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H meson and deviations from ideal mixing

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A prediction based on dual topological unitarization that the H meson, the $I=0$ partner to the $I=1$, 1^{+-} B meson, should be *less* massive than the B meson has been confirmed. The observed mass $m_H \approx 1.13-1.19$ GeV implies that nonplanar effects for unnatural-parity trajectories die out at roughly the same rate as for natural-parity trajectories.

The H meson, the $I=0$ partner of the long-established B meson, has recently been discovered.[1] The success of the naive quark model left little doubt that the H existed, but its discovery is nevertheless interesting. In particular we have one more laboratory in which to test ideas about Zweig-rule violation and deviations from ideal mixing. The mass of the H is reported to be between 1.13 and 1.19 GeV/c^2.

Except for the pseudoscalars, all known nonets of mesons exhibit a high degree of ideal mixing with an $I=0$ meson almost degenerate with the $I=0$ meson. The anomalous pattern in the pseudoscalars leads to many questions. Are the η and η' the unique counterexamples to ideal mixing? Is the mechanism for Zweig-rule violation in the pseudoscalars connected in any way to such violation in other mesons? The H meson is especially interesting in this respect because of its close relationship to the η. In the quark model the H is just the P-wave excitation of the η. In duality schemes[2] the H trajectory is exchange degenerate (EXD) with the η trajectory. Is there a relationship between the deviations from ideal mixing of these two mesons? Is the H-B system analogous to the π-η system, or to the ρ-ω system, or to neither, or to both?

The most popular model for treating Zweig-rule violations has been an annihilation model where the $q\bar{q}$ annihilate into gluons.[3] The difference between the pseudoscalar and vector mesons is then the difference between two- or three-gluon annihilation. This model however, does not really solve the U(1) problem in QCD nor can it be simply extended to P-wave mesons such as the f-A_2 multiplet or the H-B multiplet.[4] Another proposal[5] which has recently been revived[6] has Zweig-rule-violating interactions proceeding via physical glueball states. Unless we know the masses and widths of these states we cannot make the kinds of predictions we are interested in here.

Dual topological unitarization[7] (DTU) gave rise to yet another model whereby Zweig-rule violations occur because of nonplanar diagrams such as Fig. 1. In such an approach the nonplanar diagrams were evaluated in a Regge model[8] and were successfully applied to the η-η' mixing problem.[9,10] Witten[11] has used the $1/N_c$ expansion (N_c is the number of colors) to argue that the topological approach to Zweig-rule violation is in fact more appropriate to QCD than the gluon-annihilation model. This in turn has led to successful models for the pseudoscalar nonet which incorporate all the known QCD features of the problem.[12] It is possible if one prefers to do so, to view the DTU calculations as a specific, realistic, evaluation of the nonplanar diagrams in a Regge model. The DTU approach has the advantage that it is universal, applies to all the particles on a Regge trajectory, and in principle to all Regge trajectories. Relationships between different trajectories have been found.[10] The success of the topological approach prompted us, several years ago, to make a prediction for the properties of the H meson.[13] The properties of the recently discovered H are consistent with these predictions.

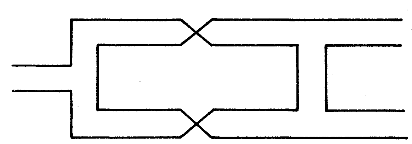

FIG. 1. A nonplanar, nonperturbative diagram responsible for violating the Zweig rule and causing deviations from ideal mixing.

Most discussions of Zweig-rule violation have focused on predicting the magnitude of the effect. There exists however a striking pattern in the sign of the $I=0$, $I=1$ mass splittings which begs for explanation. We refer to the fact that for natural parity the odd-charge-conjugation $I=0$ state is *more* massive than the $I=1$ (the ω is more massive than the ρ) while the even-charge-conjugation $I=0$ state is *less* massive than the $I=1$ (the f is less massive than the A_2). For unnatural parity the situation is reversed. The $C=-H$ is less massive than B while the η with $C=+$ is more massive than the π. The topological model of Refs. 8 and 14 predicts not just the relative signs of the mass splittings but their absolute signs. The model predicts that the sign of the mass splitting alternates with the signature (or charge conjugation) of the state. The sign for the f-A_2 splitting is given by a unitarity argument for the trajectories at $t=0$[14] (Ref. 14) and a relative minus sign has been discovered for unnatural parity.[10] Thus the sign of all mass splittings is fixed and the newly discovered H meson follows exactly the pattern $[\mathrm{sgn}(m_H-m_B)=-\mathrm{sgn}\times(m_\eta-m_\pi)]$ as predicted. Furthermore the DTU approach fixes the order of the intercepts of the various trajectories at $t=0$ to be $\alpha_f(0)>\alpha_\rho(0)>\alpha_\omega(0)$ in accord with experiment (the f is here taken to be the Pomeron according to the Pomeron-f identity) and predicts $\alpha_H(0)>\alpha_\pi(0)$. The results about particle masses and trajectory intercepts are contained in the inequalities

$$\begin{aligned}&\alpha_f(t)>\alpha_\rho(t)>\alpha_\omega(t)\ ,\\&\alpha_H(t)>\alpha_\pi(t)>\alpha_\eta(t)\ ,\end{aligned}\tag{1}$$

valid within the limits of the model, for all t.

Now that the H meson has been seen we can check how well the qualitative predictions of (13) hold. The DTU approach provides a model for the shifting of Regge trajectories with one t dependent parameter $k(t)$. Let $\alpha_0(t)$ be the EXD π-B trajectory with $\alpha_0(t)=-0.01+0.66t$, $\alpha_K(t)$ be the EXD K-Q trajectory $\alpha_{K(t)}=-0.15+0.66t$, and assume the equal spacing rule

$$\alpha_0(t)-\alpha_K(t)=\alpha_K(t)-\alpha_3(t)\tag{2}$$

to determine the pure $\lambda\bar{\lambda}$ trajectory $\alpha_3(t)$,

$$\alpha_3(t)=0.3+0.66t\ .\tag{3}$$

The DTU-model prediction for $k(t)$ is

$$K(t)\frac{[\alpha_0(t)-\alpha_H(t)][\alpha_H(t)-\alpha_3(t)]}{2\alpha_3(t)+\alpha_0(t)-3\alpha_H(t)}\ .\tag{4}$$

The mass range of H (1.13−1.19 GeV) implies that

$$-0.07<k(m_H{}^2)<-0.03\ .\tag{5}$$

This may be compared to the value $k(m_{\eta'}{}^2)\approx-0.15$. If we assume $k(t)\propto e^{-t/t_c}$ these values imply that t_c lies between 0.3 and 0.5. This estimate for t_c is very close to the value we estimated for the quenching interval ($t_c\approx0.5$) for the natural-parity trajectories.[8] It differs, however, from our previous estimate that t_c for unnatural-parity trajectories should be greater than 2.[13] The later estimate implied $m_H\approx1$ GeV. If we take the values of $k(t)$ as calculated at $t=m_\eta{}^2$, $m_{\eta'}{}^2$, $m_H{}^2$, from the simple formula (4) a value of $t_c\approx0.5$ is consistent with the data and naively predicts $\alpha_H(0)\approx1$. The unnatural-parity trajectories would then be more similar to the natural-parity trajectories than even we had imagined.

When $k(t)$ is as small as that in Eq. (5) corrections which might otherwise be negligible can become important. The formalism of DTU as here applied is only for the real parts of $\alpha(t)$ and of $k(t)$. Our analysis has tacitly assumed that the real part of the mass pole is identical with the Breit Wigner pole, but this is not the case. For relatively broad ($\Gamma\gtrsim100$ MeV) resonances the difference can be significant. It is probably these finite width effects which are responsible for the only known deviation from our qualitative prediction on the sign of the mass difference. In the 3^{--} nonet $\omega_g(1667)$ is probably less massive than the $g(1690)$ rather than the other way around as we would expect. $k(m_g{}^2)$ is expected to be so small at these masses that the finite width effects dominate and our qualitative prediction breaks down. The H meson is rather broad ($\Gamma_H\approx300$ MeV) and quantitative comparison with our naive theory may therefore not be warranted. If we, somewhat arbitrarily, allow for a further uncertainty in the real part of the pole position of the H of ±50 MeV we find

$$-0.10<k(m_H{}^2)<0.0\tag{6}$$

which still requires $t_c\lesssim0.6$. Within the limits of our analysis, we conclude that the quenching interval for natural- and unnatural-parity trajectories is similar. Equation (6) implies $\alpha_H(0)\geq0.9$. Such a high-lying trajectory has not been seen, but unitarity corrections will become very important and can push the intercept lower to a more acceptable value. The prediction for a high-lying, SU_3-singlet H trajectory with small slope at $t=0$ is reinforced.

The topological approach to deviations from ideal mixing is remarkably successful. It correctly accounts for the observed pattern for $0 < t < 2$ GeV^2. Extensions of the model also explain the fact that $\alpha_\rho(0) > \alpha_{A_2}(0)$.[15] Thus when the deviations from ideal mixing of the particle states are viewed in the broader context of mixing of trajectories a definite pattern emerges. The explanation of how this pattern arises is a great success of the topological approach, and as far as we know is not shared by any other model.

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