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Current algebra, $\pi\pi$ scattering lengths, and the $S^*(980)$ background contribution

Richard J. Jacob
Arizona State University, Tempe, Arizona 85287

Michael D. Scadron
University of Arizona, Tucson, Arizona 85721
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We demonstrate that the recently measured $S^*(980) \to \pi\pi$ decay width implies an S^* background contribution in addition to the Weinberg low-energy $\pi\pi$ amplitude which significantly improves the agreement between the current-algebra theory for $\pi\pi$ scattering lengths and experiment.

Attempts to improve upon the Weinberg low-energy current-algebra—PCAC (partial conservation of axial-vector current) analysis of $\pi\pi$ scattering[1] have met with little success. In particular, the overall picture given by unitarity and final-state-interaction corrections[2] does not appear to be in as good qualitative agreement with the data as is the soft-pion Weinberg theory. Yet recent extractions of the isospin-zero s-wave $\pi\pi$ scattering lengths from K_{e4} decay and $\pi N \to \pi N$ partial wave analysis,

$$a_0^{(0)} = (0.27 \pm 0.04) m_\pi{}^{-1} \tag{1}$$

[from K_{e4} (Ref. 3)] and

$$a_0^{(0)} = (0.27 \pm 0.03) m_\pi{}^{-1}$$

[from $\pi N \to \pi N$ (Ref. 4)], are nearly twice the soft-pion prediction[1]

$$a_0^{(0)} = 7 m_\pi / 32 \pi f_\pi{}^2 = 0.156 m_\pi{}^{-1} \tag{2}$$

for $f_\pi = 93.3$ MeV. This suggests that a modification of the Weinberg theory may be required.

At the same time, but in the context of hard-pion corrections, it is now clear that Δ-isobar corrections to the Weinberg soft-pion πN analysis[1] are necessary to explain the threshold πN scattering lengths. Indeed, the accurate low-energy πN data leading to an analytic *subthreshold* expansion[5] can only be understood via the πN current algebra and σ terms in conjunction with the *nonsoft* axial-vector background amplitude $q'_\mu \overline{M}^{\mu\nu} q_\nu$, the latter also Δ-isobar dominated.[6] Isobar corrections also play an important role in the current-algebra analyses of many other soft-pion processes such as $NN \to NN\pi$ and nonleptonic hyperon decays.[7]

A very recent measurement of the $S^*(980) \to \pi\pi$ decay width[8]

$$\Gamma_{S^*\pi\pi} = \frac{\frac{3}{2} g_{S^*}{}^2 P_{\text{c.m.}}}{8\pi m_{S^*}{}^2} = 24 \pm 8 \text{ MeV} \tag{3}$$

provides us with another test of a significant isobar background contribution to a current-algebra amplitude. In particular for the process $\pi^a \pi^b \to \pi^c \pi^d$, the nonsoft generalization of the Weinberg current-algebra amplitude is

$$M_{\pi\pi} = f_\pi{}^{-2} [(s_{ab} - m_\pi{}^2)\delta_{ab}\delta_{cd} + (s_{ac} - m_\pi{}^2)\delta_{ac}\delta_{bd} + (s_{ad} - m_\pi{}^2)\delta_{ad}\delta_{bc}] + f_\pi{}^{-2} q'_\mu \overline{M}^{\mu\nu} q_\nu \,, \tag{4}$$

where $s_{ab} = (q_a + p_b)^2$, etc. Owing to the spinless natures of π and S^*, the divergence of the axial-vector coupling, $\langle S^* | A^a_\mu | \pi^b \rangle = \alpha q^a_\mu + \beta p^b_\mu$, is then equivalent to $\langle S^* | \pi^a \pi^b \rangle$. There are then no differences[9] between the *nonsoft* S^* pole graphs in the $A^a_\mu \pi^b \to A^c_\nu \pi^d$ background current-algebra amplitude $\overline{M}_{\mu\nu}(S^*)$ and the $\pi\pi$ S^* pole amplitude itself, $M_{\pi\pi}(S^*)$. That is, once the amplitude $q'_\mu M^{\mu\nu} q_\nu$ is computed and the αq^2 term neglected, we obtain

$$f_\pi{}^{-2} q'_\mu \overline{M}^{\mu\nu}(S^*) q_\nu \big|_{q^2 \sim 0} \simeq M_{\pi\pi}(S^*) = g_{S^*}{}^2 \left[\frac{\delta_{ab}\delta_{cd}}{M_{S^*}{}^2 - s_{ab}} + \frac{\delta_{ac}\delta_{bd}}{M_{S^*}{}^2 - s_{ac}} + \frac{\delta_{ad}\delta_{bc}}{M_{S^*}{}^2 - s_{ad}} \right] . \tag{5}$$

The coupling $g_{S^*} S^* \vec{\pi} \cdot \vec{\pi}$ in (5) can be inferred directly from (3), using the value $m_{S^*} = 972$ MeV as measured by Gidal *et al.*,[8] to have the mass-shell value $g_{S^*}{}^2 / m_{S^*}{}^2 = 0.9 \pm 0.3$. Letting q, $q' \to 0$ in (3) makes no difference of course, because of the smallness of $m_\pi{}^2 / m_{S^*}{}^2 \simeq 0.02$.

Proceeding to an extraction of s-channel, s-wave scattering lengths from (4) and (5) requires the isospin decomposition

$$M=\frac{2}{3}\delta_{ab}\delta_{cd}M^{(0)}+(\delta_{ac}\delta_{bd}-\delta_{ad}\delta_{bc})M^{(1)}+(\delta_{ac}\delta_{bd}+\delta_{ad}\delta_{bc}-\frac{2}{3}\delta_{ab}\delta_{cd})M^{(2)}\ . \tag{6}$$

At threshold, $s_{ab}=4m_\pi{}^2$, $s_{ac}=s_{ad}=0$, and $M^{(i)}=16\pi m_\pi a_0^{(i)}$. Equations (4) and (5) predict

$$a_0^{(0)}=\frac{7m_\pi}{32\pi f_\pi{}^2}+\frac{g_{S^*}{}^2/m_{S^*}{}^2}{32\pi m_\pi}\left[\frac{5-8(m_\pi{}^2/m_{S^*}{}^2)}{1-4(m_\pi{}^2/m_{S^*}{}^2)}\right] \tag{7a}$$

$$=0.156m_\pi{}^{-1}+(0.045\pm0.015)m_\pi{}^{-1} \tag{7b}$$

$$=(0.201\pm0.015)m_\pi{}^{-1}\ , \tag{7c}$$

$$a_0^{(2)}=-\frac{m_\pi}{16\pi f_\pi{}^2}+\frac{g_{S^*}{}^2/m_{S^*}{}^2}{16\pi m_\pi} \tag{8a}$$

$$=-0.045m_\pi{}^{-1}+(0.017\pm0.006)m_\pi{}^{-1} \tag{8b}$$

$$=-(0.028\pm0.006)m_\pi{}^{-1}\ , \tag{8c}$$

along with the usual $I=1$ current-algebra prediction

$$2a_0^{(0)}-5a_0^{(2)}=3m_\pi/4\pi f_\pi{}^2=0.534m_\pi{}^{-1}\ ,$$

which is altered insignificantly by the S^* contribution.

We see from (7) and (8) that while $a_0^{(2)}$ is changed very little, the $a_0^{(0)}$ scattering length is enhanced by 30% in the direction of the experimental results (1). While other higher-spin isobars such as the $f(1270)$ also add to (7c), the contributions are much smaller because of $l=2$ and higher angular momentum threshold suppression. To consider the effects of the $\rho(776)$, $\epsilon(700)$, and their possible radial excitations, $\rho'(1600)$, $\epsilon(1400)$, *etc.*, in (7) and (8), we follow Sakurai[10] and observe that the ρ simulates the $I=1$ current-commutator contribution to the Weinberg amplitude in (4), while the ϵ simulates the σ-term combination in (4) according to the σ-model.[1] As such we argue that the ρ and ϵ are already accounted for in (7) and (8) via (4).

Thus we believe that to a good approximation, the current algebra-plus the S^* background contributions to the $\pi\pi$ s-wave scattering lengths represent the best theoretical estimates at the present time. A comparison between (1), (2), and (7c) shows clearly that the S^* isobar contribution should not be neglected if theory is to match experiment, as it indeeed does for every other current-algebra process.

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[2]See, e.g., S. C. Prasad and J. J. Brehm, Phys. Rev. D 6, 3216 (1972).

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[7]For a recent review, see M. D. Scadron, Rep. Prog. Phys. 44, 213 (1981).

[8]G. Gidal *et al.*, LBL-SLAC collaboration, LBL report, 1981 (unpublished).

[9]There is such a difference between, for example, the spin-$\frac{1}{2}$ nucleon pole contribution to $q'_\mu \overline{M}^{\mu\nu}(N)q_\nu$ in $\pi N\to\pi N$ or $\gamma N\to\pi N$ and the nucleon poles themselves $M_{\pi\pi}(N)$ as measured by the πN Adler term or the photoproduction Fubrini-Furlan-Rossetti term. For another example of a spinless process where no difference arises, consider $\eta'\to\eta\pi\pi$. [N. G. Deshpande and T. N. Truong, Phys. Rev. Lett. 41, 1579 (1978) use Lagrangian techniques to reach a similar conclusion.]

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