}\right]^2 \left[\frac{v_F(n)}{c_s}\right] \quad (k_B T << \hbar c_s q, \, |E_\nu - \mu_\nu| << k_B T) . \quad (5.15)$$

At low temperatures, since the number of thermally excited phonons is small the phonon absorption process gives a negligible contribution to the mean free path. At high temperatures, where $k_B T >> \hbar c_s q$, the result of the integral is given in (A9). The function I_E depends on c_s through the quantity $u \equiv c_s/c$. In the case of phonon absorption, the mean free path may be expressed in a form similar to (5.12), in which we shall denote the integral corresponding to (5.13) as I_A. It can be easily seen that there is a simple relation $I_A(c_s) = -I_E(-c_s)$, as is physically obvious. Using this relation from Eq. (A9) one has

$$I_E + I_A = \frac{2}{3} \frac{9+42u^2+5u^4}{(1-u^2)^3 ux} \frac{1}{1+e^{-x+\beta\mu_\nu}} . \quad (5.16)$$

In contrast to the low-temperature case ($k_B T << \hbar c_s q$), phonon absorption gives a contribution to the mean free path comparable to that from phonon emission, since phonons of energy $\sim \hbar c_s q$ are abundant at temperatures $k_B T >> \hbar c_s q$. The mean free path, including the contributions from both phonon emission and absorption, is

$$(1/l_D)_{\text{coll}}^{\text{high}\,T} \approx (1/l)_0 \frac{1}{32\pi B} \frac{9+42u^2+5u^4}{(1-u^2)^3} \left[\frac{T}{T_F(n)}\right] \left[\frac{v_F(n)}{c_s}\right] \quad (k_B T >> \hbar c_s q) . \quad (5.17)$$

The contributions from each of the processes calculated above are additive, and therefore one has

$$1/l_D = (1/l_D)_{\text{single pair}}^{F} + (1/l_D)_{\text{single pair}}^{\text{GT}} + (1/l_D)_{\text{coll}}^{\text{GT}} , \quad (5.18)$$

where Eqs. (5.7) and (5.8) are used for the first two terms, while only the limiting expressions (5.15) and (5.17) are available for the last term.

2. *Transport mean free path*

When momentum transport is in question the scattering rate is characterized by the transport mean free path (l_{Dt}) defined by

$$1/l_{Dt}=\frac{V}{c}\int\frac{d^3\vec{k}}{(2\pi)^3}(1+e^{-x+\beta\mu_\nu})\times[1-(q'/q)\cos\theta]W_{fi}[1-n(\vec{q}\,')] . \tag{5.19}$$

From Eq. (5.2) one has

$$1-(q'/q)\cos\theta=\tfrac{1}{2}(k/q)^2+(\omega/cq)-\tfrac{1}{2}(\omega/cq)^2 , \tag{5.20}$$

and a straightforward calculation yields

$$(1/l_{Dt})^{\rm F}_{\rm single\ pair}=\tfrac{2}{5}(1/l_D)^{\rm F}_{\rm single\ pair} , \tag{5.21}$$

$$(1/l_{Dt})^{\rm GT}_{\rm single\ pair}=\tfrac{4}{5}(1/l_D)^{\rm GT}_{\rm single\ pair} , \tag{5.22}$$

where use has been made of the same approximations as were employed to derive the expressions for the mean free paths. Since the momentum transfer (k) varies up to $2q$ in the single-pair excitations, the factor $1-(q'/q)\cos\theta$ introduces extra numerical factors in the transport mean-free-path expressions, which, however, have the same temperature dependence as the mean free paths.

As to the contribution from the collective mode, one has

$$1-(q'/q)\cos\theta=\pm uy+\tfrac{1}{2}(1-u^2)y^2 \tag{5.23}$$

for phonon emission (plus sign) and absorption (minus sign), where the notation is as in (5.14). For high temperatures ($k_BT>>\hbar c_s q$), a calculation similar to the previous one yields

$$(1/l_{Dt})^{\rm high\ T}_{\rm coll}\approx(1/l)_0\frac{1}{80\pi B}\frac{25+195u^2+55u^4-3u^6}{(1-u^2)^4}\times\left[\frac{T}{T_F(n)}\right]\left[\frac{v_F(n)}{c_s}\right] . \tag{5.24}$$

In this case we see that the transport mean free path has the same temperature dependence as the mean free path. The reason is that the variable y ranges to $O(1)$ [see (4.7)] so that the factor $1-(q'/q)\cos\theta$ is of order unity, as in the case of single-pair excitations. At low temperatures ($k_BT<<\hbar c_s q$) the situation is different. A typical phonon gives a momentum transfer $k\sim k_BT/\hbar c$ so that $uy\sim k_BT/\hbar c_s q<<1$. Thus $1-(q'/q)\cos\theta$, in fact, introduces a factor much smaller than unity. Therefore without carrying out detailed calculations one has an estimate

$$(1/l_{Dt})^{\rm low\ T}_{\rm coll}\sim(1/l)_0\times(k_BT/\hbar c_s q)^3[T/T_F(n)] , \tag{5.25}$$

whose magnitude is negligible compared with other contributions.

B. Nondegenerate neutrino case

When the neutrinos are nondegenerate no blocking factor is necessary. The neutrino mean free path (l) is simply given by

$$1/l=\frac{V}{c}\int\frac{d^3\vec{k}}{(2\pi)^3}W_{fi} . \tag{5.26}$$

Let us begin with the contribution from the Gamow-Teller part of the single-pair excitations. As has been done before, one may change the integral over the direction of $\vec{k}$ in (5.26) into an ω integral by using Eq. (5.5). At low temperatures $[k_BT<<\hbar q v_F(n)]$ the integral has the form

$$\int_0^{2q/(1+u_F)}dk\int_{-\infty}^{kv_F}d\omega+\int_{2q/(1+u_F)}^{2q}dk\int_{-\infty}^{c(2q-k)}d\omega , \tag{5.27}$$

corresponding to the ω-k domain shown in Fig. 4(b). In (5.27) the lower boundary of the ω integral is extended to $-\infty$ since the region $\omega<<-k_BT/\hbar$ gives a negligible contribution to the integral. Using Eqs. (2.9), (3.22), (5.2), and (5.27) in (5.26), one finds

$$(1/l)^{\rm GT\ low T}_{\rm single\ pair}=\frac{G^2N(0)C_A{}^2}{8\pi\hbar V}\times\frac{1}{(1+F_0^a)^2}\frac{q^3}{v_F(n)x^2}I_L(x), \tag{5.28}$$

where

$$I_L(x)\equiv\left[\int_0^{2/(1+u_F)}dy\int_{-\infty}^{u_Fxy}dz+\int_{2/(1+u_F)}^{2}dy\int_{-\infty}^{2x(1-y/2)}dz\right]\frac{z}{1-e^{-z}}\left[1-\frac{z}{x}-\frac{1}{4}\left[\frac{z}{x}\right]^2+\tfrac{1}{4}y^2\right] . \tag{5.29}$$

In (5.28) and (5.29) the dimensionless variables are defined by $x \equiv \hbar cq/k_B T$, $y \equiv k/q$, $z \equiv \hbar\omega/k_B T$, and $u_F \equiv v_F(n)/c$. The calculation of (5.29) is given in Appendix B, and one finds

$$(1/l)^{\mathrm{GT\ low}T}_{\mathrm{single\ pair}} \simeq (1/l)_0 \frac{C_A{}^2}{5} \frac{1}{(1+F_0^a)^2} \times \frac{1-u_F/8}{(1+u_F)^3} \left[\frac{q}{k_F(n)}\right] . \quad (5.30)$$

Similarly, the Fermi part of the single-pair excitations gives at low temperatures $[k_B T << \hbar q v_F(n)]$

$$(1/l)^{\mathrm{F\ low}\ T}_{\mathrm{single\ pair}} \simeq (1/l)_0 \frac{1}{20} \frac{1}{(1+F_0^s)^2} \times \frac{1+u_F/2}{(1+u_F)^3} \left[\frac{q}{k_F(n)}\right] . \quad (5.31)$$

At high temperatures $[k_B T >> \hbar q v_F(n)]$, the region of integration becomes

$$\int_0^{2q/(1+u_F)} dk \int_{-kv_F}^{kv_F} d\omega + \int_{2q/(1+u_F)}^{2q/(1-u_F)} dk \int_{-kv_F}^{2cq-ck} d\omega . \quad (5.32)$$

A straightforward calculation yields

$$(1/l)^{\mathrm{GT\ high}T}_{\mathrm{single\ pair}} = (1/l)_0 \frac{C_A{}^2}{(1+F_0^a)^2} \frac{9}{32} \times \frac{1+\frac{5}{9}u_F{}^2}{(1-u_F{}^2)^2} \left[\frac{T}{T_F(n)}\right] \quad (5.33)$$

from the Gamow-Teller part, and

$$(1/l)^{\mathrm{F\ high}T}_{\mathrm{single\ pair}} = (1/l)_0 \frac{1}{(1+F_0^s)^2} \frac{3}{32} \times \frac{1-\frac{1}{3}u_F{}^2}{(1-u_F{}^2)^2} \left[\frac{T}{T_F(n)}\right] \quad (5.34)$$

from the Fermi part of the single-pair excitations.

Next we calculate the contributions from collective excitations. For phonon emission ($\omega = c_s k$), Eqs. (2.9), (3.17), (5.2), and (5.27) in (5.26) yield

$$(1/l)^{\mathrm{em}}_{\mathrm{coll}} = \frac{G^2 N(0) C_A{}^2 v_F(n) q^3}{8\pi^2 V \hbar B c^2} I_{E1} , \quad (5.35)$$

where

$$I_{E1}(x) \equiv \int_0^{2/(1+u)} dy \frac{y^2}{1-e^{-uxy}} \times [1-uy+\tfrac{1}{4}(1-u^2)y^2] , \quad (5.36)$$

and $u \equiv c_s/c$. The range of the y integral $[0 \le k \le 2q/(1+u)]$ in Eq. (5.36) comes from the ω-k domain allowed for the upper phonon branch ($\omega = c_s k$) in Figs. 4(a) or 4(b). In the low-temperature case ($k_B T << \hbar c_s q$)

$$I_{E1}(x) \simeq \int_0^{2/(1+u)} dy [y^2 - uy^3 + \tfrac{1}{4}(1-u^2)y^4] = \frac{64}{15} \frac{1-11u/16}{(1+u)^4} . \quad (5.37)$$

Since the phonon absorption process ($\omega < 0$) gives a negligible contribution at these temperatures Eqs. (5.35) and (5.37) yield a mean free path

$$(1/l)^{\mathrm{low}\ T}_{\mathrm{coll}} \simeq (1/l)_0 \frac{2C_A{}^2}{5\pi B} \frac{1-11u/16}{(1+u)^4} \times \left[\frac{q}{k_F(n)}\right] . \quad (5.38)$$

At high temperatures ($k_B T >> \hbar c_s q$) the calculation turns out to be essentially the same as in the degenerate neutrino case. The result is

$$(1/l)^{\mathrm{high}\ T}_{\mathrm{coll}} \simeq (1/l)_0 \frac{9C_A{}^2}{32\pi B} \frac{u_F(1+\frac{14}{3}u^2+\frac{5}{9}u^4)}{u(1-u^2)^3} \left[\frac{T}{T_F(n)}\right] . \quad (5.39)$$

VI. DISCUSSIONS AND CONCLUSION

In this section we first explain the physical meaning of the various mean-free-path expressions obtained in the previous section in greater detail. Next, we discuss Fermi-liquid effects, and compare some of our results with previous calculations.

A. Physical explanation of the magnitude of the mean free paths

Let us explain the physical meaning of those mean free paths obtained in the previous section. The fundamental quantity is $(1/l)_0$ which characterizes the mean free path of a neutrino of energy E_ν in noninteracting nondegenerate neutrons of density n. When the neutrons are degenerate with Fermi temperature $T_F(n)$ the mean free path contains additional factors. As in the previous section we assume that the neutrino momentum is smaller than the neutron Fermi momentum $\hbar q < p_F(n)$.

Arguments based on the available space may be used to explain the factors in those mean-free-path expressions, Eqs. (5.7), (5.31), and (5.34), or Eqs.

(5.8), (5.30), and (5.33) which come from single-pair excitations. We first discuss the nondegenerate neutrino case. In the two-body collision of a neutrino and a neutron, the energy transfer is characterized by $\hbar q v_F(n)$. The amount of phase space available to this scattering depends on whether this energy transfer is greater or smaller than $k_B T$. When the system temperature is high such that $\hbar q v_F(n) << k_B T$, a fluctuation in the neutron energy of the order of $k_B T$ is enough to supply this characteristic energy transfer. Thus the energy and momentum transfers exhaust the region in the ω-k plane which is kinematically allowed. The reaction rate is proportional to the number of neutrons which can participate in the scattering. This is $\sim (dn/dE)\Delta E = N(0)\Delta E$, where dn/dE is the density of states. In the present case since $N(0) \sim 1/k_B T_F(n)$ and $\Delta E \sim k_B T$, one has $N(0)\Delta E \sim T/T_F(n)$, which explains the factor $T/T_F(n)$ appearing in Eq. (5.34) [or (5.33)].[19] This is a consequence of the fact that only neutrons whose momenta lie within $\sim k_B T/c$ of the Fermi surface can participate in the process. On the other hand, when the temperature is low, such that $k_B T << \hbar q v_F(n)$, the scattering occurs mainly by exciting the neutrons by an amount $\sim \hbar q v_F(n)$. The neutrinos can, indeed, absorb energy of order $k_B T$ from the neutron system ($-k_B T/\hbar \lesssim \omega < 0$), but this process has negligible phase space compared with that for positive-energy transfer $\hbar\omega \sim \hbar q v_F(n)$. Thus, the phase-space factor is $N(0)\Delta E \sim [1/T_F(n)]\hbar q v_F(n) \sim q/k_F(n)$, which explains the factor in Eq. (5.31) [or (5.30)]. Therefore, the difference arises whether the energy transfer associated with scattering is thermal $\sim k_B T$ or nonthermal $\sim \hbar q v_F(n) (>> k_B T)$.

When the neutrinos are degenerate ($\mu_\nu >> k_B T$, where μ_ν is the neutrino chemical potential) the energy transfer is restricted to the range $-k_B T/\hbar \lesssim \omega \lesssim k_B T/\hbar$ [see Fig. 4(c)]. As far as the neutrons are concerned the available phase space gives a factor $q/k_F(n)$, as in the low-temperature nondegenerate neutrino case, since the same condition $k_B T << \hbar q v_F(n)$ (for $E_\nu = \hbar c q \sim \mu_\nu$) is satisfied. The neutrino phase-space integral has the form $\int d^3\vec{k} \sim \int k^2 dk \int d(\cos\theta) \sim \int k\, dk \int d\omega$. When $k_B T << \hbar q v_F(n)$, the range of the k integral is $0 \lesssim k \lesssim 2q$ in the cases of both nondegenerate [Fig. 4(b)] and degenerate [Fig. 4(c)] neutrinos. However, the range of the ω integral differs. For nondegenerate neutrinos it is $-k_B T/\hbar \lesssim \omega \lesssim q v_F(n)$ [Fig. 4(b)], while for degenerate neutrinos it is $-k_B T/\hbar \lesssim \omega \lesssim k_B T/\hbar$ [Fig. 4(c)]. In addition, since the scattering rate [Eq. (2.9)] is proportional to $S(\vec{k},\omega)$ [Eq. (3.21)] and $\mathscr{S}(\vec{k},\omega)$ [Eq. (3.22)], it has one power of ω. Therefore, the phase-space integral with respect to ω is smaller for degenerate neutrinos by a factor $\sim [k_B T/\hbar q v_F(n)]^2 (<< 1)$. This is the origin of the difference between Eqs. (5.7) and (5.31) or between Eqs. (5.8) and (5.30) of a factor $\sim [k_B T/\hbar q v_F(n)]^2$.

Let us now turn to the collective excitations. For nondegenerate neutrinos one sees that the mean free paths due to scattering from collective excitations [Eqs. (5.39) and (5.38)] have the same factors as those from single-pair excitations {Eqs. (5.34) [or (5.33)] and (5.31) [or (5.30)]} in both the high-temperature [$k_B T >> \hbar q v_F(n)$] and low-temperature [$k_B T << \hbar q v_F(n)$] cases. This is because the energy and momentum transfers lie essentially in the same range ($-k_B T/\hbar \lesssim \omega \lesssim 2c_s q$ and $0 < k \lesssim 2q$ for low temperatures; and $-2c_s q \lesssim \omega \lesssim 2c_s q$ and $0 < k \lesssim 2q$ for high temperatures) for both single-pair excitations and collective excitations.

Let us next compare the mean free paths in the following two cases: (1) The collective-mode contribution at low temperatures ($k_B T << u E_\nu$) in the nondegenerate neutrino case [Eq. (5.38)], and (2) the collective-mode contribution at low temperatures in the degenerate neutrino case [Eq. (5.15)]. The scattering rate is proportional to the phase space available to the phonons $\int d^3k \propto \langle\omega\rangle^3$, where $\langle\omega\rangle$ is the characteristic phonon frequency. In case (1), the phonons emitted spontaneously have a frequency $\langle\omega\rangle \sim c_s q$, while those which can be absorbed have $\langle\omega\rangle \sim k_B T/\hbar << c_s q$. Therefore, the spontaneous emission process dominates the absorption process in terms of phase space, and the reaction rate is proportional to $(\hbar c_s q)^3$. Since this phase-space factor comes from spontaneous emission of nonthermal phonons, it has no direct relationship to the actual phonon number.

In case (2), the phonons which can be emitted or absorbed have to have a frequency $\lesssim k_B T/\hbar$; in other words, only the thermal phonons are of importance. Therefore, the phase-space factor is proportional to $(k_B T)^3$.

These considerations naturally explain the relative difference between the mean free paths, Eqs. (5.15) and (5.38), by a factor $(\hbar c_s q/k_B T)^3$, in terms of the difference in phase space. In case (2), an alternative way of understanding the factor $(k_B T)^3$ is in terms of the phonon number, which has more direct physical meaning. In this case spontaneous emission of nonthermal phonons is strongly suppressed, so that the reaction rate is simply proportional to the number of phonons present.

One may use the Debye model to estimate the phonon number. In solids the maximum phonon frequency is determined by $\omega_D = c_s(6\pi^2 n_i)^{1/3}$, where c_s is the sound velocity and n_i is the number density of ions. In neutron liquids phonons exist up to the critical wave number k_L, where the phonon branch ($\omega = c_s k$) merges into the single-pair excitation region ($-kv_F + \hbar k^2/2m \leq \omega \leq kv_F + \hbar k^2/2m$). For $k > k_L$ a phonon can decay into a single-pair excitation; in other words it is Landau damped. Thus, in this case $\omega_D \sim c_s k_L$, where $k_L = \delta p_F(n)/\hbar$ and $\delta \equiv [c_s - v_F(n)]/v_F(n)$. Then, as in the case of solids, the phonon number at low temperatures ($T << T_D \equiv \hbar\omega_D/k_B$) is

$$N_{ph} = \frac{9N}{\omega_D{}^3}\int_0^{\omega_D} d\omega \frac{\omega^2}{e^{\hbar\omega/k_B T}-1} \sim \frac{9N}{\omega_D{}^3}\int_0^{\infty} d\omega \frac{\omega^2}{e^{\hbar\omega/k_B T}-1} = 18\zeta(3)N(T/T_D)^3 . \tag{6.1}$$

This is the reason the Riemann ζ function appears in Eq. (5.15).

Finally let us compare the mean free paths from collective excitations at high temperatures ($k_B T >> \hbar c_s q$) in the degenerate and nondegenerate neutrino cases [Eqs. (5.17) and (5.39)]. In both cases the energy and momentum transfers lie in the same range ($-2c_s q \lesssim \omega \lesssim 2c_s q$ and $0 < k \lesssim 2q$), so that the phonon phase-space factors are the same, which gives the same expression for the mean free path for the two cases.

An additional factor $k_B T/\hbar c_s q$ in Eq. (5.15) [degenerate neutrino case, collective excitations $k_B T << \hbar c_s q$) relative to Eq. (5.7) [or (5.8)] (degenerate neutrino case, single-pair excitations) for $E_\nu \sim \mu_\nu$ may be understood in a similar way: the energy transfer is the same for both cases ($-k_B T/\hbar \lesssim \omega \lesssim k_B T/\hbar$), while the momentum transfer is $0 < k \lesssim 2q$ for single-pair excitations and $0 < k \lesssim k_B T/\hbar c_s$ for collective excitations, which explains the relative factor $k_B T/\hbar c_s q$.

B. Effects of neutron-neutron interactions (Fermi-liquid effects)

In this subsection we discuss how Fermi-liquid effects modify the mean free paths, and compare the magnitude of the contributions from the Fermi part and the Gamow-Teller part of the single-pair excitations and the collective excitations. We use the Fermi-liquid parameters listed in Table I to compare the mean free paths obtained in Sec. V.

The consequences of the Fermi-liquid effects may be summarized as follow: Since F_0^a is large (~ 0.97), the Fermi-liquid effects are especially significant for the dynamic form factor for the spin density. As a result, the single-pair excitation spectrum is reduced by a factor ~ 4 and the collective mode (spin-zero sound) appears. In the case of nondegenerate neutrinos the temperature dependences of the contribution to the mean free paths coming from the collective mode [Eqs. (5.38) and (5.39)] are the same as those of the contributions from the single-pair excitations [Eqs. (5.30), (5.31); and (5.33) and (5.34)]. Thus the reduction of the single-pair spin-density excitation is compensated by the collective mode, which results in a relatively minor change in the mean free path. More specifically, if we define

$$R \equiv \frac{(1/l)\text{with Fermi-liquid effects}}{(1/l)\text{without Fermi-liquid effects}} , \tag{6.2}$$

we have $R \sim 0.6$ at low temperatures,

$$k_B T << \hbar q v_F(n) \tag{6.3}$$

and $R \sim 1.7$ at high temperatures,

$$k_B T >> \hbar q v_F(n) . \tag{6.4}$$

In contrast to this, in the case of degenerate neutrinos the temperature dependence of the mean free paths is such that the contribution from the collective mode is negligible in general $[(1/l_D)_{\text{single pair}} >> (1/l_D)_{\text{coll}}]$. We find for the ratio of the mean free paths (R_D) defined similarly to (6.2) for the degenerate neutrino case

$$R_D \sim 0.51 \sim \tfrac{1}{2} \tag{6.5}$$

and the ratio of the transport mean free paths (R_{Dt}),

$$R_{Dt} \sim 0.34 \sim \tfrac{1}{3} . \tag{6.6}$$

Therefore, the inclusion of the Fermi-liquid effects increases the mean free path to some extent.

The contribution from the collective mode is important in the case when the neutrinos are nondegenerate. The ratio of the contribution from the collective mode to that from single-pair excitations is

$$(1/l)_{\rm coll}^{\rm low\ T}/(1/l)_{\rm single\ pair}^{\rm low\ T}\sim 0.58 \qquad (6.7)$$

at low temperatures ($k_B T << \hbar c_s q$), and

$$(1/l)_{\rm coll}^{\rm high\ T}/(1/l)_{\rm single\ pair}^{\rm high\ T}\sim 3.1 \qquad (6.8)$$

at high temperatures ($k_B T >> \hbar c_s q$). The collective-mode contribution is negligible in the case of degenerate neutrinos.

In the calculations discussed above we neglected quasiparticle collisions. The collision rate $1/\tau_{\rm coll}$ is of order $k_B T^2/\hbar T_F(n)$,[10] and zero sound should suffer little damping provided the phonon frequency is large compared with $1/\tau_{\rm coll}$. Since the phonon frequencies of importance in our calculations are of order $k_B T/\hbar$ or greater, damping is unimportant since $T << T_F(n)$. Collisions will cause a first-sound mode to exist at long wavelengths, irrespective of the values of the Landau parameters, but at low temperature the range of wave numbers for which this mode exists is small.

Let us finally make some remarks on Sutherland and Flowers's paper.[2] Our calculations support their conclusion that scattering from phonons can be significant. However, from our work it is clear that the phonons of interest for neutron matter at about nuclear matter density are spin-zero-sound phonons, rather than ordinary density-zero-sound phonons. Sutherland and Flowers took into account only the density fluctuation channel (Fermi transitions) and did not consider the spin-density fluctuation channel (Gamow-Teller transitions). We note that they did not make specific assumptions about the properties of the matter, but assumed that the density fluctuation mode exhausts the f-sum rule, and calculated the dynamic form factor from a hydrodynamic model. Our calculations show that for degenerate neutrons the f-sum rule is not exhausted by the collective mode (in fact, for the density channel a collective mode is unlikely to exist at all at nuclear densities), and the hydrodynamic model is a poor approximation.

One interesting conclusion from our calculations is that in general one must consider scattering from both quasiparticles and collective modes. When the quasiparticle interaction is such as to give rise to a collective mode, the scattering from single-pair excitations is reduced. In fact, for degenerate neutrinos the total scattering is reduced if the interaction is such as to produce a collective mode, since the collective mode itself gives little scattering while the scattering from single-pair excitations decreases.

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APPENDIX A: EVALUATION OF EQ. (5.13)

In this appendix we evaluate the integral I_E [Eq. (5.13)] in the following limiting cases:

(1) low-temperature limit $k_B T << \hbar c_s q$ ($ux >> 1$) and

(2) high-temperature limit $k_B T >> \hbar c_s q$ ($ux << 1$). (In the latter case, it is to be understood that the temperature is still much lower than the Fermi energy [$T << T_F(n)$].)

(1) Low-temperature limit $k_B T << \hbar c_s q$ ($ux >> 1$). Introducing a dummy variable v along with a δ function in the integral representation, one has

$$\begin{aligned} I_E &= \int_0^{2/(1+u)} dy \int_{-\infty}^{\infty} dv\, \delta(v+x-uxy-\beta\mu_\nu)\frac{1}{1+e^v}\frac{1}{1-e^{-uxy}}f(y) \\ &= \int_0^{2/(1+u)} dy \int_{-\infty}^{\infty} dv \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i(v+x-uxy-\beta\mu_\nu)t} \cdots \\ &= \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i(x-\beta\mu_\nu)t} I_1 I_2 , \end{aligned} \qquad (A1)$$

where

$$I_1 \equiv \int_{-\infty}^{\infty} dv \frac{e^{iv(t-i0)}}{1+e^v} , \qquad (A2)$$

$$I_2 \equiv \int_0^{2/(1+u)} dy \frac{e^{-iuxyt}}{1-e^{-uxy}} f(y) , \qquad (A3)$$

and

$$f(y) = y^2 - uy^3 + \tfrac{1}{4}(1-u^2)y^4 . \qquad (A4)$$

The integrand of (A2) has poles at $v=(2n+1)\pi i$ with residues $-e^{-(2n+1)\pi t}$ ($n=0,\pm1,\ldots$). The contour of integration with respect to v may be modified such that the poles in the upper or lower halves of the complex v plane are encircled according to whether $t>0$ or $t<0$. Then we have

$$I_1 = -2\pi i e^{\pi t}/(e^{2\pi t}-1) . \quad \text{(A5)}$$

Since $ux >> 1$ in the present case, we may omit the function e^{-uxy} in the integrand of (A3). Then the integral is straightforward and to leading order in ux

$$I_2 \sim \frac{i}{ux}\left[\frac{2}{(ux)^2 t^3}+\frac{8(1-u)}{(1+u)^3}\frac{e^{-2iuxt/(1+u)}}{t}\right] . \quad \text{(A6)}$$

The function I_1 [Eq. (A5)] has poles at $t=ni$ with residues $-i(-1)^n$ ($n=0,\pm 1,\ldots$). Modifying the t-integration contour in (A1) in order to encircle some of these poles, we have

$$I_E \approx \begin{cases} \frac{3}{2}\frac{\zeta(3)}{(ux)^3} & \text{for } |x-\beta\mu_\nu| << 1 \quad \text{(A7)} \\ \frac{2}{(ux)^3}e^{-|x-\beta\mu_\nu|} & \text{for } |x-\beta\mu_\nu| \gtrsim 1 , \quad \text{(A8)} \end{cases}$$

where $\zeta(3)=1.20\ldots$ is the Riemann ζ function.

(2) High-temperature limit $k_B T >> \hbar c_s q$ ($ux << 1$). In this case we approximate

$$\frac{1}{1+e^{-x+uxy+\beta\mu_\nu}} \simeq \frac{1}{1+e^{-x+\beta\mu_\nu}} ,$$

$$\frac{1}{1-e^{-uxy}} \simeq \frac{1}{uxy}$$

in Eq. (5.13) and find

$$I_E \sim \frac{9-5u}{3(1+u)^3}\frac{1}{ux}\frac{1}{1+e^{-x+\beta\mu_\nu}} . \quad \text{(A9)}$$

APPENDIX B: CALCULATION OF EQ. (5.29)

Dividing the z integral into four parts, $-\infty<z<0$, $0<z<u_F xy$, $-\infty<z<0$, and $0<z<2x(1-y/2)$ in (5.29), one has

$$\begin{aligned} I_L(x) &= \int_0^2 dy \int_0^\infty dz \frac{z}{e^z-1}\left[1+\frac{z}{x}-\frac{1}{4}\left[\frac{z}{x}\right]^2+\frac{1}{4}y^2\right] \\ &\quad + \left[\int_0^{2/(1+u_F)} dy \int_0^{u_F xy} dz + \int_{2/(1+u_F)}^{2} dy \int_0^{2x(1-y/2)} dz\right] \\ &\quad \times \left[\left[1+\tfrac{1}{4}y^2\right]z - \frac{z^2}{x} - \frac{1}{4}\frac{z^3}{x^2}\right] \\ &\equiv I_{L1}(x)+I_{L2}(x)+I_{L3}(x) . \end{aligned} \quad \text{(B1)}$$

These integrals are straightforward and one finds

$$I_{L2}(x) = \frac{2x^2 u_F^2}{15(1+u_F)^5}(16+10u_F-3u_F^2) , \quad \text{(B2)}$$

$$I_{L3}(x) = \frac{2x^2 u_F^3}{15(1+u_F)^5}(20+15u_F-2u_F^2) , \quad \text{(B3)}$$

and for $x >> 1$, which is of particular interest to us,

$$I_{L1}(x) \sim \tfrac{4}{9}\pi^2 + O\left[\frac{1}{x}\right] \quad (x >> 1) , \quad \text{(B4)}$$

which turns out to be negligible compared with $I_{L2}(x)$ and $I_{L3}(x)$.

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[15]This dispersion relation is understood to be valid up to the value of k where (4.2) crosses the single-pair excitation region (4.5) (i.e., within the range of k where the collective mode is not Landau damped).
[16]Since the general argument in this section holds irrespective of whether the collective mode is the first sound or (spin-) zero sound, we shall often use the terminology "a phonon" to denote a quantum of the collective mode.
[17]It is to be noted that the system temperature is still assumed to be low compared with the neutron Fermi temperature $T_F(n)$, as well as the neutrino Fermi temperature $T_F(\nu)$ (when the neutrinos are degenerate).
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[19]At high temperatures $[k_B T >> \hbar q v_F(n)]$ characteristic energy transfer is $\hbar\omega \sim \hbar q v_F(n)$ [Fig. 4(a)]. Therefore, the Bose function in the mean-free-path expression takes on a value $n(\omega) = [\exp(\hbar\omega / k_B T) - 1]^{-1} \simeq k_B T / \hbar q v_F(n)$, which gives rise to one power of $k_B T$. This is what is meant by $\Delta E \sim k_B T$. Note that the induced process $[\propto n(\omega)]$ completely dominates the spontaneous process $(\propto 1)$ in this case.