Application of unitarity to nonleptonic D decays

J. H. Reid
Department of Physics, University of Nairobi, Nairobi, Kenya

N. N. Trofimenkoff
Heritage Campus, Collège d'Enseignement Général et Professionnel de l'Outaouais, Hull, Canada
(Received 1 December 1981)

Coupled-channel unitarity is used to relate real parts to magnitudes of weak amplitudes for nonleptonic D-meson decays. Sizable corrections to the original weak amplitudes are found.

Most models[1] give purely real values for weak decay matrix elements. However, it is well known that because of strong final-state interactions the physical matrix element must also have an imaginary part. It is widely assumed that models which yield real values are giving the *magnitude* of the physical matrix element. Trofimenkoff[2] pointed out that there is no compelling reason to make this assumption, and explored the consequences of taking instead the model values to be the *real* parts of the physical matrix elements in $K\to 2\pi$ decays. It is the intention of this note to extend these considerations to nonleptonic D-meson decays.

Since one has to handle strong coupling between several final-state channels, the well known coupled-channel K-matrix formalism[3] is used. The strong scattering matrix is

$$S=1+2i\rho^{1/2}(1-KC)^{-1}K\rho^{1/2}\ , \tag{1}$$

where C is the diagonal matrix of Chew-Mandelstam functions, $\rho=\mathrm{Im}C$, and K is a real, symmetric matrix whose elements are meromorphic in the Mandelstam variable s. If the column vector of weak amplitudes is denoted by F, then the generalized Watson theorem, or unitarity relation, takes the form

$$\mathrm{Im}F=\tilde{K}\,\mathrm{Re}F\ , \tag{2}$$

where

$$\tilde{K}=(1-K\,\mathrm{Re}C)^{-1}K\,\mathrm{Im}C\ . \tag{3}$$

Specializing to the two-channel case which is needed here one gets

$$|F_1|=[(\mathrm{Re}F_1)^2+(\tilde{K}_{11}\mathrm{Re}F_1+\tilde{K}_{12}\mathrm{Re}F_2)^2]^{1/2}\ ,$$
$$|F_2|=[(\mathrm{Re}F_2)^2+(\tilde{K}_{21}\mathrm{Re}F_1+\tilde{K}_{22}\mathrm{Re}F_2)^2]^{1/2}\ , \tag{4}$$

where

$$\tilde{K}_{11}=\Delta^{-1}(K_{11}\mathrm{Im}C_1-\det K\,\mathrm{Im}C_1\mathrm{Re}C_2)\ ,$$
$$\tilde{K}_{22}=\Delta^{-1}(K_{22}\mathrm{Im}C_2-\det K\,\mathrm{Im}C_2\mathrm{Re}C_1)\ ,$$
$$\tilde{K}_{21}=\Delta^{-1}K_{12}\mathrm{Im}C_1\ , \tag{5}$$
$$\tilde{K}_{12}=\Delta^{-1}K_{12}\mathrm{Im}C_2\ ,$$
$$\Delta=1-K_{11}\mathrm{Re}C_1-K_{22}\mathrm{Re}C_2+\mathrm{Re}C_1\mathrm{Re}C_2\det K\ .$$

Note that in the one-channel case one recovers the

TABLE I. Amplitudes and isospin decompositions for Cabibbo-favored decays (without FSI). Reduced matrix elements are $A_1=\langle\frac{1}{2}||1||\frac{1}{2}\rangle$, $A_3=\langle\frac{3}{2}||1||\frac{1}{2}\rangle$.

Process	Amplitude	Isospin decomposition
$D^+\to\bar{K}^0\pi^+$	$2(X_++X_-)$	A_3
$D^0\to K^-\pi^+$	$2X_+$	$\frac{2A_1+A_3}{3}$
$D^0\to\bar{K}^0\pi^0$	$\sqrt{2}X_-$	$\frac{\sqrt{2}(A_3-A_1)}{3}$
$D^0\to\bar{K}^0\eta$	X_-	A_η
$D^0\to\bar{K}^0\eta'$	X_-	$A_{\eta'}$

TABLE II. Amplitudes and isospin decompositions for Cabibbo-suppressed decays (without FSI). All amplitudes are to be multiplied by $\tan\theta$. Reduced matrix elements are $A_{11}=\langle 1\|\frac{1}{2}\|\frac{1}{2}\rangle$, $A_{31}=\langle 1\|\frac{3}{2}\|\frac{1}{2}\rangle$, $A_{10}=\langle 0\|\frac{1}{2}\|\frac{1}{2}\rangle$, $A_0=\langle 0\|\frac{1}{2}\|\frac{1}{2}\rangle$, and $A_2=\langle 2\|\frac{3}{2}\|\frac{1}{2}\rangle$.

Process	Amplitude	Isospin decomposition
$D^+\to K^+\bar{K}^0$	$2X_+$	$A_{11}+\frac{A_{31}}{2}$
$D^0\to K^+K^-$	$2X_+$	$\frac{(A_{31}-A_{11}+A_{10})}{2}$
$D^0\to K^0\bar{K}^0$	0	$\frac{(-A_{31}+A_{11}+A_{10})}{2}$
$D^+\to\pi^+\pi^0$	$-\sqrt{2}(X_+ + X_-)$	$\frac{-\sqrt{3}}{2}A_2$
$D^0\to\pi^+\pi^-$	$-2X_+$	$\frac{A_0}{\sqrt{3}}-\frac{A_2}{\sqrt{6}}$
$D^0\to\pi^0\pi^0$	$\sqrt{2}X_-$	$\frac{A_0}{\sqrt{6}}+\frac{A_2}{\sqrt{3}}$

familiar relation

$$|F|=\sec\delta\,\mathrm{Re}F\,, \tag{6}$$

where $\tilde{K}=\tan\delta$, and δ is the strong scattering phase shift in the relevant channel.

Equation (4) is our basic result. In order to apply it, we take the K-matrix fits of Sorensen[4] for the $K\pi/K\eta'$ and $\pi\pi/K\bar{K}$ systems, and we use the QCD-improved spectator model[5] for the real parts of the weak amplitudes. We are aware of the limitations[6] of the spectator model, which have been much discussed lately. This model is used here only by way of illustration. Our general considerations apply as readily to any other weak model which would yield purely real amplitudes.

The quark-model amplitudes[5] and isospin decompositions are displayed in Tables I and II. Here $X_\pm$ stand for $\frac{1}{3}(2f_+ \pm f_-)$ where $f_+=(f_-)^{-1/2}$ and

$$f_-=\left[1+\frac{(33-2f)}{12\pi}\alpha_s(m_c{}^2)\ln\left[\frac{M_W{}^2}{m_c{}^2}\right]\right]^{12/(33-2f)}. \tag{7}$$

The numerical results are given in Tables III and IV. In the QCD model we have taken $X_+=1.2$ and $X_-=-0.3$ which corresponds to the choice $\alpha_s(m_c{}^2)=0.7$, $f=6$, $m_c=1.5$ GeV, and $M_W=85$ GeV used in.[5] For comparison, the weak amplitudes with and without final-state interactions (FSI) are tabulated side by side.

Since the QCD spectator model is now believed to be inadequate, we do not draw precise conclusions from our results. Instead, we make a few qualitative remarks on the effects of FSI in our scheme.

First, it is clear that the effects may be large. For example, compare $D^0\to\bar{K}^0\pi^0$ and $D^0\to\bar{K}^0\eta'$; or, more dramatically, $D^0\to\bar{K}^0K^0$ and $D^0\to\pi^0\pi^0$ with and without FSI. Note especially $D^0\to\bar{K}^0K^0$,

TABLE III. Numerical results for Cabibbo-favored decays.

Process	Amplitude (without FSI)	Amplitude (with FSI)	Magnitude (with FSI)
$D^+\to\bar{K}^0\pi^+$	1.8	$2.02e^{-i27^\circ}$	2.02
$D^0\to K^-\pi^+$	2.4	$1.862e^{i196^\circ}+0.673e^{-i27^\circ}$	1.445
$D^0\to\bar{K}^0\pi^0$	-0.424	$0.952e^{-i27^\circ}-1.317e^{i196^\circ}$	2.115
$D^0\to\bar{K}^0\eta'$	-0.3	$1.603e^{-i19^\circ}$	1.603
$D^0\to\bar{K}^0\eta$	-0.3	(-0.3)	(-0.3)

TABLE IV. Numerical results for Cabibbo-suppressed decays (to be multiplied by $\tan\theta$).

Process	(Without FSI)	Amplitude (with FSI)	Magnitude (with FSI)
$D^+ \to K^+ \bar{K}^0$	2.4	$2.554e^{-i20°}$	2.554
$D^0 \to \bar{K}^0 K^0$	0	$-1.277e^{-i20°}+2.019e^{i124°(-40°)}$	3.143(.928)
$D^0 \to K^+ K^-$	2.4	$1.277e^{-i20°}+2.019e^{i124°(-40°)}$	1.239(3.248)
$D^0 \to \pi^+\pi^-$	-2.4	$2.453e^{i405°}-0.662e^{-i25°}$	2.312
$D^+ \to \pi^+\pi^0$	-1.273	$1.622e^{-i25°}$	1.622
$D^0 \to \pi^0\pi^0$	-0.424	$1.734e^{i405°}+0.936e^{-i25°}$	2.235

which is *exactly* zero without FSI, but which becomes sizable with their inclusion. The strong coupling between $K\bar{K}$ and $\pi\pi$ channels is essential here. A nonzero amplitude cannot be generated from a zero one in the single-channel case [see Eq. (6)]; only Eq. (4), the coupled-channel case, can do this.

Second, FSI can considerably alter the ratio of amplitudes for the Cabibbo-suppressed decays $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$. In the QCD spectator model[1] or in models with SU(3) symmetry, this ratio is unity. Using Sorensen's[4] phase shifts for the $\pi\pi/K\bar{K}$ system, this ratio is ~ 0.5. However, there is recent work by Wicklund *et al.*[7] suggesting that the $I=0$ $K\bar{K}$ phase shift is *not* large and positive near the D mass (Sorensen[4] has $\delta^0_{K\bar{K}}=124°$), but small and negative. Taking instead the value $\delta^0_{K\bar{K}}=-40°$ suggested by the favored β solution of Martin and Pennington,[7] we obtain the *bracketed* values in Table IV. The ratio is now ~ 1.4, within one standard deviation of the experimental value of ~ 1.8 (Lüth *et al.*[8]).

Finally, there is a large enhancement in the ratio of the decay rates for $D^0 \to \bar{K}^0\pi^0$ and $D^0 \to K^-\pi^+$. Without FSI this ratio is 0.03, with FSI it is 2.14, and experimentally[8] it is 0.75. The main effect here is that noted by Lipkin,[9] namely that FSI can introduce a relative sign, so that two amplitudes, which formerly cancelled against each other, now add, resulting in a large enhancement. Our calculation introduces the refinement of a coupled-channel treatment of inelasticity, ignored by Lipkin.

In conclusion, it should also be noted that our results on the enhancement of the $\bar{K}^0K^0$ and $\pi^0\pi^0$ modes, and on the sensitivity of the ratio of the K^+K^- and $\pi^+\pi^-$ modes to the effects of final-state interactions, are in basic agreement with the earlier remarks of Donoghue and Holstein.[10]

[1] M. Suzuki, Phys. Rev. Lett. 15, 986 (1965); Y. Hara, Y. Nambu, and J. Schechter, *ibid.* 16, 380 (1966); J. J. Sakurai, Phys. Rev. 156, 1508 (1967); J. Katz and S. Tatur, Phys. Rev. D 16, 3281 (1977); N. Cabibbo and L. Maiani, Phys. Lett. 73B, 418 (1979).

[2] N. N. Trofimenkoff, Phys. Rev. D 20, 808 (1979).

[3] O. Babelon, J. L. Basdevant, D. Caillerie, and G. Mennessier, Nucl. Phys. B113, 445 (1976).

[4] C. Sorensen, Phys. Rev. D 23, 2618 (1981).

[5] N. Cabibbo and L. Maiani, Phys. Lett. 73B, 418 (1979).

[6] W. Bernreuther, O. Nachtmann, and B. Stech, Z. Phys. C 4, 257 (1980).

[7] A. B. Wicklund *et al.*, Phys. Rev. Lett. 45, 1469 (1980); A. D. Martin and M. R. Pennington, Ann. Phys. (N. Y.) 114, 1 (1978).

[8] V. Lüth *et al.*, in *Proceedings of the 1979 International Symposium on Lepton and Photon Interactions at High Energies, Fermilab,* edited by T. B. W. Kirk and H. D. I. Abarbanel (Fermilab, Batavia, Illinois, 1980), p. 78; J. Kirkby, *ibid.* p. 187.

[9] H. J. Lipkin, Phys. Rev. Lett. 44, 710 (1980).

[10] J. F. Donoghue and B. R. Holstein, Phys. Rev. D 21, 1334 (1980).