Electromagnetic form factor of the pion: Vector mesons or quarks?

C. A. Dominguez
Department of Physics, Texas A & M University, College Station, Texas 77843
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An updated analysis of the pion form-factor data in the spacelike region up to $q^2=-10$ GeV^2 is performed using the extended vector-meson-dominance (EVMD) model, with parameters fixed by the dual model, as well as asymptotic quantum-chromodynamics (QCD) expressions. EVMD provides a good fit to the data and at the same time it predicts a deviation from universality in very good agreement with experiment. Perturbative QCD results to first order in α_s do very poorly while the nonperturbative QCD prediction of Pagels and Stokar gives an excellent fit to the data in the entire region $1\leq -q^2 \leq 10$ GeV^2.

It has become gradually clear that quantum chromodynamics[1] (QCD) provides an economical and successful description of the electromagnetic structure of hadrons.[2,3] Nevertheless, the traditional approach based on vector-meson dominance (VMD) or its extended version (EVMD) still provides a useful complementary framework[2,3] in the large-distance domain where perturbative QCD breaks down. This is particularly true of the elastic electromagnetic form factors of hadrons where EVMD has been most successful in predicting their q^2 dependence both in the spacelike and timelike regions.[3]

The purpose of this paper is to update the analysis of the pion form factor $F_\pi(q^2)$ in the spacelike region using the latest data[4] extending up to $q^2=-10$ GeV^2. Previous analyses[5,6] of $F_\pi(q^2)$ based on EVMD only covered the region below $q^2=-4$ GeV^2 and used an uncorrected version of the data base.[4] These analyses,[5,6] however, were quite successful in the timelike region where models based on EVMD receive their ultimate test.

Other related issues of current interest that will be considered here are (i) the breakdown of universality as evidenced by the measured departure of $g_{\rho\pi\pi}/\gamma_\rho$ from unity and (ii) the possibility that the pion form-factor data may be suggesting important nonperturbative QCD effects.[7,8]

In the framework of EVMD the pion form factor is built from the contribution of the ρ meson together with all its radial excitations,[2] i.e.,

$$F_\pi(q^2)=\sum_{n=0}^{N}\frac{M_{\rho_n}{}^2}{\gamma_{\rho_n}}\frac{g_{\rho_n\pi\pi}}{M_{\rho_n}{}^2-q^2}\ , \tag{1}$$

where γ_{ρ_n} is the electromagnetic coupling of the photon to ρ_n and $g_{\rho_n\pi\pi}$ is the strong $\rho_n\pi\pi$ coupling constant normally assumed to have no additional q^2 dependence. Experimentally two radial excitations of the ρ meson have been observed, viz., the $\rho'(1250)$ (Ref. 9) and the $\rho''(1550)$ (Ref. 10), with masses in good agreement with the dual-model prediction: $M_{\rho_n}{}^2=M_\rho{}^2+n/\alpha'$, where

$$\alpha'=1/2M_\rho{}^2=0.83\ \text{GeV}^{-2}$$

is the universal Regge slope. Using the normalization of $F_\pi(q^2)$ at $q^2=0$ in Eq. (1) one finds that EVMD leads trivially to a breakdown of universality. From the measured[10] decay rates of $\rho\to\pi\pi$ and $\rho\to e^+e^-$ one obtains $g_{\rho\pi\pi}{}^2/4\pi=3.01\pm0.06$ and $\gamma_\rho{}^2/4\pi=2.03\pm0.06$, respectively, giving

$$\left(\frac{g_{\rho\pi\pi}}{\gamma_\rho}\right)_{\text{exp}}=1.22\pm0.02\ . \tag{2}$$

This suggests that a successful model based on EVMD should not only account for the data on $F_\pi(q^2)$ but also reproduce Eq. (2).

An attractive possibility is to fix the couplings and masses in Eq. (1) using the dual model, in which case the form factor becomes a ratio of gamma functions with a single free parameter (in the zero-width approximation) that controls the asymptotic power behavior of the form factor. This model was first introduced more than a decade ago[11] and has been quite successful in accounting for the data on $F_\pi(q^2)$ below $q^2=-2$ GeV^2 as well as in the timelike region.[5] It has also been shown to describe quite well the electromagnetic form factors of the nucleon,[12] the $\Delta(1236)$ (Ref. 13), the kaon,[14] and purely hadronic vertex functions.[15] The resulting expression for $F_\pi(q^2)$ in this model may be written as

$$F_\pi(q^2)=\frac{M_\rho^2}{\gamma_\rho^2}\left[\frac{g_{\rho\pi\pi}}{M_\rho^2-q^2}\right]F_{\rho\pi\pi}(q^2)\ , \tag{3}$$

where

$$F_{\rho\pi\pi}(q^2)=\Gamma(\beta-1)\frac{\Gamma(1-\alpha'(q^2-M_\rho^2))}{\Gamma(\beta-1-\alpha'(q^2-M_\rho^2))} \tag{4}$$

is the $\rho\pi\pi$ hadronic form factor for an off-mass-shell ρ meson, normalized to $F_{\rho\pi\pi}(M_\rho^2)=1$. In the asymptotic limit $q^2\to-\infty$, $F_\pi(q^2)\approx(-\alpha' q^2)^{1-\beta}$ and $\beta=2$ corresponds to ρ dominance (VMD). The unitarization of Eqs. (3) and (4) has been discussed in Ref. 5 but it has little effect in the space-like region that is of interest here.

A least-squared fit to the data on $F_\pi(q^2)$ as corrected in Ref. 4 has been performed using Eq. (3) with the results shown in Table I and illustrated in Figs. 1 and 2. The single-ρ-dominance ($\beta=2$) result is shown for comparison together with a simple monopole fit of the form

$$F_\pi(q^2)=(1-q^2/M_V^2)^{-1}\ . \tag{5}$$

It should be clear from the results that Eq. (3) provides a much better fit to the data than single-ρ dominance or Eq. (5). On the other hand, using Eq. (3) at $q^2=0$ one predicts

$$\frac{g_{\rho\pi\pi}}{\gamma_\rho}=2\frac{\Gamma(\beta-\frac{1}{2})}{\Gamma(\frac{1}{2})\Gamma(\beta-1)}=1.2\ , \tag{6}$$

in good agreement with the experimental result, Eq. (2). Finally, the electromagnetic radius of the pion predicted by Eq. (3) turns out to be $\langle r_\pi^2\rangle^{1/2}=0.66$ fm, in reasonable agreement with various experimental extractions[16] in the range 0.6–0.7 fm.

Turning now to the QCD description of $F_\pi(q^2)$, if one assumes that the asymptotic behavior of the form factor is governed entirely by short-distance dynamics then to first order in the QCD running coupling constant $\alpha_s(Q^2)$, one has[17]

$$F_\pi(Q^2)=\left[\frac{1}{Q^2}\right]16\pi f_\pi^2\alpha_s(Q^2)\ , \tag{7}$$

with

$$\alpha_s(Q^2)=\frac{12\pi}{(33-2n_f)\ln(Q^2/\Lambda^2)}\ , \tag{8}$$

where $f_\pi=93.24$ MeV is the pion decay constant $Q^2=-q^2$ and n_f is the number of quark flavors. This point of view is not widely shared[7,8] and in fact it has been argued that the first term in the asymptotic expansion of $F_\pi(q^2)$ is of order zero in α_s and given by[8]

$$F_\pi(Q^2)=\left[\frac{1}{Q^2}\right]\frac{4M_D^4\ln 2}{\sqrt{3}\sqrt[3]{2}\pi f_\pi^2}\ , \tag{9}$$

where M_D is the dynamical quark mass estimated to be[8,18] $M_D\simeq 244-300$ MeV, with a large error.

The data base for $F_\pi(q^2)$ contains a total of 21 data points with 16 data points above $Q^2=1$ GeV^2,

TABLE I. Results of the various fits to the data on $F_\pi(q^2)$. $Q_{\min}^2$ is the minimum value of $Q^2=-q^2$ considered in the fits and χ_F^2 is the χ^2 per degree of freedom.

Model	$Q_{\min}^2$ (GeV^2)	Parameter	χ_F^2
EVMD [Eq. (3)]	All data	$\beta=2.33$	1.36
ρ dominance	All data	$\beta=2$(Fixed)	10.90
Monopole [Eq. (5)]	All data	$M_V=679.6$ MeV	1.94
Perturbative QCD [Eq. (7)]	1	$\Lambda=433$ MeV	3.42
	2	$\Lambda=576$ MeV	1.77
Nonperturbative QCD [Eq. (9)]	1	$M_D=289$ MeV	1.28
	2	$M_D=292$ MeV	0.92
QCD expansion [Eqs. (7) and (9)]	1	$\Lambda=0.05$ MeV; $M_D=282$ MeV	1.38
	2	$\Lambda=0.40$ MeV; $M_D=283$ MeV	1.11

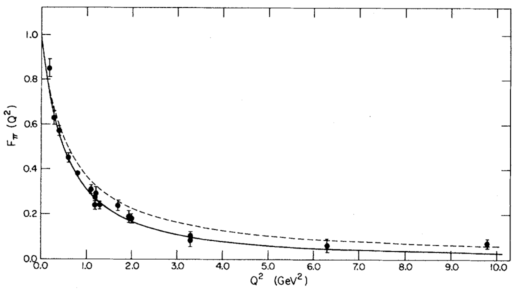

FIG. 1. Solid line is the best fit to the data base (as corrected in Ref. 4) using EVMD together with the dual model, Eq. (3). Broken line is the parameter-free prediction of ρ dominance (see Table I).

7 above 2 GeV^2, 4 above 3 GeV^2, and only 2 above 6 GeV^2. It is obvious then that a comparison of the above asymptotic QCD expressions with these data is not going to be entirely fair, although it is common practice to make such comparisons for momentum transfers as low as $Q^2=1$ GeV.[2] To minimize this to some extent, two separate fits were made to all the data above $Q^2=1$ GeV^2 and then above $Q^2 \simeq 2$ GeV^2. The free parameters in these fits were Λ for Eq. (8), M_D for Eq. (9), and both Λ and M_D in a combined fit. The results are listed in Table I and illustrated in Fig. 3, and they

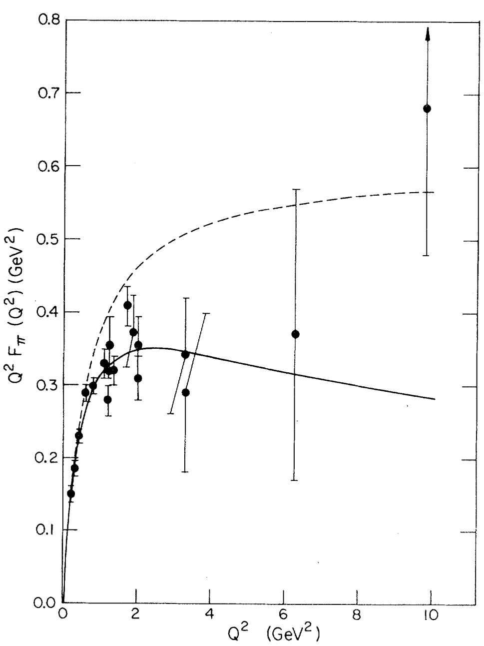

FIG. 2. Plot of $Q^2F_\pi(Q^2)$ versus Q^2. Solid and broken lines have the same meaning as in Fig. 1.

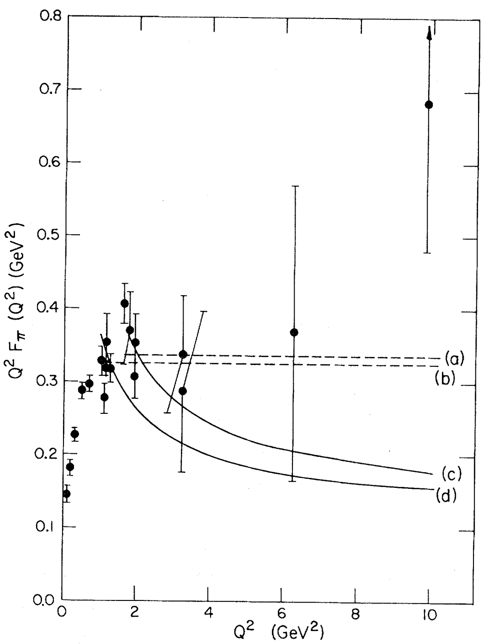

FIG. 3. Broken lines (a) and (b) are the best fits to all the data above $Q^2 \simeq 2$ GeV^2 and above $Q^2=1$ GeV^2, respectively, using the nonperturbative QCD result, Eq. (9). Solid lines (c) and (d) are the corresponding best fits using the perturbative QCD result, Eq. (7) (see Table I).

show that perturbative QCD provides a very poor fit to the data while Eq. (9) does remarkably well and also leads to a value of M_D in line with other estimates.[8,18] This conclusion is evidenced again in the results of the simultaneous fit using Eqs. (7) and (9) together, viz., Λ comes out very close to zero while M_D remains basically unchanged. The number of flavors in Eq. (8) was fixed to $n_f=3$ but an increase in n_f does not improve the quality of the fits.

It must be pointed out, for purposes of comparison, that if instead of searching for the value of Λ that gives the best fit, one fixes it in the range suggested by other processes ($\Lambda \simeq 100-300$ MeV) then the resulting $\chi_F{}^2$ increases considerably. For example, for $\Lambda=100$ MeV one finds $\chi_F{}^2=62$ and $\chi_F{}^2=22$ for $Q_{\min}{}^2=1$ and 2 GeV^2, respectively. Finally, mass corrections (which in principle should be important in nonasymptotic regions) were incorporated into Eq. (7) as regulators of the leading $(1/Q^2)$ term as well as of the nonleading terms.[17] Taking the mass as an additional free parameter, new fits to the data were carried out but their quality was slightly worse than with no mass corrections, with $\chi_F{}^2$ increasing typically to

$\chi_F^2 \simeq 2-4$.

In conclusion, the EVMD expression with couplings and masses fixed by the dual model, Eq. (3), provides a reasonable description of the pion form factor up to $q^2=-10$ GeV^2 and at the same time it predicts, at $q^2=0$, the corrrect deviation from universality. On the other hand, although there is little reason to expect the asymptotic QCD results to account for the present data, Eq. (9) leads to a remarkably good fit in the entire region 1 GeV$^2 \leq Q^2 \leq 10$ GeV. With all due reservations, this result may be taken as an indication of the importance on nonperturbative QCD effects in the electromagnetic structure of the pion. It also illustrates the complementary nature of the description of the pion form factor in terms of vector mesons (EVMD) and of quarks (QCD).

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