Prospects for a second neutral vector boson at low mass in SO(10)

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(Received 28 December 1981)

Symmetries $SU(3)_c \times SU(2)_L \times U(1) \times U(1)$ arising from a specific class of models involving the spontaneous breakdown of SO(10) are considered. In these, two neutral vector bosons Z_1 and Z_2 arise. The Z_1 is expected to be very similar to the Z_0 of the standard $SU(2)_L \times U(1)$ model. The most restrictive constraints, arising from the Novosibirsk experiment on parity violation in atomic bismuth, imply $0.98 \leq M(Z_1)/M(Z_0) \leq 1$. The Z_2 can be as light as $(2.5-3)M(Z_0)$. Its effects in neutral-current interactions, couplings, and production mechanisms are examined.

I. INTRODUCTION

Since the discovery of neutral-current interactions of neutrinos in 1973,[1] a wide range of neutral-current phenomena has shown remarkable agreement with the standard SU(2)×U(1) unified theory[2,3] of the weak and electromagnetic interactions.[4] Nonetheless, a key feature of this theory remains to be confirmed: The masses of the vector bosons are predicted to be[5]

$$M_{W^\pm}=83.0\pm2.4 \text{ GeV}/c^2 , \tag{1.1}$$

$$M_{Z_0}=93.8\pm2.0 \text{ GeV}/c^2 , \tag{1.2}$$

when radiative corrections at the one-loop level are taken into account. It is quite conceivable that the prediction (1.2) could be checked within the next year or two. In this article we would like to examine a scheme in which small deviations from the result (1.2) can point the way to interesting physics at higher energies which, however, are still accessible to the present generation of accelerators. Even if (1.2) turns out to be accurate to within about 2 GeV/c^2, we will show that there is a natural class of theories, based on SO(10), permitting a second Z (Z_2) at as low a mass as about 2.5 times the Z_0 mass. In this article we examine some of the properties and consequences of such a boson.

Our starting point is the analysis of Georgi and Weinberg,[6] in which it was shown that expanded gauge theories with extra U(1) symmetries can lead to results for neutral-current neutrino interactions at $Q^2=0$ identical to those of SU(2)×U(1). (Others have analyzed models more recently in which charged-lepton interactions at $Q^2=0$ also are similar to those in the standard model.[7]) There has also been considerable effort in the more general phenomenology of models with extra neutral vector bosons.[6–18] What we report here is a result which appears to go beyond those known previously, and indicates that extra Z's could be somewhat more accessible in the 200−500 GeV range, without distorting the picture at lower energies appreciably, than may have been anticipated earlier.

The present discussion is a particularly conservative version of two-Z models. First, it is based on the enlargement of the minimal grand unified theory [based on SU(5)] to SO(10). This is a relatively modest extension, from a group of rank 4 to one of rank 5. The extra Z is an immediate consequence of this increase in rank. Any grand unified group beyond SU(5) will have at least one such extra Z. We are simply asking how low in mass the lowest such Z can be. Secondly, in contrast to some models,[7–10] the "extra U(1)" is the *only* piece of new physics in comparison with the standard SU(2)×U(1) theory accessible at energies up to several hundred GeV. This is a consequence in the present model of the assignments of both fermions and certain Higgs bosons [those breaking SU(2) ×U(1)×U(1) down to $U(1)_{em}$] to the lowest-dimensional spinor representation of SO(10). As a result, we find that the neutral-current-to-charged-current ratio of effective coupling strengths is naturally the same as in the standard model. Results of this type also have been obtained by Deshpande and Iskandar[11] and Smirnov.[14] A consequence of our approach is that the demands upon experimental precision are considerably more exacting than in many other models.[7–10]

The Georgi-Weinberg analysis can be applied directly to a class of theories based on $SU(3)_{color} \times SU(2)_L \times U(1) \times U(1)$ derived from the successive symmetry breakdowns

$$SO(10) \rightarrow SO(6) \times SO(4) , \qquad (1.3)$$

$$SO(6) \sim SU(4) \rightarrow SU(3)_{color} \times U(1)_{B-L} , \qquad (1.4)$$

$$SO(4) \sim SU(2)_L \times SU(2)_R \rightarrow SU(2)_L \times U(1)_R . \qquad (1.5)$$

We find very similar results also for the chain

$$SO(10) \rightarrow SU(5) \times U(1) , \qquad (1.6)$$

$$SU(5) \rightarrow SU(3)_{color} \times SU(2)_L \times U(1) . \qquad (1.7)$$

There is, in fact, a limit in which the two chains are equivalent. For properties of neutral weak currents at $Q^2=0$, the results of (1.6) and (1.7) lie between limits allowed for (1.3)–(1.5). We present results for both chains which indicate that the lightest neutral (massive) vector boson (Z_1) could be observed at a mass a few percent below Eq. (1.2) without violating any constraints based on low-energy experiments. Moreover, in such situations there is a heavier boson (Z_2) which should appear at no more than several times the value (1.2).

As an example of the type of constraints we encounter, the most restrictive piece of information in the schemes considered here turns out to be the magnitude of parity violation in atomic-physics experiments.[19–25] Neutrino interactions at $Q^2=0$ provide no constraint whatsoever, as in Ref. 6. Moreover, the interactions of polarized electrons with protons and deuterons,[26] while providing an important constraint leading to the confirmation of the "standard model", turn out to be affected very little by the modifications made here for the observed range of $\sin^2\theta$.

One constraint which could in principle be useful with more precise experiments comes from weak-electromagnetic interference effects in $e^+e^- \rightarrow \mu^+\mu^-$. At present, these experiments[27] provide less of a constraint than the experiment of Ref. 19 on parity violation in atomic bismuth.

We discuss some reasons for believing in the existence of an "extra" U(1) in Sec. II. These are primarily based on the usefulness of the group SO(10) as a grand unified model of the weak, electromagnetic, and strong interactions.[28] All of our considerations based on grand unified models will be confined to this section. Subsequently, we shall discuss only the $SU(2) \times U(1) \times U(1)$ structures that result from various forms of symmetry breaking in SO(10). In Sec. II, we also estimate coupling constants associated with each U(1) depending on assumed forms of symmetry breaking.

The neutral-boson mass spectrum is analyzed in Sec. III. Two Z's are discussed, one (Z_1) a few percent lighter than the standard Z_0 of Eq. (2), and the other (Z_2) heavier. In one variant of the models we consider, we find the relation

$$(1-M_1{}^2/M_0{}^2)(M_2{}^2/M_0{}^2-1)=\tfrac{3}{2}\sin^2\theta \approx 0.34 ,$$

where M_0, M_1, and M_2 stand for the masses of Z_0, Z_1, and Z_2. Other variants involve Z_2 masses of this same general magnitude: several times a Z_0 mass if Z_1 lies a few percent below the standard value.

The effective form of the weak Hamiltonian (both at $Q^2=0$ and for nonzero momentum transfer) is discussed in Sec. IV. Here we show that very simple results hold at $Q^2=0$; these results can be seen immediately to lead to a form for neutrino interactions identical (at $Q^2=0$) to the standard $SU(2) \times U(1)$ ones.

The most stringent constraint on the present models comes from experiments on parity violation in atomic physics. These, and other effects in charged-lepton–nucleon interactions, are treated in Sec. V. Weak asymmetries in the process $e^+e^- \rightarrow \mu^+\mu^-$ are discussed in Sec. VI.

The mixing of two U(1)'s can have small but possibly measurable effects on branching ratios of the Z_1, in comparison with standard predictions for Z_0. The branching ratios for Z_2 are quite different; most striking is the prediction

$$\Gamma(Z_2 \rightarrow d\bar{d}) > 5\Gamma(Z_2 \rightarrow u\bar{u}) ,$$

which holds in most variants discussed here. These branching ratios are discussed in Sec. VII. If there exists a right-handed neutrino N and $M(N) << M(Z_i)/2$, we find an amusing relation

$$\Gamma(Z_i \rightarrow N\bar{N}+\nu\bar{\nu}+u\bar{u})=\Gamma(Z_i \rightarrow e\bar{e}+d\bar{d})$$

independent of mixing effects. A heavy Z_2 also can decay, via mixing, to W^+W^- and we find the corresponding branching ratio to be surprisingly large.

Because of the relatively weak predicted coupling of Z_2 to $u\bar{u}$, production of Z_2 in pp and $\bar{p}p$ collisions is expected to be nearly an order of magnitude more difficult than production of a Z_0 coupled in the standard way, but with comparable mass. These estimates are set forth in Sec. VIII.

In Sec. IX, we summarize our results and suggest further experimental and theoretical studies. An appendix deals with technical matters concerning combinations of U(1) symmetries, which may arise in different reductions of SO(10).

II. GRAND UNIFICATION AND A SECOND U(1)

A. Why an extra U(1) is likely

The standard SU(2)×U(1) model of the weak and electromagnetic interactions[2,3] may be combined with color SU(3) into a minimal grand unifying model based on SU(5).[29] This group, of rank 4, is the only acceptable group of that rank for a synthesis of the strong and electroweak interactions. The rank counts the number of gauge bosons which do not change diagonal quantum numbers of the particle absorbing or emitting them: two color gluons (corresponding, e.g., to color isospin and hypercharge), the photon, and the Z_0.

The SU(5) model has the virtue of simplicity. Furthermore, it leads to a prediction of $\sin^2\theta$ in accord with experiment,[30,31,5] and to the successful prediction[31] (if an additional assumption is made) $m_b \approx 3m_\tau$. However, there are several reasons why one might expect the SU(5) description to be only part of the story.

In the SU(5) model, left-handed and right-handed charged leptons or charge $-\frac{1}{3}$ quarks belong to different representations, so that parity or charge conjugation has complicated properties under the group. The lightest quarks and leptons belong to the representations

$$5:\ (d,e^+,\bar{\nu}_e)_R\ , \tag{2.1}$$

$$10^*:\ (\bar{d},u,\bar{u},e^-)_R\ , \tag{2.2}$$

$$5^*:\ (\bar{d},e^-,\nu_e)_L\ , \tag{2.3}$$

$$10:\ (d,\bar{u},u,e^+)_L\ . \tag{2.4}$$

Here the subscript denotes helicity: $\psi_{R,L} \equiv (1\pm\gamma_5) \times\psi/2$.

There is no right-handed neutrino or left-handed antineutrino in the set (2.1)−(2.4). Thus the minimal SU(5) model, without additional SU(5) singlets, would provide a natural explanation for the absence of neutrino mass. We reserve judgment on whether this is a liability or an asset, merely noting that claims exist[32,33] that $m(\bar{\nu}_e)\neq 0$, and further experiments are planned to settle the question.[34,35]

The sets (2.1), (2.2), and (2.3), (2.4) can be combined very simply into single representations of a group of one higher rank. This is the group SO(10).[36] The orthogonal groups of the form SO($4n+2$) have two 2^{2n}-dimensional spinors transforming as complex conjugates of one another.[37] Thus, SO(6) [∼SU(4)] has a 4 and a 4* representation, while SO(10) has a 16 and a 16*. With the convention adopted in Ref. 37, we may incorporate the set (2.1), (2.2) into a 16* and (2.3), (2.4) into a 16,

$$16^* = \begin{cases} 5=(d,e^+,\bar{\nu}_e)_R\ , \\ 10^*=(\bar{d},u,\bar{u},e^-)_R\ , \\ 1=N_R\ , \end{cases} \tag{2.5}$$

$$16 = \begin{cases} 5^*=(\bar{d},e^-,\nu_e)_L\ , \\ 10=(d,\bar{u},u,e^+)_L\ , \\ 1=\bar{N}_L\ , \end{cases} \tag{2.6}$$

by merely introducing one additional two-component object N. As a consequence, the neutrino can acquire a mass, though it is plausible that this mass can be very tiny.[38]

The group SO(10) has rank 5, one higher than SU(5). Consequently, it has one more neutral boson. The mass of this extra boson is not at all well known. It could be as heavy as $\gtrsim 10^{15}$ GeV, or as light as a couple of hundred GeV. In the latter case, which is the one we shall consider here, this boson could be within reach of experiments contemplated in the next few years.

The class of models we shall examine here is one in which the *only* structure of SO(10) which survives down to "low" energies is SU(3)×SU(2) ×U(1)×U(1). By low energies we mean values below ∼1 TeV. The second U(1) is that associated with a generator of SO(10) not contained in SU(5) and reflects the difference in rank of the two groups. This class of models is perhaps the most demanding of experiment; there are other versions of SO(10) with charged vector bosons coupling to right-handed currents which make much more spectacular predictions for physics in the several-hundred-GeV range,[39] and for low-energy charged-current processes.[40]

B. SO(10) and its reduction to subgroups

We shall adopt a convenient language for SU(n) and SO($2n$) algebras which makes use of a geometric representation of generators and representation members.[41,42]

Generators outside the Cartan subalgebra may be represented by linear combinations of unit vectors $\vec{e}_i$ pointing along the ith axis in Cartesian n-dimensional coordinates. These combinations are

$$\pm(\vec{e}_i-\vec{e}_j)\quad (i<j=1,\ldots,n)\ \text{for SU}(n)\,, \tag{2.7}$$

and

$$\pm(\vec{e}_i\pm\vec{e}_j)\quad (i<j=1,\ldots,n)\ \text{for SO}(2n)\,. \tag{2.8}$$

Members of the Cartan subalgebra ("charges") may be represented by vectors in this n-dimensional space whose scalar product with an element changes the corresponding charge. Thus, for example, the isospin-raising and -lowering operators in SU(2) correspond to

$$I_\pm\leftrightarrow\pm(\vec{e}_1-\vec{e}_2)\,, \tag{2.9}$$

while the third component of isospin corresponds to a vector with components

$$I_3\leftrightarrow(\tfrac{1}{2},-\tfrac{1}{2})\,. \tag{2.10}$$

Representation members also may be denoted by vectors in the n-dimensional Cartesian space. Their charges may be ascertained by taking scalar products with the corresponding vectors associated with members of the Cartan subalgebra.

Members of the fundamental representation of SU(n) may be denoted by vectors with components

$$n\sim(1,0,0,\ldots);\ \ (0,1,0,\ldots);\cdots;(0,\ldots,0,0,1), \tag{2.11}$$

$$n^*\sim(-1,0,0,\ldots);\ \ (0,-1,0,\ldots);\cdots;\ (0,\ldots,0,0,-1)\,. \tag{2.12}$$

The action of all generators of SU(n) (including members of the Cartan subalgebra) on these vectors is not affected by a shift of these vectors by a constant amount in each coordinate: $\sim$const $\times(1,1,1,\ldots,1)$. For the set (2.7) this is readily apparent, while for members of the Cartan subalgebra it follows from the property that the sum of the components of vectors representing these members vanishes for SU(n). Thus, for SU(2), the sum of the components in (2.10) vanishes, while for SU(3) the corresponding vectors are

$$I_3\leftrightarrow(\tfrac{1}{2},-\tfrac{1}{2},0)\,, \tag{2.13}$$

$$Y\leftrightarrow(\tfrac{1}{3},\tfrac{1}{3},-\tfrac{2}{3})\,, \tag{2.14}$$

and both sums of components vanish.

The group SU(n) is contained in SO($2n$) as one may see by a simple construction. The group SU(n) preserves the scalar product $a^*\cdot b$ for complex vectors in n-dimensional Euclidean space. This product is

$$\begin{aligned} a^*\cdot b &= (\text{Re}a-i\,\text{Im}a)\cdot(\text{Re}b+i\,\text{Im}b)\\ &= \text{Re}a\cdot\text{Re}b+\text{Im}a\cdot\text{Im}b\\ &\quad +i(\text{Re}a\cdot\text{Im}b-\text{Im}a\cdot\text{Re}b)\,. \end{aligned} \tag{2.15}$$

Now imagine a $2n$-dimensional real vector composed of (Rea,Ima). An SO($2n$) transformation preserves the product

$$(\text{Re}b,\text{Im}b)\cdot(\text{Re}a,\text{Im}a)=\text{Re}(a^*\cdot b)\,, \tag{2.16}$$

so every SU(n) transformation preserving (2.15) is an SO($2n$) transformation preserving (2.16). Thus SU(n)$\subset$SO($2n$).

Since SO($2n$) is of rank n while SU(n) is of rank $n-1$, the Cartan subalgebra of SO($2n$) contains one additional generator in addition to those of SU(n). This generator may be represented by a vector proportional to $(1,1,\ldots,1)$, i.e., by a vector all of whose components are equal.

Spinor representations of SO($2n$) also have a simple geometric interpretation in the present language. Their members correspond to vectors in the n-dimensional vector space pointing to vertices of a hypercube with coordinates

$$\text{Spinor of dimension } 2^{n-1}\begin{cases}(\tfrac{1}{2},\tfrac{1}{2},\ldots,\tfrac{1}{2})\,,\\ (-\tfrac{1}{2},-\tfrac{1}{2},\tfrac{1}{2},\ldots,\tfrac{1}{2})+\text{perms.}\,,\\ (-\tfrac{1}{2},-\tfrac{1}{2},\ldots,-\tfrac{1}{2},\pm\tfrac{1}{2})+\text{perms.}\,,\\ (\text{even no. of}-\text{signs})\,,\end{cases} \tag{2.17}$$

$$(\text{Spinor of dimension } 2^{n-1})' \begin{cases} (-\frac{1}{2},\frac{1}{2},\ldots,\frac{1}{2})+\text{perms.}\,, \\ (-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},\frac{1}{2},\ldots,\frac{1}{2})+\text{perms.}\,, \\ (-\frac{1}{2},-\frac{1}{2},\ldots,-\frac{1}{2},\mp\frac{1}{2})+\text{perms.}\,, \\ (\text{odd no. of}-\text{signs})\,. \end{cases} \quad (2.18)$$

If n is even, the two spinors are inequivalent and real (equivalent to their complex conjugates). Here complex conjugation may be thought of as reversal of sign of all the components, as in (2.11) and (2.12). If n is odd, the spinors (2.17) and (2.18) are complex conjugates of one another.

Let us adopt a normalization for generators in the Cartan subalgebra such that the vectors V_i representing them obey

$$V_i \cdot V_j = \tfrac{1}{2}\delta_{ij} \quad (i,j=1,\ldots,r)\,, \quad (2.19)$$

where r is the rank of the group. Then a convenient set for SU(5) is

$$\text{color SU(3): } V(I_{3c})=(\tfrac{1}{2},-\tfrac{1}{2},0,0,0)\,, \quad (2.20)$$

$$V(Y_c)=\frac{1}{2\sqrt{3}}(1,1,-2,0,0)\,, \quad (2.21)$$

$$\text{SU(2)}_L\text{: } V(I_{3L})=(0,0,0,\tfrac{1}{2},-\tfrac{1}{2})\,, \quad (2.22)$$

$$\text{U(1)}_{Y_W}\text{: } V(Y_W)=\frac{1}{2\sqrt{15}}(-2,-2,-2,3,3)\,. \quad (2.23)$$

For SO(10), we may take in addition

$$\text{U(1)}_\chi\text{: } V(\chi)=\frac{1}{\sqrt{10}}(1,1,1,1,1)\,. \quad (2.24)$$

Other familiar charges may be expressed in terms of the vectors (2.20)–(2.24). For example, with the convention we have adopted,

$$V(I_{3R})=(0,0,0,\tfrac{1}{2},\tfrac{1}{2})\,. \quad (2.25)$$

The parity operation thus corresponds here to inversion of the fifth component of vectors denoting representation members. The charge may be represented by a vector $\tilde{V}(Q)$ [the tilde denotes a normalization other than (2.19)]:

$$\tilde{V}(Q)=(-\tfrac{1}{3},-\tfrac{1}{3},-\tfrac{1}{3},1,0) = V(I_{3L})+\sqrt{5/3}\,V(Y_W)\,. \quad (2.26)$$

Thus charge conjugation corresponds to inversion of the first four components of vectors denoting representation members. The (baryon number)–(lepton number) may be denoted by

$$\tilde{V}(B-L)=(-\tfrac{2}{3},-\tfrac{2}{3},-\tfrac{2}{3},0,0)\,. \quad (2.27)$$

An alternative expression for the charge is

$$\tilde{V}(Q)=V(I_{3L})+V(I_{3R})+\frac{\tilde{V}(B-L)}{2}\,, \quad (2.28)$$

well known from the study of left-right-symmetric models of the electroweak interactions.[43]

As we have mentioned in Eqs. (2.17) and (2.18), the members of the 16-dimensional spinor of SO(10) may be represented by alternate vertices of a hypercube with Cartesian coordinates $\pm\frac{1}{2}$ in a five-dimensional space. The conjugate 16-dimensional spinor is represented by the other sixteen vertices of this hypercube. The members, and their corresponding charges, are shown in Table I.

The decomposition of SO(10) spinors into a five-dimensional space may have deeper significance in terms of composite models.[42] These models envision quarks and leptons as composed of five distinct objects, each of two varieties, which can be identified with the five coordinates in the Cartesian space.

The decomposition SO(10)→SU(5)×U(1)$_\chi$ has been performed in Table I. Notice that $2\sqrt{10}\chi$ just counts the number of positive signs minus the number of negative signs in the spinor $\frac{1}{2}(\pm1,\pm1,\pm1,\pm1,\pm1)$. Different SU(5) representations have different values of χ, and the sum of χ over all members of an SO(10) representation vanishes as it should. SU(5) has the further decomposition

$$\text{SU(5)}\to\text{SU(3)}_c\times\text{SU(2)}_L\times\text{U(1)}_{Y_W}\,,$$

TABLE I. Members of the 16-dimensional representation of SO(10). Shown are left-handed spinors. The *CP* operation reverses all coordinates in the five-dimensional weight space and all charges.

SU(5) representation	$2\sqrt{10}\chi$	Particle	Signs in weight vector $\frac{1}{2}(\pm1,\pm1,\pm1,\pm1,\pm1)$	I_{3L}	Q
1	-5	$\bar{N}_L$	$-----$	0	0
10	-1	$\bar{u}_L$	$++---$ $+-+--$ $-++--$	0	$-\frac{2}{3}$
		u_L	$+--+-$ $-+-+-$ $--++-$	$\frac{1}{2}$	$\frac{2}{3}$
		d_L	$+---+$ $-+--+$ $--+-+$	$-\frac{1}{2}$	$-\frac{1}{3}$
		e_L^+	$---++$	0	$+1$
5*	3	$\bar{d}_L$	$-++++$ $+-+++$ $++-++$	0	$\frac{1}{3}$
		e_L^-	$+++-+$	$-\frac{1}{2}$	-1
		ν_{eL}	$++++-$	$\frac{1}{2}$	0

a breakdown which should occur around 10^{15} GeV.[5]

The decomposition $SO(10)\to SO(6)\times SO(4)$ also is immediate in terms of the five-dimensional vector space utilized here. We simply take the first three components to represent SO(6) and the last two to represent SO(4) in accord with (2.8). The Cartan subalgebra of SO(6) contains I_{3c} (2.20), Y_c (2.21), and $B-L$ (2.27). The Cartan subalgebra of SO(4) contains I_{3L} (2.22) and I_{3R} (2.25). We note that $SO(6)\sim SU(4)$ has the further decomposition $SU(4)\to SU(3)\times U(1)_{B-L}$ and $SO(4)\sim SU(2)_L \times SU(2)_R$ is expected to break down to no higher a symmetry than $SU(2)_L\times U(1)_R$ by energies of several hundred GeV.[40]

Thus, whether SO(10) breaks down initially via $SU(5)\times U(1)$ or $SO(6)\times SO(4)$, the structure surviving at an energy of several hundred GeV should be no richer than $SU(3)\times SU(2)\times U(1)\times U(1)$.

C. Higgs-boson representations

We now discuss the specific choices of patterns of symmetry breaking which led to a very simple form of the effective neutral-current Hamiltonian at $Q^2=0$. As a result of this choice, we shall find that neutrino neutral-current interactions at $Q^2=0$ turn out to be identical to those in the standard model.

The group SO(10) could be broken all the way down to SU(5) at a mass scale M if gauge bosons are coupled to a 16-dimensional representation of SO(10) whose SU(5) singlet piece acquires a large vacuum expectation value ($\gtrsim M$). We shall not consider this possibility further; it leads to the standard model at low energy.

A Higgs-boson multiplet belonging to the adjoint (45-dimensional) representation of SO(10) has the decomposition

$$\begin{array}{ccc} SO(10) \rightarrow & SU(5) & \\ \underline{45} & = \underline{1}+\underline{10}+\underline{10}^*+\underline{24} & . \end{array} \tag{2.29}$$

The SU(5) singlet may be used to break SO(10) down to $SU(5)\times U(1)_\chi$, and the 24-plet has a member which can break $SU(5)\rightarrow SU(3)_c\times SU(2)_L \times U(1)_{Y_W}$.

The breakdown $SO(10)\rightarrow SO(6)\times SO(4)$ can utilize a 54- or 210-dimensional Higgs representation.[44] At this point our symmetry $[SU(4) \times SU(2)_L\times SU(2)_R]$ must be broken down to $SU(3)_c\times U(1)_{B-L}\times SU(2)_L\times U(1)_R$. We assume, in accord with constraints derived in Refs. 45 and 46, that this occurs somewhere above 10^9 GeV. The 210-dimensional representation contains a (15,1,3) of $SU(4)\times SU(2)_L\times SU(2)_R$, which is capable of performing the necessary breaking.

At a mass of several hundred GeV, then, we have either the symmetry

$$S(A)\equiv SU(3)_c\times SU(2)_L\times U(1)_{Y_W}\times U(1)_\chi \tag{2.30}$$

or

$$S(B)\equiv SU(3)_c\times SU(2)_L\times U(1)_{B-L}\times U(1)_R\ . \tag{2.31}$$

To break the symmetry down further we may choose[47] a Higgs field belonging to the 16-dimensional representation of SO(10). We wish to preserve $SU(3)_c\times U(1)_{em}$, so this member must be colorless and neutral. From Table I, we see there are two candidates analogous to ν and N. We may denote them by

$$\phi_1\leftrightarrow\tfrac{1}{2}(1,1,1,1,-1)\in\underline{5}^*\ , \tag{2.32}$$

$$\phi_2\leftrightarrow\tfrac{1}{2}(-1,-1,-1,-1,-1)\in\underline{1}\ . \tag{2.33}$$

We denote the corresponding vacuum expectation values by v_1 and v_2. The quantum numbers of these fields are shown in Table II.

The Higgs field in the "standard" $SU(2)\times U(1)$ model is ϕ_1. It has $I_{3L}=\frac{1}{2}$, $Y_W\equiv 2(Q-I_{3L}) =-1$. By virtue of its belonging to the 5*-dimensional representation of SU(5) contained in the 16-dimensional SO(10) spinor, the field ϕ_1 is taken to have charge $2\sqrt{10}\chi=3$. This is a very specific assumption which we can only motivate by reference to SO(10) at present.

The Higgs field ϕ_2 has $I_{3L}=0$ and $Y_W=0$, so it does not affect charged-current phenomenology. Its charge $2\sqrt{10}\chi$ is taken to be -5, as is appropriate for an SU(5) singlet belonging to the SO(10) 16-spinor.

In the decomposition

$$SO(10)\rightarrow SO(6)\times SO(4)\sim SU(4)\times SU(2)_L\times SU(2)_R$$

which leads to manifest left-right symmetry, the 16-spinor reduces to

$$\begin{array}{ccc} SO(10) \rightarrow & SU(4)\times SU(2)_L\times SU(2)_R & \\ \underline{16} = & (4,2,1)+(4^*,1,2) & . \end{array} \tag{2.34}$$

The boson ϕ_1 is the only neutral particle belonging to (4,2,1), while ϕ_2 is the only neutral particle belonging to (4* ,1,2). The left-right symmetry between ϕ_1 and ϕ_2 is apparent from a comparison of their I_{3L} and I_{3R} assignments in Table II.

One could ask whether Higgs bosons belonging to another low-dimension representation of SO(10) could have been chosen in place of 16-plet members to break $SU(2)\times U(1)\times U(1)$ down to $U(1)_{em}$. We have not performed an exhaustive study, but find that the simplest alternative, members of the vector 10-dimensional representation of SO(10), will not be satisfactory, as the end result of this choice remains an unbroken $U(1)\times U(1)$ symmetry.

D. Symmetry breaking, mass scales, and coupling constants

We shall examine four limits, two based on the breakdown of SO(10) to $S(A)\equiv SU(3)_c\times SU(2)_L \times U(1)_{Y_W}\times U(1)_\chi$ and two based on the breakdown to $S(B)\equiv SU(3)_c\times SU(2)_L\times U(1)_{B-L}\times U(1)_R$. These proceed via the chains (1.6) and (1.7) and

TABLE II. Assumed Higgs bosons leading to $SU(2)\times U(1)\times U(1)\rightarrow U(1)_{em}$.

Field ϕ_i	I_{3L}	$Y_W\equiv 2(Q-I_{3L})$	$2\sqrt{10}\chi$	I_{3R}	$B-L$
ϕ_1	$+\frac{1}{2}$	-1	3	0	-1
ϕ_2	0	0	-5	$-\frac{1}{2}$	$+1$

(1.3) and (1.5), respectively, and are illustrated in Figs. 1 and 2.

The breakdown to $S(A)$ illustrated in Figs. 1 leads to two U(1) subgroups, only one of whose charges (Y_W) contributes to the electric charge. The coupling of $U(1)_{Y_W}$ at $Q \cong 2M_W$ thus is fixed by standard electroweak phenomenology. As we shall see, neutrino physics at $Q^2=0$ leads to a view of this phenomenology indistinguishable from the standard one.

The coupling constant associated with $U(1)_X$ at $Q \simeq 2M_W$ depends on the mass scales associated with successive symmetry breakdowns. The breakdown $SO(10) \rightarrow SU(5) \times U(1)_X$ may take place at any mass (defined to be M_U) greater than that (M_X) at which $SU(5) \rightarrow SU(3)_c \times SU(2)_L \times U(1)_{Y_W}$.

By extrapolation of known couplings to higher Q^2 with the help of the renormalization group,[30] one finds at the one-loop level

$$M_X = M_W \exp\left[\frac{\pi}{11}(\alpha^{-1} - \tfrac{8}{3}\alpha_3^{-1})\right]$$

$$= 3.2 \times 10^{14} \text{ GeV} \qquad (2.35)$$

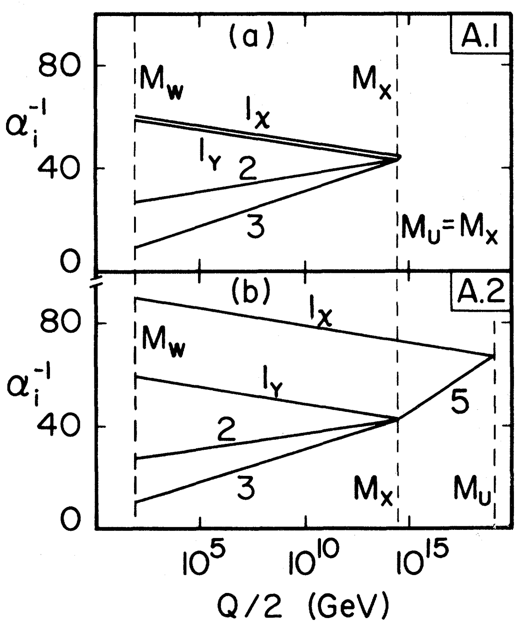

FIG. 1. Coupling-constant behavior as function of momentum Q when SO(10) breaks down via $SU(5) \times U(1)_X$ to $SU(3)_c \times SU(2)_L \times U(1)_{Y_W} \times U(1)_X$. Labels: $1_X \leftrightarrow U(1)_X$, $1_Y \leftrightarrow U(1)_{Y_W}$, $2 \leftrightarrow SU(2)_L$, $3 \leftrightarrow SU(3)_c$. (a) Case A.1 noted in the text (here the first and second breakdowns take place at the same mass). (b) Case A.2. Here $5 \leftrightarrow SU(5)$.

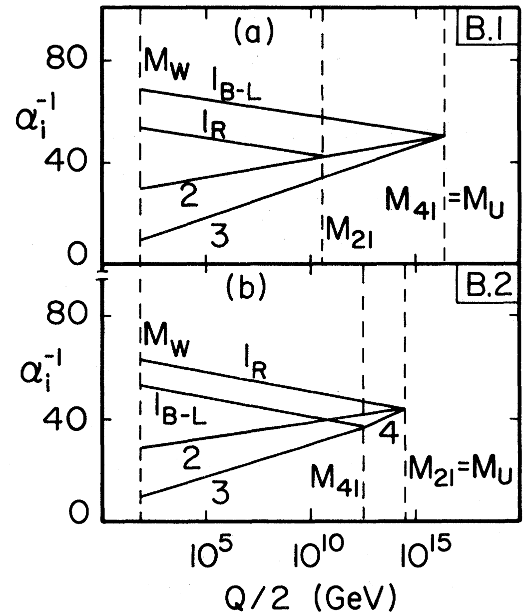

FIG. 2. Coupling-constant behavior when SO(10) breaks down via $SO(6) \times SO(4)$ to $SU(3)_c \times SU(2)_L \times U(1)_{B-L} \times U(1)_R$. Labels: $1_{B-L} \leftrightarrow U(1)_{B-L}$, $1_R \leftrightarrow U(1)_R$, $2 \leftrightarrow SU(2)_L$, $3 \leftrightarrow SU(3)_c$. (a) Case B.1; (b) case B.2. Here $4 \leftrightarrow SU(4)$.

for[46,48]

$$\alpha^{-1} \equiv \alpha^{-1}(2M_W) = 128.3 ,$$
$$M_W = 80 \text{ GeV} , \qquad (2.36)$$
$$\alpha_3^{-1} \equiv \alpha_3^{-1}(2M_W) = 10 ,$$

and

$$x \equiv \sin^2\theta = \frac{1}{6} + \frac{5}{9}\frac{\alpha_3^{-1}}{\alpha^{-1}} = 0.21 . \qquad (2.37)$$

The experimental value (before application of radiative corrections[5]) is a little higher,[49] and we shall use the nominal value $x = 0.23$ subsequently.

We examine two limits of the symmetry chain leading to S(A):

A.1 [Fig. 1(a)]

$$M_U = M_X , \qquad (2.38)$$

$$\alpha_X^{-1} = \alpha_1^{-1} = \tfrac{3}{5}\alpha^{-1}\cos^2\theta , \qquad (2.39)$$

A.2 [Fig. 1(b)]

$$M_U = \text{Planck mass}$$
$$= 1.2 \times 10^{19} \text{ GeV} , \qquad (2.40)$$

$$\alpha_{\chi}{}^{-1}=\alpha_1{}^{-1}+\frac{55}{12\pi}\ln\frac{M_U{}^2}{M_{\chi}{}^2} \;. \tag{2.41}$$

Here $\alpha_i \equiv g_i{}^2/4\pi$ $(i=\chi,1)$, where g_i are the couplings associated with $U(1)_{\chi}$ and $U(1)_{Y_W}$ as normalized in Eqs. (2.24) and (2.23), measured at $Q=2M_W$. The resulting coupling constants are shown in Table III.

The breakdown to $S(A)$ illustrated in Fig. 1, leads to two U(1)'s, both of which contribute to the electric charge. The initial breakdown of SO(10) to SO(6)×SO(4) can occur at a unification mass M_U confined within the limits[46,48]

$$M_W\exp\left[\frac{\pi}{11}(\alpha^{-1}-\tfrac{8}{3}\alpha_3{}^{-1})\right]=3.2\times10^{14}\text{ GeV}\le M_U\le M_W\exp\left[\frac{6\pi}{11}(\alpha^{-1}x-\alpha_3{}^{-1})\right]=2.6\times10^{16}\text{ GeV} \;. \tag{2.42}$$

[The limits here are somewhat different than those quoted in Refs. 46 and 48 as a result of the choice $\alpha_3{}^{-1}(2M_W)=10$, motivated by a recent analysis[50] of Υ decays.] The mass scales M_{41}, M_{21} of the subsequent breakdowns $SU(4)\to SU(3)_{\text{color}}\times U(1)_{B-L}$ and $SU(2)_R\to U(1)_R$ are governed by the constraints[46,48]

$$\alpha^{-1}x-\alpha_3{}^{-1}=\frac{11}{6\pi}\ln\frac{M_U{}^2}{M_W M_{41}} \tag{2.43}$$

and

$$\alpha^{-1}-\tfrac{8}{3}\alpha_3{}^{-1}=\frac{11}{3\pi}\ln\frac{M_U{}^2M_{21}}{M_W{}^3} \;. \tag{2.44}$$

They are hence correlated. If we eliminate M_U from (2.43) and (2.44), we find

$$(M_{21}M_{41})^{1/2}=M_W\exp\left\{\frac{3\pi}{22}[\alpha^{-1}(1-2x)-\tfrac{2}{3}\alpha_3{}^{-1}]\right\}$$
$$=3.6\times10^{13}\text{ GeV} \;. \tag{2.45}$$

The upper bound in (2.42) comes from (2.43) with $M_{41}=M_U$ (the largest value of M_{41}). Then M_{21} is as small as possible. This case, which we call (B.1), is illustrated in Fig. 2(a).

The lower bound in (2.42) comes from (2.44) with $M_{21}=M_U$ (the largest value of M_{21}). Then M_{41} is as small as possible. The corresponding behavior of coupling constants for this situation, denoted (B.2), is shown in Fig. 2(b).

We may summarize the behavior of coupling constants in cases B.1 and B.2 in Table IV. The couplings shown in this table are related to masses by

$$\alpha_{41}{}^{-1}=\alpha_3{}^{-1}+\frac{33}{12\pi}\ln\frac{M_{41}{}^2}{M_W{}^2} \;, \tag{2.46}$$

$$\alpha_R{}^{-1}=\alpha^{-1}x+\frac{22}{12\pi}\ln\frac{M_{21}{}^2}{M_W{}^2} \;. \tag{2.47}$$

The unification mass in case B.2 [and also in the SU(5) cases, Eq. (2.35)] is low enough to cause detectable proton instability. We summarize previous calculations by the expression[51]

$$\tau_{\text{nucleon}}\simeq(5\times10^{30}\text{ yr})(20)^{\pm1}\left[\frac{M_U}{6\times10^{14}\text{ GeV}}\right]^4 \;, \tag{2.48}$$

where the factor of 20 comes from uncertainties in the t-quark mass, Higgs meson structure, phase-space effects, and the value of $|\Psi(0)|^2$ for two quarks inside a proton. Present limits are $\tau_{\text{nucleon}}\gtrsim 10^{30}-10^{31}$ yr,[52–54] very close to the upper limit based on (2.48) and (2.35).

III. MASS MATRIX

A. Formalism

The source of gauge-boson masses in an $SU(2)\times U(1)_a\times U(1)_b$ Lagrangian with two Higgs fields ϕ_1 and ϕ_2 is a kinetic term of the form

$$\mathscr{L}_K=(D^{\mu}\phi_1)^{\dagger}(D_{\mu}\phi_1)+(D^{\mu}\phi_2)^{\dagger}(D_{\mu}\phi_2) \;. \tag{3.1}$$

Here D^{μ} is a covariant derivative, expressed in terms of SU(2) and U(1) generators $\vec{T}, T_a, T_b$ and

TABLE III. Couplings $\alpha_{\chi}{}^{-1}$ in SO(10)→SU(5)×$U(1)_{\chi}$.

Case	M_U	$\alpha_{\chi}{}^{-1}$
A.1	$M_{\chi}=3.2\times10^{14}$ GeV	59.3
A.2	$M_{\text{Planck}}=1.2\times10^{19}$ GeV	90.0

TABLE IV. Masses and couplings in $SO(10)\to SO(6)\times SO(4)\to SU(3)\times SU(2)_L\times U(1)_{B-L}\times U(1)_R$.

Case	M_U (GeV)	M_{41} (GeV)	M_{21} (GeV)	α_{41}^{-1}	α_R^{-1}
B.1	2.6×10^{16}	2.6×10^{16}	4.8×10^{10}	68.5	53.1
B.2	3.2×10^{14}	3.9×10^{12}	3.2×10^{14}	53.1	63.4

couplings g_2, g_a, g_b as

$$D^\mu=\partial^\mu+i(g_2\vec{T}\cdot\vec{W}^\mu+g_aT_aB_a^\mu+g_bT_bB_b^\mu)\ . \tag{3.2}$$

The fields $\vec{W}^\mu$, B_a^μ, and B_b^μ are the SU(2), $U(1)_a$, and $U(1)_b$ gauge fields.

(1) For the symmetry chain S(A) based on $SU(5)\times U(1)$, we obtain a mass matrix of the form

$$\mathcal{M}^2=M_0{}^2\mu^2\ , \tag{3.3}$$

where

$$M_0\equiv(g_2{}^2+g'^2)^{1/2}v_1/2 \tag{3.4}$$

is the mass of the standard Z_0;

$$g'\equiv(\tfrac{3}{5})^{1/2}g_1 \tag{3.5}$$

is the conventionally normalized[2] coupling associated with $U(1)_{Y_W}$ and

$$\mu^2=\begin{bmatrix}\cos^2\theta & -\sin\theta\cos\theta & \frac{3\hat{g}}{\sqrt{10}}\cos\theta\\ -\sin\theta\cos\theta & \sin^2\theta & \frac{-3\hat{g}}{\sqrt{10}}\sin\theta\\ \frac{3\hat{g}}{\sqrt{10}}\cos\theta & \frac{-3\hat{g}}{\sqrt{10}}\sin\theta & \hat{g}^2(\frac{9}{10}+\frac{5}{2}R)\end{bmatrix}\ . \tag{3.6}$$

Here

$$\hat{g}\equiv g_\chi/(g_2{}^2+g'^2)^{1/2} \tag{3.7}$$

and

$$R\equiv v_2{}^2/v_1{}^2\ . \tag{3.8}$$

The matrix (3.6) is expressed in the basis space

$$\begin{bmatrix}I_{3L}\\ Y_W/2\\ \chi\end{bmatrix}\ . \tag{3.9}$$

(2) For the symmetry S(B) based on $SO(6)\times SO(4)$, the mass matrix takes the form

$$\mathcal{M}^2=\frac{v_1{}^2}{4}\begin{bmatrix}g_2{}^2 & 0 & -g_2g_{B-L}\\ 0 & g_R{}^2R & -g_Rg_{B-L}R\\ -g_2g_{B-L} & -g_Rg_{B-L}R & g_{B-L}{}^2(1+R)\end{bmatrix} \tag{3.10}$$

in the basis space

$$\begin{bmatrix}I_{3L}\\ I_{3R}\\ (B-L)/2\end{bmatrix}\ . \tag{3.11}$$

The coupling constant $g_{41}\equiv(\frac{2}{3})^{1/2}g_{B-L}$ is associated with a generator of SO(10) normalized in the manner (2.19). The subscript denotes the origin of the corresponding U(1) in the breakdown SU(4) $\to SU(3)_c\times U(1)$. We have already discussed the behavior of $\alpha_{41}{}^{-1}\equiv4\pi/g_{41}{}^2$ in Eq. (2.46).

B. Eigenvectors and eigenvalues

(1) For the symmetry S(A), the normalized eigenvectors of (3.6) may be written

$$|\gamma\rangle=\begin{bmatrix}\sin\theta\\ \cos\theta\\ 0\end{bmatrix};\quad |Z_1\rangle=\begin{bmatrix}\cos\phi\cos\theta\\ -\cos\phi\sin\theta\\ -\sin\phi\end{bmatrix};\quad |Z_2\rangle=\begin{bmatrix}\sin\phi\cos\theta\\ -\sin\phi\sin\theta\\ \cos\phi\end{bmatrix}\ . \tag{3.12}$$

The photon $|\gamma\rangle$ corresponds to zero eigenvalue. If we define ϵ and η by

$$\mu^2|Z_1\rangle=(1-\epsilon)|Z_1\rangle=(M_1{}^2/M_0{}^2)|Z_1\rangle\ , \tag{3.13}$$

$$\mu^2|Z_2\rangle=(1+\eta)|Z_2\rangle=(M_2{}^2/M_0{}^2)|Z_2\rangle\ , \tag{3.14}$$

then

$$\tan\phi=\sqrt{10}\epsilon/3\hat{g}\ , \tag{3.15}$$

$$\cot\phi = \sqrt{10}\eta/3\hat{g} , \quad (3.16)$$

so

$$\epsilon\eta = \tfrac{9}{10}\hat{g}^2 = \frac{9}{10}\frac{g_\chi^2}{g_2^2+g'^2} . \quad (3.17)$$

The masses of Z_1 and Z_2 hence are correlated with one another. Moreover the trace of μ^2 is equal to the sum of its eigenvalues, so that

$$\eta-\epsilon=\hat{g}^2[\tfrac{9}{10}+\tfrac{5}{2}R]-1 . \quad (3.18)$$

The masses resulting from Eqs. (3.17) and (3.18) are shown in Fig. 3(a) for the two choices of g_χ noted in Table III. Also shown are tick marks corresponding to values of R. For large R we have $\eta \simeq \frac{5}{2}\hat{g}^2 R$ +const. and hence $\epsilon \approx \frac{9}{25}R$, independent of $\hat{g}^2$. Thus, for large R, in the $S(A)$ scheme, we have

$$M_1/M_0 \approx 1-\tfrac{9}{50}R . \quad (3.19)$$

The lighter Z can be very close to the standard mass for not very large values of R, while the mass of the heavier one grows only as $\sqrt{R}$. As we shall see, present experiments (parity violation in atomic bismuth is the most restrictive) exclude values of R below about 10. This implies typical values of

$$M_1/M_0 \gtrsim 0.98 , \quad (3.20)$$

$$M_2/M_0 \gtrsim 2.5-3 . \quad (3.21)$$

There is a particularly simple limit of (3.17) corresponding to case A.1. When SO(10) breaks down to $SU(5)\times U(1)_\chi$ at the same mass at which SU(5) breaks down to $SU(3)\times SU(2)\times U(1)$, we have $g_\chi = g_1 = (\frac{5}{3})^{1/2}g'$, so

$$\hat{g}^2 = \tfrac{5}{3}g'^2/(g_2^2+g'^2) = \tfrac{5}{3}\sin^2\theta = \tfrac{5}{3}x$$

and

$$\left[1-\frac{M_1^2}{M_0^2}\right]\left[\frac{M_2^2}{M_0^2}-1\right] = \epsilon\eta = \tfrac{3}{2}x \simeq 0.34 . \quad (3.22)$$

Thus, when $M_1/M_0 \gtrsim 0.98$, $M_2/M_0 \gtrsim 3.1$. For smaller values of g_χ (as in case A.2), $\epsilon\eta$ may be smaller and the Z_2 mass need not be quite as high. Nonetheless, in many of our applications we shall take nominal values $M_1/M_0=0.98$, $M_2/M_0=3.1$ as examples to determine the precision of experiments needed to detect the second Z.

An extreme limit (outside the bounds set in Table III) occurs when $\hat{g}\to 0$, so that $\epsilon \to 1-\frac{5}{2}\hat{g}^2 R$, $\eta \to \frac{9}{10}\hat{g}^2$. This leads to a solution[16] with Z_1 very light and Z_2 just above the standard Z_0 mass.

(2) For the symmetry S(B), diagonalization of the mass matrix (3.10) leads to the eigenvalues depicted in Fig. 3(b). Two limits are shown, corresponding to the coupling constants $\alpha_{B-L}^{-1} = 4\pi/g_{B-L}^2$ and $\alpha_R^{-1} = 4\pi/g_R^2$ enumerated in Table IV. The qualitative behavior of the masses resembles that in the previous two cases.

For case S(B), a simple relation [reducing to (3.22) in case A.1] which holds in general is

$$\epsilon\eta = \frac{3}{2}x\frac{g_{41}^2}{g_R^2} . \quad (3.23)$$

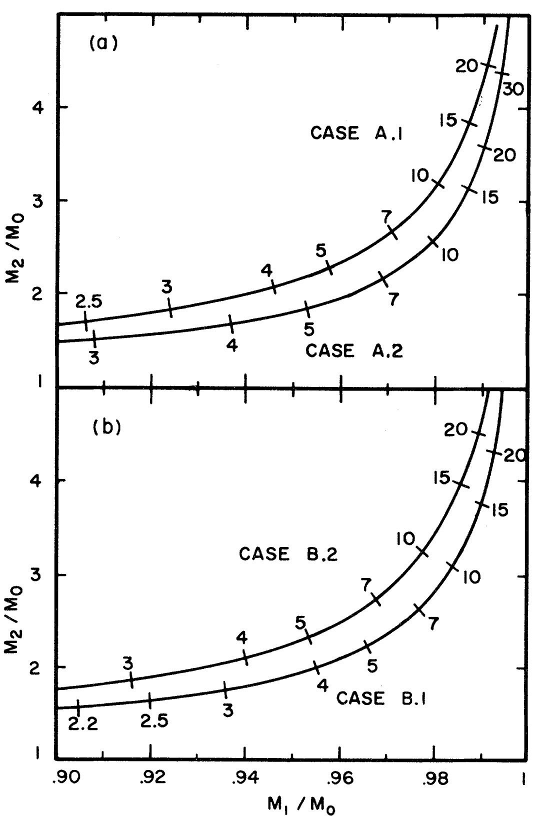

FIG. 3. Relations between M_2/M_0 and M_1/M_0 as functions of $R \equiv (v_2/v_1)^2$ (labels on curves). (a) Cases A [SU(5)×U(1)]. (b) Cases B [SO(6)×SO(4)].

IV. NEUTRAL-CURRENT COUPLINGS AT $Q^2=0$

The standard $SU(2)_L \times U(1)_{Y_W}$ model has a weak neutral-current—current effective Hamiltonian of the simple form

$$\mathscr{H}_N=\mathscr{H}_N{}^0\equiv \frac{2}{v_1{}^2}[\bar\psi\gamma^\mu(I_{3L}-\sin^2\theta\, Q)\psi]\times[\bar\psi\gamma_\mu(I_{3L}-\sin^2\theta\, Q)\psi]\ , \tag{4.1}$$

when we assume only the Higgs field ϕ_1 is present.

Georgi and Weinberg[6] have shown how to generalize (4.1) to $SU(2)_L\times U(1)_a\times U(1)_b$ models. Let $U(1)_a$ be a U(1) contributing to the electric charge

$$Q=I_{3L}+C_aT_a+C_bT_b,\ \ C_a\neq 0\ . \tag{4.2}$$

Specifically for S(A),

$$Q=I_{3L}+\frac{Y_W}{2}\ \ (\text{choose } a=Y_W)\ , \tag{4.3}$$

while for S(B),

$$Q=I_{3L}+I_{3R}+\frac{B-L}{2}\ \ (\text{choose } a=B-L)\ . \tag{4.4}$$

Then, let Σ_{ij} denote the submatrix of $\mathscr{M}^2$ corresponding to $SU(2)_L\times U(1)_b$. At $Q^2=0$ the weak neutral-current effective Hamiltonian takes the form

$$\mathscr{H}_N=\frac{1}{2}\sum_{i,j=2,b}(\bar\psi\gamma^\mu n_i\psi)(\bar\psi\gamma_\mu n_j\psi)(\Sigma^{-1})_{ij}\ , \tag{4.5}$$

where

$$n_2=g_2\left[I_{3L}-\frac{e^2}{g_2{}^2}Q\right]=g_2(I_{3L}-\sin^2\theta\, Q) \tag{4.6}$$

and

$$n_b=g_b\left[T_b-\frac{e^2}{g_b{}^2}C_bQ\right]\ . \tag{4.7}$$

The specific forms of (4.7) are

$$S(\text{A}):\ n_b=g_\chi\chi \tag{4.8}$$

and

$$S(\text{B}):\ n_b=g_R\left[I_{3R}-\frac{e^2}{g_R{}^2}Q\right]\ . \tag{4.9}$$

The submatrix Σ_{ij} has an inverse Σ^{-1}, which is

$$S(\text{A}):\ \Sigma^{-1}=\frac{32}{5g_2{}^2g_\chi{}^2v_1{}^2v_2{}^2}\times\begin{bmatrix}(\frac{9}{40}v_1{}^2+\frac{5}{8}v_2{}^2)g_\chi{}^2 & -\frac{3v_1{}^2g_2g_\chi}{4\sqrt{10}}\\ -\frac{3v_1{}^2g_2g_\chi}{4\sqrt{10}} & \frac{g_2{}^2v_1{}^2}{4}\end{bmatrix} \tag{4.10}$$

or

$$S(\text{B}):\ \Sigma^{-1}=\begin{bmatrix}\frac{4}{g_2{}^2v_1{}^2} & 0\\ 0 & \frac{4}{g_R{}^2v_2{}^2}\end{bmatrix}\ . \tag{4.11}$$

The specific forms of the weak neutral-current Hamiltonian (4.5) are then found to be

$$S(\text{A}):\ \mathscr{H}_N=\mathscr{H}_N^0+\frac{2}{v_2{}^2}[\bar\psi\gamma^\mu(I_{3R}-\tfrac{3}{5}\cos^2\theta\, Q)\psi]\times[\bar\psi\gamma_\mu(I_{3R}-\tfrac{3}{5}\cos^2\theta\, Q)\psi] \tag{4.12}$$

and

$$S(\text{B}):\ \mathscr{H}_N=\mathscr{H}_N{}^0+\frac{2}{v_2{}^2}\left[\bar\psi\gamma^\mu\left[I_{3R}-\frac{e^2}{g_R{}^2}Q\right]\psi\right]\times\left[\bar\psi\gamma_\mu\left[I_{3R}-\frac{e^2}{g_R{}^2}Q\right]\psi\right] \tag{4.13}$$

Equation (4.13) also follows immediately from the manifest left-right symmetry of the model based on $SU(4)\times SU(2)_L\times SU(2)_R$. To derive (4.12), it is helpful to use the relation

$$\tfrac{6}{5}(I_{3L}-\sin^2\theta\, Q)-(\tfrac{8}{5})^{1/2}\chi=-2(I_{3R}-\tfrac{3}{5}\cos^2\theta\, Q) \tag{4.14}$$

to combine terms which appear at intermediate stages of the calculation. The identity (4.14) can be easily verified with the help of expressions given in Sec. II. [We thank C. N. Leung for showing us this simple way to derive Eq. (4.12).]

In the limit in which SO(10) breaks down at a single mass all the way to $SU(3)_c\times SU(2)_L\times U(1)\times U(1)$, the schemes S(A) and S(B) are equivalent, and the Hamiltonians (4.12) and (4.13) should coincide. They do, since in this limit all the U(1) couplings are equal,

$$g_R{}^{-2}=g_1{}^{-2}=\tfrac{3}{5}g'^{-2}=\tfrac{3}{5}e^{-2}\cos^2\theta\ . \tag{4.15}$$

The schemes S(A) and S(B) are *not* equivalent when the couplings g_R, g_{41}, g_1, and g_χ are not all equal. This may be shown by an application of Schur's lemma, noted in the Appendix.

An immediate consequence of Eqs. (4.12) and (4.13) is that the low-energy neutral-current interactions of neutrinos are not affected by the additional U(1). This has been known for some time for theories based on $SU(2)_L\times U(1)_R\times U(1)_{B-L}$, but it does not seem to have been noticed for

$SU(2)_L \times U(1)_{Y_W} \times U(1)_\chi$, and some contrary results appear in the literature.[13]

The form (4.12) based on S(A) may be thought of as lying between extreme limits specified by (4.13) and S(B). These limits are depicted in Fig. 2 (cases B.1 and B.2). Notice that the form (4.12) is independent of g_χ, and hence applies for cases A.1 and A.2, and all intermediate versions of S(A).

The derivation of Eqs. (4.12) and (4.13) is a major result of this paper. We reiterate that their form follows from specific choices of fermion and Higgs representations set forth in Sec. II. In the next section we apply the results at $Q^2=0$ (and in Secs. VI–VIII also at $Q^2>0$) to several reactions using standard phenomenological analyses.

V. COUPLINGS IN ELECTRON-NUCLEON EXPERIMENTS

The parity-violating electron-quark neutral-current Hamiltonian at $Q^2=0$ may be written as[55]

$$\mathscr{H}_N^{(eq)}=\frac{G_F}{\sqrt{2}}\{(\bar{e}\gamma^\mu\gamma_5 e)[C_{1u}(\bar{u}\gamma_\mu u)+C_{1d}(\bar{d}\gamma_\mu d)]+(\bar{e}\gamma^\mu e)[C_{2u}(\bar{u}\gamma_\mu\gamma_5 u)+C_{2d}(\bar{d}\gamma_\mu\gamma_5 d)]\} . \tag{5.1}$$

Isovector and isoscalar combinations can be defined as

$$\begin{aligned}\tilde{\alpha}&\equiv C_{1u}-C_{1d} ,\\ \tilde{\beta}&\equiv C_{2u}-C_{2d} ,\\ \tilde{\gamma}&\equiv C_{1u}+C_{1d} ,\\ \tilde{\delta}&\equiv C_{2u}+C_{2d} .\end{aligned} \tag{5.2}$$

For theories based on the Hamiltonians (4.12) and (4.13), we find

$$\begin{aligned}\tilde{\alpha}&=-1+2x+(1-2\tilde{x})/R ,\\ \tilde{\beta}&=-1+4x+(1-4\tilde{x})/R ,\\ \tilde{\gamma}&=2x/3-(2\tilde{x}/3)/R ,\\ \tilde{\delta}&=0 ,\end{aligned} \tag{5.3}$$

where $x=\sin^2\theta=e^2/g_2{}^2$, $R=(v_2/v_1)^2$, and

$$\tilde{x}=\begin{cases}\frac{3}{5}\cos^2\theta & \text{for } S(\text{A})\ [SU(5)\times U(1)] ,\\ \dfrac{e^2}{g_R{}^2} & \text{for } S(\text{B})\ [SO(6)\times SO(4)] .\end{cases} \tag{5.4}$$

Using one-loop renormalization arguments we found in Sec. II that $\alpha_R{}^{-1}$ was allowed to lie between 53.1 and 63.4 (see Table IV). This forces $e^2/g_R{}^2$ to lie in the region 0.414–0.494, with the $SU(5)\times U(1)_\chi$ value at $\tilde{x}=\frac{3}{5}\cos^2\theta=0.462$ comfortably in the middle. In the $SU(5)\times U(1)_\chi$ case, the eN couplings (in fact, all the $Q^2=0$ couplings) are seen to depend solely on $\sin^2\theta$ and R and not at all on $g_\chi{}^2$, in fact consistent with a theorem due to Kim and Zee.[12]

A. Parity violation in heavy atoms

Parity-violation experiments in heavy atoms measure, after many arcane atomic-physics calculations, the so-called "weak charge"

$$Q_W(Z,N)=-[\tilde{\alpha}(Z-N)+3\tilde{\gamma}(Z+N)] , \tag{5.5}$$

which depends on the number of protons Z and neutrons N in the nucleus used.

A series of experiments on bismuth[19] ($Z=83$, $N=126$) yields the result

$$Q_W(\text{Bi})=-135\pm17.5 , \tag{5.6}$$

while a more recent experiment on thallium[20] finds that

$$Q_W(\text{Tl})=-155\pm63 . \tag{5.7}$$

The model predictions are readily found to be

$$Q_W^{\text{th}}(\text{Bi})=-(43+332x)+\frac{1}{R}(43+332\tilde{x}) , \tag{5.8}$$

$$Q_W^{\text{th}}(\text{Tl})=-(42+324x)+\frac{1}{R}(42+324\tilde{x}) . \tag{5.9}$$

For the $SU(5)\times U(1)$ case [S(A)], the values of these quantities are

$$Q_W^{\text{th}}(\text{Bi})=-119+196/R , \tag{5.10}$$

$$Q_W^{\text{th}}(\text{Tl})=-117+192/R . \tag{5.11}$$

A value

$$R\geq10 \tag{5.12}$$

allows Eq. (5.10) to agree with (5.6) to within two standard deviations. The bound on R based on Eqs. (5.7) and (5.11) is less restrictive at present: at the 2σ level, $R \gtrsim 2.2$. A bound based on the bismuth experiment of Ref. 21 is consistent with Eq. (5.12).

Equation (5.12), as mentioned earlier, restricts the mass M_1 of the light Z to be within about 2% of the standard value M_0 as one can see from Fig. 3(a) or Eq. (3.19). The corresponding lower bound on M_2 ranges from 3.2 M_0 (case A.1) to 2.6 M_0 (case A.2), as one sees from Fig. 3(a).

Allowing a range in $\tilde{x}$ as in the SO(6)$\times$SO(4) cases B.1 and B.2 changes the results very little. The bounds on R range from $R \gtrsim 9$ for case B.1 to $R \gtrsim 11$ for case B.2. The Z_1 continues to be within 2% of the standard Z_0, as in cases based on SU(5)$\times$U(1)$_\chi$, while the lower bound on M_2 ranges from 2.9 M_0 (B.1) to 3.4 M_0 (B.2).

The bound $R \geq 10$, arising from the Novosibirsk experiment on parity violation in atomic bismuth, is the most restrictive source of information on R at present. In view of the sometimes contradictory results and interpretations in such experiments [for a discussion, see Ref. 14], we wish to look further for constraints on R, even though they will turn out not to be as tight as Eq. (5.12) at present.

B. Polarized-electron scattering on deuterium and hydrogen

The asymmetry in the scattering of polarized electrons on deuteron and proton targets measures parity-violating effects in electron-quark interactions and can provide further information on $\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\gamma}$, and $\tilde{\delta}$. The asymmetry in the inelastic cross section for electrons polarized parallel and antiparallel to the beam is given by[56]

$$\frac{A(x,y,Q^2)}{Q^2}=a_1(x)+a_2(x)\left[\frac{1-(1-y)^2}{1+(1-y)^2}\right] . \quad (5.13)$$

For the isoscalar deuteron, $a_1(x)$ is given by

$$a_1=\frac{1}{5}\frac{3G}{\sqrt{2}e^2}(3\tilde{\alpha}+\tilde{\gamma}) , \quad (5.14)$$

with the experimentally determined value

$$a_1^{\exp}=(-9.7\pm2.6)\times10^{-5}\ (\text{GeV}/c)^{-2} , \quad (5.15)$$

giving the constraint[57]

$$3\tilde{\alpha}+\tilde{\gamma}=-1.8\pm0.48 . \quad (5.16)$$

For the proton, x-dependent quark distribution functions are required in the definitions of $a_1(x)$ and $a_2(x)$ and their values can be estimated at the values of the kinematic variables appropriate to experiment.[58] The SLAC ep experiment finds

$$\frac{A(x\simeq0.2,y\simeq0.21)}{Q^2}=(-9.7\pm2.7)\times10^{-5}\ (\text{GeV}/c)^{-2} , \quad (5.17)$$

so that including the estimated distribution functions, we find the constraint

$$C_p\equiv2.76\tilde{\alpha}+1.24\tilde{\gamma}+0.278\tilde{\beta}+0.167\tilde{\delta}=-1.71\pm0.48 . \quad (5.18)$$

The model predictions are

$$3\tilde{\alpha}+\tilde{\gamma}=(-3+\tfrac{20}{3}x)+(3-\tfrac{20}{3}\tilde{x})/R \quad (5.19)$$

for deuterium, and

$$C_p=-3.04+7.46x+(3.04-7.46\tilde{x})/R \quad (5.20)$$

for hydrogen. These expressions are evaluated for the various cases of interest in Table V. As a result of the very small coefficients of $1/R$, the constraints (5.16) and (5.17) are not very restrictive.

In other words, present polarized electron-deuteron and electron-proton deep-inelastic scattering data provide very little constraint on the models presented here. Indeed, the coefficient of $1/R$ in $3\tilde{\alpha}+\tilde{\gamma}$ vanishes identically when $\tilde{x}=\frac{9}{20}$.

TABLE V. Constraints on R from parity violation in deuterium and hydrogen. Here $x=0.23$ is taken.

Case	$\tilde{x}$	$3\tilde{\alpha}+\tilde{\gamma}$	C_p [Eq. (5.18)]
A	0.462	$-1.467-0.080/R$	$-1.323-0.408/R$
B.1	0.414	$-1.467+0.240/R$	$-1.323-0.050/R$
B.2	0.494	$-1.467-0.293/R$	$-1.323-0.647/R$
Expt.	(Ref. 26)	$-1.8\ \pm0.48$	$-1.71\ \pm0.48$

This corresponds in $SU(5)\times U(1)_\chi$ to $x=1-(5\tilde{x}/3)=\frac{1}{4}$, a situation very close to the true value of x.

It has been suggested[14] that more precise measurements of the y-dependent terms in Eq. (5.13) could yield some additional information on the types of models suggested here. At present these measurements are too crude to provide much of a constraint.

We have begun preliminary investigations of the behavior of the asymmetry at much higher values of Q^2 [$\approx 10^4$ $(\mathrm{GeV}/c)^2$], such as would be probed with colliding electron-proton beams. This asymmetry is sensitive to the structure of the exchanged bosons and should in principle be different for the class of two-Z models considered here. Asymmetries for e^-p scattering which are about -0.5 in the standard model are modified by about 10% of their value in case A.1. The effects are considerably less dramatic than those encountered in the model considered in Ref. 59.

C. Parity violation in atomic hydrogen

Experiments have been proposed,[24,60] but not yet performed, to detect parity violation in atomic hydrogen. The effective parity-violating electron-nucleon neutral-current Hamiltonian may be written[61]

$$\mathscr{H}_N^{(eN,P\text{ violating})}=-\frac{G_F}{\sqrt{2}}[\,C_{1p}(\bar{e}\gamma_\mu\gamma_5 e)(\bar{p}\gamma^\mu p)+C_{2p}(\bar{e}\gamma_\mu e)(\bar{p}\gamma^\mu\gamma_5 p)+(p\to n)]\ . \qquad (5.21)$$

In the present model one finds the results shown in Table VI. The coefficient of $1/R$ is much larger in hydrogen than the R-independent term, which would vanish if $x=\frac{1}{4}$. For a value $R\approx 10$ not excluded by other experiments, one could envision a doubling of the effect in comparison with that predicted by the standard model. However, extreme care must be taken with regard to this prediction, since it is very sensitive to the precise value of x. The value of x extracted from deep-inelastic scattering experiments is only affected by a little more than ± 0.01 by weak radiative corrections,[5] but this is enough to make a substantial difference in predictions of the standard model for C_{1p}, C_{2p}, and C_{2n}.

D. Summary of charged-lepton—nucleon constraints

In Fig. 4 we show the constraints on $\tilde{\alpha}$ and $\tilde{\gamma}$ that arise from the measurements described in Secs. V A—V C. [For parity violation in atoms we imagine $C_{1p}=-(2C_{1u}+C_{1d})=-(\tilde{\alpha}+3\tilde{\gamma})/2$ is measured.] Parity violation in heavy atoms and in polarized electron-deuteron scattering give almost orthogonal information. Also shown are predictions of the $SU(5)\times U(1)_\chi$ model, with tick marks denoting values of R. As mentioned, the trajectory in the $\tilde{\alpha}$-$\tilde{\gamma}$ plane of these predictions lies almost parallel to the lines defined by polarized-electron—deuteron scattering. Parity violation in hydrogen can provide constraints lying midway between the other two types of experiment.

VI. WEAK EFFECTS IN $e^+e^-\to\mu^+\mu^-$

We shall discuss three types of experiment. At low energies, the angular asymmetry in e^+e^-

TABLE VI. Predictions for parameters in Hamiltonian (5.21) affecting parity violation in atomic hydrogen and deuterium. We take $x=0.23$.

Quantity	Expression	Value A ($\tilde{x}=0.462$)	B.1 ($\tilde{x}=0.414$)	B.2 ($\tilde{x}=0.494$)
C_{1p}	$\frac{1-4x}{2}-\frac{1-4\tilde{x}}{2R}$	$0.04+0.42/R$	$0.04+0.33/R$	$0.04+0.49/R$
C_{2p}	$g_A\left[\frac{1-4x}{2}-\frac{1-4\tilde{x}}{2R}\right]$	$g_A\ (0.04+0.42/R)$	$g_A\ (0.04+0.33/R)$	$g_A\ (0.04+0.49/R)$
C_{1n}	$-\frac{1}{2}\left[1-\frac{1}{R}\right]$	$-0.5\ +0.5/R$	$-0.5\ +0.5/R$	$-0.5\ +0.5/R$
C_{2n}	$-g_A\left[\frac{1-4x}{2}-\frac{1-4\tilde{x}}{2R}\right]$	$-g_A\ (0.04+0.42/R)$	$-g_A\ (0.04+0.33/R)$	$-g_A\ (0.04+0.49/R)$

$\to\mu^+\mu^-$ grows with s. At higher energies the s dependence is governed by the position and couplings of the Z pole(s) in the amplitude. One can also measure polarization effects as a function of energy.

A. Low-energy limit

The weak neutral-current interaction Hamiltonian for leptons at $Q^2=0$ may be written[4]

$$\mathscr{H}_N^{e\mu}=\frac{G_F}{\sqrt{2}}[h_{VV}(\bar{e}\gamma_\mu e+\bar{\mu}\gamma_\mu\mu)(\bar{e}\gamma^\mu e+\bar{\mu}\gamma^\mu\mu)+2h_{VA}(\bar{e}\gamma_\mu e+\bar{\mu}\gamma_\mu\mu)(\bar{e}\gamma^\mu\gamma_5 e+\bar{\mu}\gamma^\mu\gamma_5\mu)$$
$$+h_{AA}(\bar{e}\gamma_\mu\gamma_5 e+\bar{\mu}\gamma_\mu\gamma_5\mu)(\bar{e}\gamma^\mu\gamma_5 e+\bar{\mu}\gamma^\mu\gamma_5\mu)]\ . \qquad (6.1)$$

In the models described by the Hamiltonians (4.12) and (4.13), we find

$$h_{VV}=(-\tfrac{1}{2}+2x)^2+\frac{1}{R}(-\tfrac{1}{2}+2\tilde{x})^2\ , \qquad (6.2)$$

$$h_{VA}=-\tfrac{1}{2}(-\tfrac{1}{2}+2x)+\frac{1}{R}\left[\frac{1}{2}\right](-\tfrac{1}{2}+2\tilde{x})\ , \qquad (6.3)$$

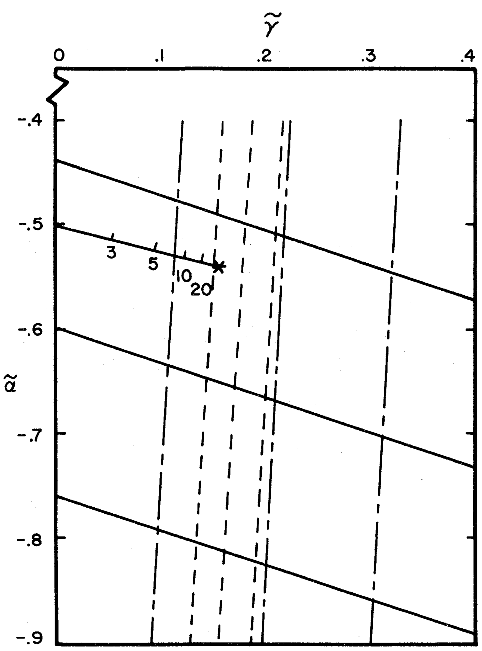

FIG. 4. Constraints on weak-neutral-current parameters $\tilde{\alpha}$ and $\tilde{\gamma}$ from parity violation in atomic bismuth (dashed lines: Ref. 19; dash-dotted lines: Ref. 20), and in polarized-electron—deuteron scattering (solid lines: Ref. 26). The asterisk shows the prediction of the standard model. The solid line emanating from this point shows the predictions of the present class of models (cases A.1, A.2), with tick marks denoting the allowed values of $R\equiv(v_2/v_1)^2$.

$$h_{AA}=\tfrac{1}{4}\left[1+\frac{1}{R}\right]\ , \qquad (6.4)$$

where $\tilde{x}$ was defined in Eq. (5.4).

At energies where weak effects begin to be detectable the differential cross section for $e^+e^-\to\mu^+\mu^-$ behaves as[27]

$$\frac{d\sigma}{d\Omega}(e^+e^-\to\mu^+\mu^-)=\frac{\alpha^2}{4s}[(1+2h_{VV}\rho)(1+\cos^2\theta)$$
$$+4h_{AA}\rho\cos\theta]\ , \qquad (6.5)$$

where

$$\rho\equiv-\sqrt{2}G_F s/e^2\ .$$

Thus h_{VV} is measurable in terms of deviations of the total cross section from its expected value, while h_{AA} is probed by forward-backward asymmetries.

A recent experiment[27] at PETRA in the range $30\leq\sqrt{s}\leq 36$ GeV finds

$$h_{VV}=0.01\pm0.08\ , \qquad (6.6)$$

$$h_{AA}=0.18\pm0.16\ . \qquad (6.7)$$

At the 1σ level, for $\tilde{x}=\frac{3}{5}\cos^2\theta$, Eqs. (6.6) and (6.7) imply only

$$R\geq 2\ \ (h_{VV}) \qquad (6.8)$$

and

$$R\geq 2.8\ \ (h_{AA})\ , \qquad (6.9)$$

when combined with (6.2) and (6.4). The numerical value of the h_{VV} constraint is slightly different in models based on SU(6)$\times$SO(4), but the h_{AA} constraint is unchanged. Both constraints are considerably less severe than those imposed by parity violation in atomic bismuth or thallium.

B. Energy dependence of cross section and of forward-backward asymmetry in $e^+e^-\to\mu^+\mu^-$

Future e^+e^- accelerators might well attain energies beyond the standard Z_0 mass. Direct searches for additional Z's are then possible. In addition, forward-backward asymmetries in $e^+e^-\to\mu^+\mu^-$ typically exhibit variation over a wider energy range than cross sections, and could in principle be used to search for Z's beyond the highest attainable energy at a given accelerator.

At this point we make explicit use of the interaction Lagrangian based on the covariant derivative introduced in Eq. (3.2):

$$\mathscr{L}_{\rm int}^{\rm neut}=-\bar\psi\gamma_\mu(g_2\vec{\rm T}\cdot\vec{\rm W}^\mu+g_aT_aB_a^\mu+g_bT_bB_b^\mu)\psi\ , \qquad (6.10)$$

where $\vec{\rm W}^\mu$, B_a^μ, and B_b^μ are the SU(2), U$(1)_a$, and U$(1)_b$ gauge fields. We may rewrite this Lagrangian in terms of physical fields for neutral particles by diagonalizing the mass matrix (3.6) or (3.10). Convenient parametrizations for doing this are

$$S(\mathrm{A}):\quad |W_3^\mu\rangle=\sin\theta\,|\gamma\rangle+\cos\theta\cos\phi\,|Z_1\rangle+\cos\theta\sin\phi\,|Z_2\rangle\ ,$$
$$|B_Y^\mu\rangle=\cos\theta\,|\gamma\rangle-\sin\theta\cos\phi\,|Z_1\rangle-\sin\theta\sin\phi\,|Z_2\rangle\ , \qquad (6.11)$$
$$|B_\chi^\mu\rangle=-\sin\phi\,|Z_1\rangle+\cos\phi\,|Z_2\rangle$$

or

$$S(\mathrm{B}):\quad |W_3^\mu\rangle=\sin\theta\,|\gamma\rangle+\cos\theta\cos\phi\,|Z_1\rangle+\cos\theta\sin\phi\,|Z_2\rangle\ ,$$
$$|B_R^\mu\rangle=\cos\theta\cos\psi\,|\gamma\rangle-(\cos\phi\sin\theta\cos\psi+\sin\phi\sin\psi\,|Z_1\rangle+(\cos\phi\sin\psi-\sin\phi\sin\theta\cos\psi)\,|Z_2\rangle\ , \qquad (6.12)$$
$$|B_{B-L}^\mu\rangle=\cos\theta\sin\psi\,|\gamma\rangle+(\sin\phi\cos\psi-\cos\phi\sin\theta\sin\psi)\,|Z_1\rangle-(\cos\phi\cos\psi+\sin\phi\sin\theta\sin\psi)\,|Z_2\rangle\ .$$

The angle ϕ was introduced in Sec. III. By combining Eqs. (3.15) and (3.16) we see in case S(A)

$$\tan\phi=\sqrt{\epsilon/\eta}=\left[\frac{M_0{}^2-M_1{}^2}{M_2{}^2-M_0{}^2}\right]^{1/2}\ , \qquad (6.13)$$

where we recall

$$\epsilon=1-(M_1/M_0)^2\ , \qquad (6.14)$$

$$\eta=(M_2/M_0)^2-1\ . \qquad (6.15)$$

It turns out that the parametrizations (6.13)–(6.15) also may be applied to case S(B).

The angle ψ in Eq. (6.12) is found to satisfy

$$\tan^2\psi=\frac{2}{3}\frac{g_R{}^2}{g_{41}{}^2}\quad [S(\mathrm{B})]\ , \qquad (6.16)$$

as one may verify by combining (6.12) with the mass matrix (3.10) and demanding that the photon correspond to zero eigenvalue. In the limit $g_R=g_{41}$, $\tan^2\psi=\frac{2}{3}$. This is just the limit in which S(B) is a special case of S(A). In this limit, it may be verified that (6.12) reduces to (6.11).

The rewritten Lagrangian takes the form

$$\mathscr{L}_{\rm int}^{\rm neut}=-\bar\psi\gamma_\mu(eQA^\mu+\lambda_1Z_1^\mu+\lambda_2Z_2^\mu)\psi\ . \qquad (6.17)$$

A general form for the couplings valid for both S(A) and S(B) schemes is

$$\lambda_i=\frac{e}{\sin\theta\cos\theta}\left[\frac{|M_j{}^2-M_0{}^2|}{M_2{}^2-M_1{}^2}\right]^{1/2}\left[\frac{M_i{}^2}{M_0{}^2}(I_{3L}-Q\sin^2\theta)+\left[\frac{M_i{}^2}{M_0{}^2}-1\right](I_{3R}\sec^2\psi-Q\cos^2\theta)\right]$$
$$(i=1,2\Longrightarrow j=2,1)\ , \qquad (6.18)$$

where

$$S(\mathrm{A}):\ \sec^2\psi=\tfrac{5}{3}\ , \qquad (6.19)$$

$$S(\mathrm{B}):\ \sec^2\psi=1+\frac{2}{3}\frac{g_R{}^2}{g_{41}{}^2}\ . \qquad (6.20)$$

Equations (6.18) and (6.19) are valid for all cases of S(A), not only the one which is a special case of S(B).

With the interaction Lagrangian (6.17) and (6.18), we may now calculate cross sections and asymmetries in $e^+e^- \to \mu^+\mu^-$. Let z denote $\cos\theta_{\rm c.m.}$ for this section. Then[8]

$$\frac{d\sigma}{dz}=\frac{\pi\alpha^2}{4s}[(1+z^2)(|F_-|^2+|F_+|^2+|F_0|^2) +2z(-|F_-|^2+|F_+|^2+|F_0|^2)]\,, \tag{6.21}$$

so that

$$\sigma=\frac{2\pi\alpha^2}{3s}(|F_-|^2+|F_+|^2+|F_0|^2)\,, \tag{6.22}$$

and the forward-backward asymmetry is given by

$$A_{FB}=\frac{1}{\sigma}\left[\int_0^1 \frac{d\sigma}{dz}dz-\int_{-1}^0\frac{d\sigma}{dz}dz\right] =\frac{3}{4}\left[\frac{-|F_-|^2+|F_+|^2+|F_0|^2}{|F_-|^2+|F_+|^2+|F_0|^2}\right]\,, \tag{6.23}$$

$$F_\pm(s)=1+\sum_{i=1,2}\frac{\mathcal{N}_i^2}{e^2}\frac{[(g_{V,e}^i)^2\pm(g_{A,e}^i)^2]}{1-(M_i^2-iM_i\Gamma_i)/s}\,, \tag{6.24}$$

$$F_0(s)=\sum_{i=1,2}\frac{\mathcal{N}_i^2}{e^2}\frac{2g_{V,e}^i g_{A,e}^i}{1-(M_i^2-iM_i\Gamma_i)/s}\,, \tag{6.25}$$

where

$$\mathcal{N}_1^2=\frac{8G_F}{\sqrt{2}}\frac{M_1^4}{M_0^2}\left[\frac{M_2^2-M_0^2}{M_2^2-M_1^2}\right]\,, \tag{6.26}$$

$$\mathcal{N}_2^2=\frac{8G_F}{\sqrt{2}}\frac{M_2^4}{M_0^2}\left[\frac{M_0^2-M_1^2}{M_2^2-M_1^2}\right]\,. \tag{6.27}$$

The interaction Lagrangian $\mathcal{L}^{(Z_i f\bar f)}$ takes the form

$$\mathcal{L}^{(Z_i f\bar f)}=-\mathcal{N}_i\bar f\gamma^\mu(g_{V,f}^{(i)}+g_{A,f}^{(i)}\gamma_5)fZ_{i\mu} \tag{6.28}$$

here, where f denotes the fermion field. The couplings in Eqs. (6.24) and (6.25) in the present class of models are given by

$$g_{V,e}^{(i)}=\frac{3-\sec^2\psi}{4}+\frac{M_0^2}{M_i^2}\left[\frac{\sec^2\psi}{4}-(1-x)\right]\,, \tag{6.29}$$

$$g_{A,e}^{(i)}=-\tfrac{1}{4}\tan^2\psi+\frac{M_0^2}{M_i^2}\frac{\sec^2\psi}{4}\,. \tag{6.30}$$

When $M_1=M_0$, and $\sec^2\psi=\frac{5}{3}$, these reduce to the standard model predictions

$$g_{V,e}^{(1)}=-\tfrac{1}{4}+x;\quad g_{A,e}^{(1)}=\tfrac{1}{4}\,. \tag{6.31}$$

We present sample calculations of the total cross section and forward-backward asymmetry for several cases, each chosen so that

$$M_1=0.98M_0\,. \tag{6.32}$$

As mentioned in Sec. V A, this choice avoids conflict with the Novosibirsk experiment on parity violation in atomic bismuth. It will be very hard to see deviations of the $Z_{(1)}$ mass by 2% from the standard model predictions, at least in initial experiments.

The corresponding Z_2 mass may be deduced from the relations noted in Sec. III. A useful expression is

$$S(\mathrm{B}):\ \epsilon\eta=\sin^2\theta\cot^2\psi=\tfrac{3}{2}x\frac{g_{41}^2}{g_R^2}\,, \tag{6.33}$$

which also holds for case A.1 [which is a special instance of S(B)]. We shall examine four possibilities, listed in Table VII. (The widths are calculated in Sec. VII.) In this calculation, the decay $Z_2\to W^+W^-$ was omitted. It adds several percent

TABLE VII. Parameters of Weinberg-Salam (WS) and two-Z models used in cross-section and asymmetry calculations for $e^+e^-\to\mu^+\mu^-$.

Case	WS	A.1	A.2	B.1	B.2
M_1 (GeV)	88.59	86.82	86.82	86.82	86.82
Γ_1 (GeV)	2.486	2.430	2.424	2.440	2.424
M_2 (GeV)		276.1	230.0	246.6	299.1
Γ_2 (GeV)[a]		3.016	1.678	2.636	3.469

[a]Neglecting width to W^+W^-; see Table IX.

to the total Z_2 width, as will be shown in Sec. VII. We have used a Z_0 mass based on $x = 0.23$, $M_0 = 88.6$ GeV, which is somewhat below the value expected[5] if one-loop weak radiative corrections are taken into account.

The resulting cross sections and asymmetries are shown in Figs. 5 and 6.

The forward-backward asymmetry indeed shows evidence for the second heavier Z at energies well below its pole.[62] However, the deviations from the standard model (and the distinctions among models) are most pronounced in the energy range in which the cross section is quite low. Both the dip in σ and the details of A_{FB} may be hard to see

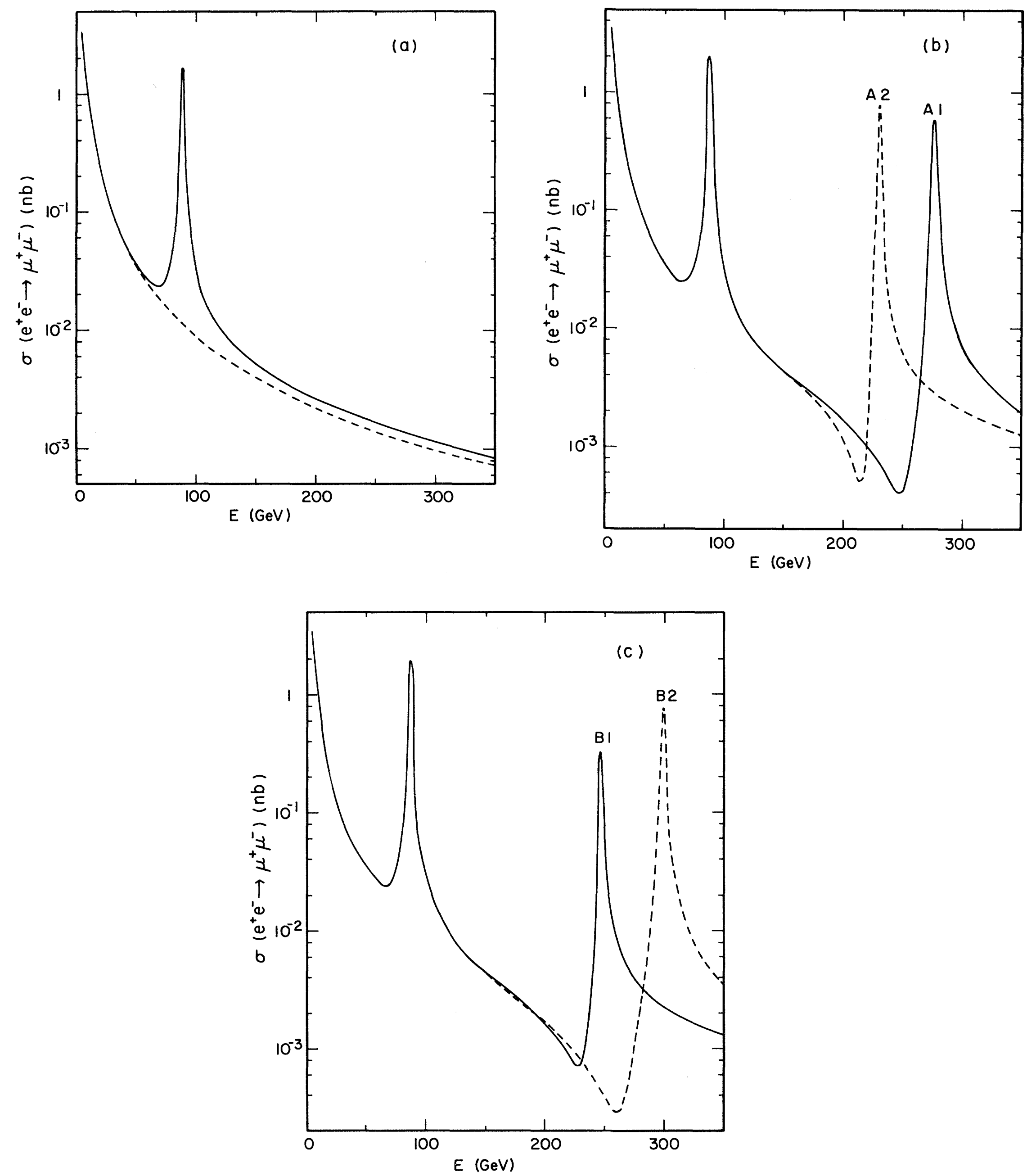

FIG. 5. Cross sections for $e^+e^- \to \mu^+\mu^-$. (a) Dashed line: point cross section; solid line: standard model (one Z_0). (b), (c) Two-Z models with $M_1 = 0.98 M_0$ fixed. (b) Cases A [SU(5)×U(1)]; (c) cases B [SO(6)×SO(4)].

as a result of practical limitations on e^+e^- interaction rates. In Fig. 6, there is no dramatic sign change in A_{FB} between the first and second Z, in contrast to the behavior in some other models.[62]

The heavier Z in the present models thus is best seen by finding a peak in $\sigma(e^+e^-\rightarrow\mu^+\mu^-)$. The magnitude of the cross section at the peak is

$$\sigma_{pk}(e^+e^-\rightarrow\mu^+\mu^-)=\frac{12\pi}{M_i^2}[B(Z_i\rightarrow l^+l^-)]^2 . \tag{6.34}$$

The heavier Z hides itself remarkably well at energies below the peak.

C. Polarization effects in $e^+e^-\rightarrow\mu^+\mu^-$

One can discuss the energy dependence of the muon helicity in $e^+e^-\rightarrow\mu^+\mu^-$ in a manner very similar to that used to discuss the forward-backward asymmetry.[63,64] An identical discussion applies to polarized initial beams.

The average positive muon longitudinal polarization $\bar{P}_L(\mu^+)$, for example, may be expressed as

$$\bar{P}_L(\mu^+)=-\frac{2\,\mathrm{Re}(F_+^*F_0)}{|F_+|^2+|F_-|^2+|F_0|^2} , \tag{6.35}$$

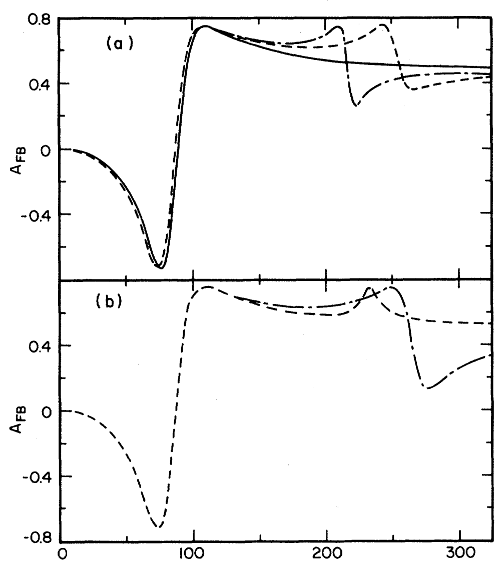

FIG. 6. Forward-backward asymmetry in $e^+e^-\rightarrow\mu^+\mu^-$. Solid line: standard model. (a) Cases A [SU(5)×U(1)]; dashed line, A.1; dash-dotted line, A.2. (b) Cases B [SO(6)×SO(4)]; dashed line, B.1; dash-dotted line, B.2.

which reduces at a Z_i peak to

$$\bar{P}_L(\mu^+)\,|_{Z_i}=\frac{-2g_{V,e}^{(i)}g_{A,e}^{(i)}}{(g_{V,e}^{(i)})^2+(g_{A,e}^{(i)})^2} . \tag{6.36}$$

The energy dependence of the μ^+ polarization for one two-Z example is compared with that of the standard model in Fig. 7. Muon polarization at the Z_i peaks are shown in Table VIII.

At the Z_1 peak the polarization is sensitive to the exact value of x, since $g_{V,e}^{(1)}$ is very nearly zero in the standard model. This behavior requires a more exact treatment, in which one-loop weak radiative corrections to x and to Z_0 properties are taken into account. Once this is done, we would expect a measurement of $\bar{P}_L(\mu^+)$ to within a few percent to give very useful information.

At the Z_2 peak, the different cases yield polarizations ranging from $<60\%$ to 90%. Here useful measurements distinguishing among models can be considerably more crude.

It may be easier to polarize one or both initial lepton beams than to measure final-muon polarization in $e^+e^-\rightarrow\mu^+\mu^-$. A very similar discussion then applies. For a fully polarized e^+ beam, for example, with helicity $\lambda=\pm1$,

$$\left.\frac{d\sigma}{dz}\right|_{\lambda(e^+)=1}-\left.\frac{d\sigma}{dz}\right|_{\lambda(e^+)=-1}=-\frac{\pi\alpha^2}{2s}(1+z)^2\mathrm{Re}(F_+^*F_0) \tag{6.37}$$

and the angular-averaged cross section obeys

$$\frac{\sigma(\lambda(e^+)=1)-\sigma(\lambda(e^+)=-1)}{\sigma(\lambda(e^+)=1)+\sigma(\lambda(e^+)=-1)}=\frac{-2\,\mathrm{Re}(F_+^*F_0)}{|F_+|^2+|F_-|^2+|F_0|^2} , \tag{6.38}$$

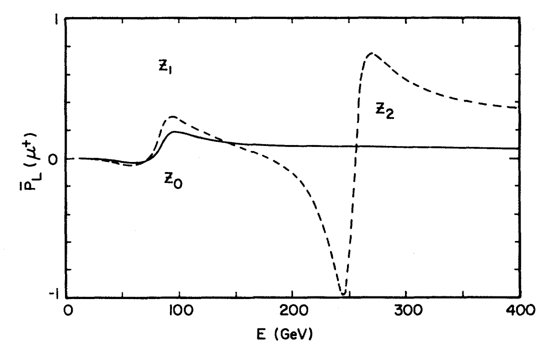

FIG. 7. Average longitudinal μ^+ polarization in $e^+e^-\rightarrow\mu^+\mu^-$ as a function of energy. Solid line: standard model; dashed line: case A.1 (two-Z model).

TABLE VIII. Values of $g_{V,e}^{(i)}$, $g_{A,e}^{(i)}$, and μ^+ polarizations [Eq. (6.33)] at the Z_i peak. Here $x=0.23$ has been assumed.

Case	WS	A.1	A.2	B.1	B.2
$g_V^{(1)}$	-0.02	-0.035	-0.035	-0.033	-0.036
$g_A^{(1)}$	0.25	0.267	0.267	0.269	0.266
$\bar{P}_L(\mu^+)\mid_{Z_1}$	0.16	0.25	0.25	0.24	0.26
$g_V^{(2)}$		0.297	0.281	0.246	0.327
$g_A^{(2)}$		-0.124	-0.105	-0.155	-0.106
$\bar{P}_L(\mu^+)\mid_{Z_2}$		0.71	0.66	0.90	0.58

the same expression as (6.35).

VII. TOTAL WIDTHS AND BRANCHING RATIOS

We can explicitly calculate the decay rates for both neutral bosons Z_1, Z_2 into fermion-antifermion pairs. The Lagrangian (6.28) yields the expression

$$\Gamma(Z_i \to f\bar{f}) = \Gamma_0 N_i [g_{V,f}^{(i)2} + g_{A,f}^{(i)2}] , \tag{7.1}$$

where

$$\Gamma_0 \equiv \frac{M_0 \alpha}{24x(1-x)} = \Gamma(Z_0 \to \nu\bar{\nu}) \tag{7.2}$$

is the partial width for a standard Z_0 to decay to $\nu\bar{\nu}$. With the parameters chosen here ($x=0.23$), we have $\Gamma_0 = 152$ MeV. The normalization constants N_i are given by

$$N_i = 8\left[\frac{M_i}{M_0}\right]^4 \frac{|M_j^2 - M_0^2|}{M_2^2 - M_1^2} \quad (i=1,2 \Longrightarrow j=2,1) . \tag{7.3}$$

The vector and axial couplings are

$$g_{V,\nu}^{(i)} = \tfrac{1}{4};\ \ g_{A,\nu}^{(i)} = -\tfrac{1}{4} ; \tag{7.4}$$

and for quarks or charged leptons with weak isospin $I_3(u) \equiv +\frac{1}{2}$, $I_3(d) \equiv -\frac{1}{2}$, and $I_3(e) \equiv -\frac{1}{2}$,

$$g_V^{(i)} = I_3 + g_A^{(i)} - Q\left[1 - \frac{M_0^2}{M_i^2}\cos^2\theta\right] , \tag{7.5}$$

$$g_A^{(i)} = I_3\left[\tan^2\psi - \frac{M_0^2}{M_i^2}\sec^2\psi\right]/2 . \tag{7.6}$$

Here the angle ψ is the same one defined in Sec. VI.

The resulting branching ratios are shown in Table IX. (The total widths already have been quoted in Table VII.) We have assumed three fermion generations and have neglected the decays $Z_i \to N\bar{N}$; presumably the N's are too heavy to be seen,[38] though the argument is not airtight.[65] (We shall discuss some consequences of detectable $Z_i \to N\bar{N}$ decays presently.) The masses of all fermions (including t quarks) are neglected in comparison with M_i.

The Z_1 differs very little from the Z_0 in its branching ratios for the range of mixing parameters considered here. Its total width also is not very different from that of the standard Z, as noted in Table VII.

The branching ratios of the Z_2 are distinctly different from those of the Z_1. Note in particular the contrast between

$$\frac{B(Z_1 \to d\bar{d})}{B(Z_1 \to u\bar{u})} \approx 1.2 - 1.3$$

and

$$\frac{B(Z_2 \to d\bar{d})}{B(Z_2 \to u\bar{u})} \gtrsim 5 . \tag{7.7}$$

The suppressed $Z_2 \to u\bar{u}$ decay is a peculiar feature of a boson which couples mainly to the χ charge [the charge of the U(1) in SO(10)$\to$SU(5)$\times$U(1)$_\chi$]. In the limit of a very heavy Z_2 in schemes based on SU(5)$\times$U(1)$_\chi$, the decays of Z_2 are in the ratio

$$\begin{aligned}
&\Gamma(Z_2 \to \nu\bar{\nu}) \propto 3^2 = 9 , \\
&\Gamma(Z_2 \to u\bar{u}) \propto 3(1^2 + 1^2) = 6 , \\
&\Gamma(Z_2 \to e\bar{e}) \propto 3^2 + 1^2 = 10 , \\
&\Gamma(Z_2 \to d\bar{d}) \propto 3(3^2 + 1^2) = 30 ,
\end{aligned} \tag{7.8}$$

where the squared integers in the brackets are just proportional to the charges listed in Table I. If

TABLE IX. Branching ratios (in percent of total width to fermions) of Z's in various models. (See Table VII for other parameters.) Values calculated for $M_W = M_Z \cos\theta = 77.7$ GeV. Branching ratios for the value (1.1) will be somewhat lower.

	WS	A.1	A.2	B.1	B.2
$Z_1 \to \nu\bar{\nu}$	6.1	5.6	5.6	5.6	5.7
$u\bar{u}$	10.6	11.0	11.0	11.0	11.0
$e\bar{e}$	3.1	3.3	3.3	3.3	3.3
$d\bar{d}$	13.6	13.4	13.4	13.4	13.4
$Z_2 \to \nu\bar{\nu}$		6.7	7.3	5.6	7.3
$u\bar{u}$		2.5	2.0	3.6	2.0
$e\bar{e}$		5.5	5.3	3.8	6.9
$d\bar{d}$		18.6	18.8	20.3	17.1
$Z_2 \to W^+W^-$		5.1	5.1	4.1	5.5

the N were light enough, we would have

$$\Gamma(Z_2 \to N\bar{N}) \propto 5^2 = 25 \ . \quad (7.9)$$

Taking the decays in (7.8) and assuming three generations, we find the branching ratios for a very heavy Z_2 coupled to χ:

$$\begin{aligned} B(Z_2 \to \nu\bar{\nu}) &= \frac{9}{(3)(55)} = 5.5\% \ , \\ B(Z_2 \to u\bar{u}) &= \frac{6}{(3)(55)} = 3.6\% \ , \\ B(Z_2 \to e\bar{e}) &= \frac{10}{(3)(55)} = 6.1\% \ , \\ B(Z_2 \to d\bar{d}) &= \frac{30}{(3)(55)} = 18.2\% \ . \end{aligned} \quad (7.10)$$

The suppressed coupling of Z_2 to $u\bar{u}$ is a distinct disadvantage in hadronic production experiments, as we shall see in Sec. VIII. On the other hand, the leptonic branching ratio is actually expected to be a little bigger than that of Z_1.

It is unlikely, but still possible, that the right-handed neutral leptons N are light enough to permit the decays $Z_{1,2} \to N\bar{N}$. Let us imagine the N mass in a generation $g \equiv e,\mu,\tau$ to be constrained by the relation[38]

$$M_{N(g)} = O\left[\frac{m_{l(g)}{}^2}{m_{\nu(g)}}\right] , \quad (7.11)$$

where l and ν are the corresponding charged lepton and neutrino. Direct experimental bounds[65] on neutrino masses then permit N masses as light as 10 or 20 GeV for each generation. However, if astrophysical bounds on neutrino masses are to be taken seriously,[66] $m_{\nu(g)} \lesssim 100$ eV, the corresponding N masses for the second and third generations are considerably larger, and only the decays $Z_{1,2} \to N(e)\bar{N}(e)$ remain possible. As mentioned earlier, the Z_2 should then have a prominent $N\bar{N}$ decay.[67]

It is also possible that astrophysical *lower* bounds may apply directly to N masses. These tend to be in the 10-GeV range.[68]

For reference we note that (6.17) and (6.18) imply

$$g_{V,N}^{(i)} = g_{A,N}^{(i)} = \tfrac{1}{4} \sec^2\psi \left[1 - \frac{M_0{}^2}{M_i{}^2}\right] . \quad (7.12)$$

An interesting feature of the standard model is that, in the limit of zero fermion mass, the Z_0 decays half the time to $\nu\bar{\nu} + u\bar{u}$ and half the time to $e\bar{e} + d\bar{d}$, independent of the value of $x = \sin^2\theta$:

$$\Gamma(Z_0 \to (\nu\bar{\nu} + u\bar{u})) = \Gamma(Z_0 \to (e\bar{e} + d\bar{d})) \ . \quad (7.13)$$

This relation is no longer satisfied exactly in the present class of models if the N is heavy. However, if the N is light in comparison with either Z, we find that for both Z's,

$$\Gamma(Z_i \to (N\bar{N} + \nu\bar{\nu} + u\bar{u})) = \Gamma(Z_i \to (e\bar{e} + d\bar{d})), \quad (7.14)$$

a generalization of (7.13). The relation (7.14) holds independent of mixing. To derive it one may use the explicit forms of the couplings (7.5), (7.6), and

(7.12). Alternatively, when N and ν masses may be neglected, we may replace N and ν by an ordinary Dirac fermion ν_D. In that case all partial widths take the form $\Gamma \sim g_V{}^2+g_A{}^2$, where

$$g_V=aI_3+bQ \ , \tag{7.15}$$

$$g_A=(a-1)I_3 \ , \tag{7.16}$$

where $I_3(\nu_D)=I_3(u)\equiv\frac{1}{2}$, $I_3(e)=I_3(d)\equiv-\frac{1}{2}$, and the magnitudes of a and b need not concern us. The relation (7.14) then holds because

$$[I_3(\nu_D)]^2+3[I_3(u)]^2=[I_3(e)]^2+3[I_3(d)]^2 \ ,$$

$$I_3(\nu_D)Q(\nu_D)+3I_3(u)Q(u) =I_3(e)Q(e)+3I_3(d)Q(d) \ , \tag{7.17}$$

$$[Q(\nu_D)]^2+3[Q(u)]^2=[Q(e)]^2+3[Q(d)]^2 \ .$$

Because after diagonalization of the mass matrix the physical Z_2 field has a small W_3 component, the decay $Z_2\to W^+W^-$ is allowed as long as $M_2>2M_W$. The decay is governed by the usual SU(2) three-gauge-boson vertex weighted by the expansion coefficient $\cos\theta\sin\phi$ in Eqs. (6.11) and (6.12). We find, in our notation, that

$$\Gamma(Z_2\to W^+W^-)=\Gamma_0 N_2(1-x)^3(M_0/M_2)^5(1-\zeta^{-2})^{1/2}(\zeta^2-1)(4\zeta^4+20\zeta^2+3) \ , \tag{7.18}$$

where $\zeta\equiv M_2/2M_W$. The factor $(1-\zeta^{-2})^{1/2}$ coming from the one-body decay phase space drives the rate to zero when $M_2=2M_W$ as it must. The zero at $\zeta^2=1$ in the additional factor comes from angular momentum conservation in the fundamental gauge coupling. The contribution of the channel to the total rate is tabulated in Table IX for the usual four cases and is seen to be roughly equivalent to an additional leptonic width. This decay to W^+W^- pairs would be seen as a small additional peak in the dilepton ($e^\pm e^\mp$, $\mu^\pm\mu^\mp$, or $e^\pm\mu^\mp$) spectrum, broadened by the motion of the W's.

VIII. $Z_{1,2}$ PRODUCTION IN $\bar{p}p$ AND pp COLLISIONS.

The photoproduction of the standard Z_0 in $\bar{p}p$ and pp collisions is eagerly anticipated. Many estimates exist[69] for the rates. These may be adapted with little difficulty to the present situation.

A simple parametrization of the cross section for $\overset{(-)}{p}p\to\mu^+\mu^-X$ has been given[8] in terms of the rapidity y and mass m of the $\mu^+\mu^-$ system and the proton structure functions. Let the quark distributions be denoted by

$$u=u_v+\xi; \ \ d=d_v+\xi \ , \tag{8.1}$$

where u_v and d_v denote valence quark distributions and ξ denotes the sea distribution. We assume that $u=\bar{u}=d=\bar{d}=s=\bar{s}$ in the sea, and that the neutral-current interactions of $s\bar{s}$ are the same as those of $d\bar{d}$. Then (we omit X)

$$\frac{d\sigma}{dy\,dm}(\bar{p}p\to\mu^+\mu^-)\,|_{y=0} =\frac{2x^2}{3m}[\,(u^2+\xi^2)\sigma(u\bar{u}\to\mu^+\mu^-) +(d^2+3\xi^2)\sigma(d\bar{d}\to\mu^+\mu^-)] \ , \tag{8.2a}$$

$$\frac{d\sigma}{dy\,dm}(pp\to\mu^+\mu^-)\,|_{y=0} =\frac{2x^2}{3m}[\,2u\xi\sigma(u\bar{u}\to\mu^+\mu^-) +(2d\xi+2\xi^2)\sigma(d\bar{d}\to\mu^+\mu^-)] \ , \tag{8.2b}$$

where the structure functions are evaluated at $x=m/\sqrt{s}$. We may integrate these expressions with respect to mass over a resonant peak, recalling that

$$\sigma_{\rm pk}(q\bar{q}\to\mu^+\mu^-)=\frac{12\pi}{M_i{}^2}\left[\frac{x_{q\bar{q}}}{3}\right]x_{\mu^+\mu^-} \ . \tag{8.3}$$

The branching ratios $x_{q\bar{q}}$ and $x_{\mu^+\mu^-}$ are given in Table IX. We then find

$$\left.\frac{d\sigma}{dy}\right|_{y=0}(\bar{p}p\to Z_i\to\mu^+\mu^-) =x^2[A\,(u^2+\xi^2)+B(d^2+3\xi^2)] \ , \tag{8.4}$$

$$\left.\frac{d\sigma}{dy}\right|_{y=0}(pp\to Z_i\to\mu^+\mu^-) =x^2[A(2u\xi)+B\,(2d\xi+2\xi^2)] \ , \tag{8.5}$$

where

$$\left.\begin{matrix} A \\ B \end{matrix}\right\} = \frac{4\pi^2 \Gamma_{\rm tot} x_{\mu^+\mu^-}}{3M_i^3} \times \left\{\begin{matrix} x_{u\bar{u}} \\ x_{d\bar{d}} \end{matrix}\right. . \qquad (8.6)$$

The structure functions are assumed to be those of Ref. 70. We evaluate them for simplicity at a fixed value of $Q^2 = 10^4$ GeV^2 and $\Lambda = 500$ MeV, and find

$$x(u_v + d_v) = 2.5x^{0.4}(1-x)^{3.96} , \qquad (8.7)$$

$$xd_v = 1.06x^{0.44}(1-x)^{4.74} , \qquad (8.8)$$

$$x\xi = 0.02(1-x)^{7.05} + 0.139(1-x)^{10.93} + 1.58e^{-37.3x} . \qquad (8.9)$$

In Fig. 8 we show the values of $d\sigma/dy$ for Z production as a function of $\sqrt{s}/M$ in two cases: (a) Weinberg-Salam Z_0, and (b) heavy-Z (Z_2) production in the case A.1, which is common to both SU(5)$\times$U(1) and SO(6)$\times$SO(4) models.

The cross-section estimates are very sensitive to the exponential term in (8.9) above $\sqrt{s}/M = 10$. We thus show estimates both with and without this term. For $3 \lesssim \sqrt{s}/M \lesssim 10$, the cross sections for ($\bar{p}p$ or pp)$\to Z_2 \to \mu^+\mu^-$ have the same shape as those for ($\bar{p}p$ or pp)$\to Z_0 \to \mu^+\mu^-$, but are about a *factor of 20–40 lower at the same value of* $\sqrt{s}/M$. This factor arises from the suppression of the $\bar{u}u$ branching ratio and from the lower value of Γ/M^3 for the higher resonance.

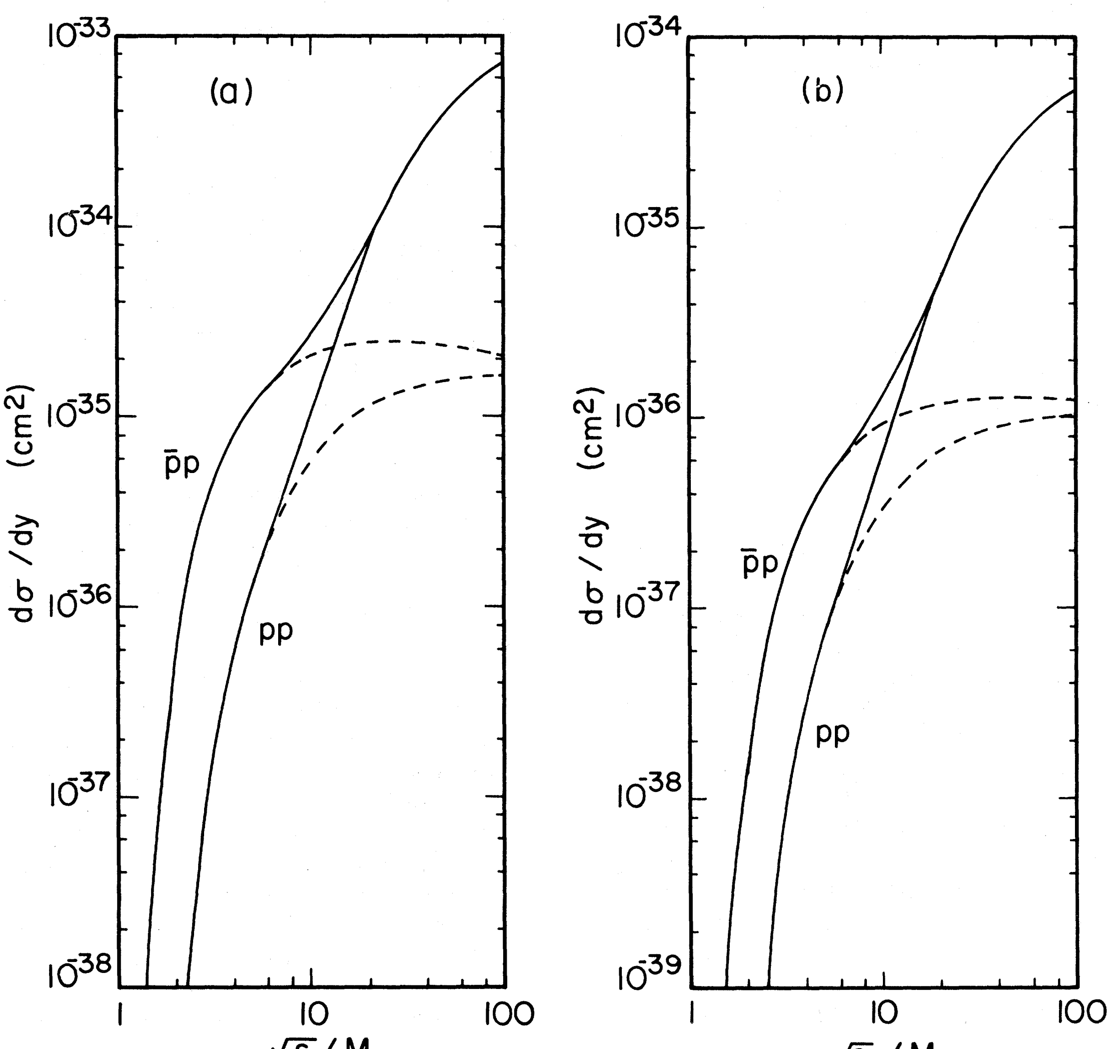

FIG. 8. Values of $d\sigma/dy$ as function of $\sqrt{s}/M$ for $\bar{p}p$ or $pp \to Z \to \mu^+\mu^-$. Solid line: with exponential term in sea quark structure function (8.9); dashed line: without this term. (a) Standard model, Z_0 production; (b) case A.1, Z_2 production (M_2 =276 GeV).

The production of the standard Z_0 in $\bar{p}p$ collisions at CERN ($\sqrt{s}/M = 540/89 \approx 6$) may be compared very closely with that of a heavier Z at the Fermilab $\bar{p}p$ collider. Thus, for $M_2 = 276$ GeV and $\sqrt{s} = 1.9$ TeV, $\sqrt{s}/M \simeq 6.9$. The resulting cross sections are

$$\frac{d\sigma}{dy}\bigg|_{y=0}(\bar{p}p \to Z_0 \to \mu^+\mu^-)\bigg|_{\sqrt{s}=540\text{ GeV}} = 1.5\times 10^{-35}\text{ cm}^2\,, \quad (8.10)$$

$$\frac{d\sigma}{dy}\bigg|_{y=0}(\bar{p}p \to Z_2 \to \mu^+\mu^-)\bigg|_{\sqrt{s}=1.9\text{ TeV}} = 7\times 10^{-37}\text{ cm}^2\text{ (case } A.1)\,. \quad (8.11)$$

Uncertainties in quark distributions should largely cancel out in the ratio between Eqs. (8.10) and (8.11). Thus the observation of Z_0 at $\sqrt{s} = 540$ GeV may provide a very useful calibration in the near future, indicating how easy it may be to detect a heavier Z at $\sqrt{s} \approx 2$ TeV.

A typical $\bar{p}p$ collision experiment at Fermilab is expected to involve an integrated luminosity of $\approx 10^{36}$ cm^{-2}.[71] The cross section (8.11) thus is only marginally detectable there, unless the luminosity exceeds present projections of $\mathscr{L} \approx 10^{30}$ cm^{-2}s^{-1}.

For pp interactions at $\sqrt{s} = 800$ GeV, the observation of Z_2 (in case A.1) may be just barely possible in an experiment of integrated luminosity $\int \mathscr{L}\, dt = 10^{39}$ cm^{-2}. We estimate in Fig. 8(b)

$$\frac{d\sigma}{dy}(pp \to Z_2 \to \mu^+\mu^-)\bigg|_{\sqrt{s}=800\text{ GeV}} = 4\times 10^{39}\text{ cm}^2\,. \quad (8.12)$$

If the Z_2 is somewhat lighter than the value considered above ($\approx 3M_0$), its observation becomes easier. As M_2 decreases, the situation improves more rapidly for pp than for $\bar{p}p$ collisions. However, one soon encounters difficulty (within the context of the models discussed here) with the constraints discussed in Sec. V B arising from parity violation in heavy atoms. There is thus *a narrow window of heavy-Z masses* [$\approx(2.5-3)M_0$] arising naturally in the context of SO(10), which is both experimentally accessible (in the next few years) and not ruled out by any other experiment.

IX. CONCLUSIONS AND DISCUSSION

We have examined some properties and consequences of a second neutral heavy vector-boson that arises naturally in various SO(10) theories. The discussion has involved both the chain starting with SU(5)$\times$U(1) and that starting with SO(6)$\times$SO(4).

We have found that a second Z arising in these theories has minimal effects on the low-energy behavior of neutral-current interactions, since neutrino interactions at zero momentum transfer are not affected at all. Polarized-electron deep-inelastic scattering near $Q^2 = 0$ is affected very little. The main information that restricts the second Z to be above about 2.5 times the standard Z_0 mass comes from one experiment[19] on parity violation in heavy atoms.

There have been numerous other investigations of two-Z models.[6–18] The present discussion is a particularly conservative version of such models in that very little of the standard picture is altered. The small effect of the boson coupled to U(1)$_\chi$ in SO(10)$\to$SU(5)$\times$U(1)$_\chi$ on phenomena up to energies of a couple of hundred GeV, even when that boson is only three times the mass of the standard Z_0, comes as a particular surprise to us. The models we consider here are thus particularly demanding alternatives to the standard one; nonetheless they were not constructed with any artificially small couplings. All the couplings follow from the group structure and the symmetry-breaking pattern.

Notable among the properties of a gauge boson coupled mainly to U(1)$_\chi$ is its small coupling to $u\bar{u}$. This makes its production in $\bar{p}p$ and pp collisions difficult. It should couple strongly to pairs of right-handed neutral leptons $N\bar{N}$ if these leptons are light enough. Finally, the physical Z_2 boson should have a small W_3 admixture, enabling it to decay to W^+W^- with a surprisingly large (several percent) branching ratio.

Many of our illustrative calculations were performed for a light Z (Z_1) mass only two percent below the prediction (1.2) of the standard model. This choice was based on constraints from atomic parity violation in bismuth.[19] (Our "one-loop" standard-model Z_0 weighs 88.6 GeV/c^2, but its corrected mass—when $x = \sin^2\theta$ is lowered to ≈ 0.21 by radiative corrections and other corrections are taken into account—should be about 5 GeV/c^2 higher[5].) Thus if experiments in $p\bar{p} \to Z \to \mu^+\mu^-$ in the next year find a peak below about 90 GeV/c^2, either more radical alterations of the standard picture must be considered, or the atomic-physics results of Ref. 19 must be wrong. If the lowest peak is *above* the value (1.2), even the rather general assumptions of Ref. 6 will have to be reexamined, and the present discussion certainly will be moot.[72] For example, a model based on $SU(2)\times U(1)\times SU(2)'$, but with couplings not related to our schemes (*B*), can accommodate masses of all the lightest gauge bosons larger than their values in the standard picture.[9]

If the Z_0 mass is found to be within about 2 GeV of the prediction (1.2), we have shown that there are still many interesting possibilities for a heavy *Z* above a mass of a couple of hundred GeV. These are best addressed by direct experimental search. We have considered the possibility that the second *Z* alters weak effects in such processes as $e^+e^-\to\mu^+\mu^-$ or deep-inelastic *ep* scattering, but in the class of models considered here there seems to be little alternative to actually *producing and detecting the second Z.* This appears to be in contrast to some left-right-symmetric models in which more radical modifications of couplings are made in comparison to those of the standard model.[10,18,60,62,63]

An early motivation for the present work was to gain some idea of the magnitude of the coupling g_χ in $SO(10)\to SU(5)\times U(1)_\chi$. The results of Sec. IV indicate that the low-energy behavior of the theory is remarkably insensitive to this coupling. The mass of the second *Z* provides some information, but only if the light-*Z* mass (M_1) is known very well. The best information on g_χ is provided by the total decay width of the second *Z*.

What if a heavy *Z* is not found around $(2.5-3)M_0$? Certainly nothing in the SO(10) theory requires it to be so light. Searches in e^+e^- annihilations at higher energies (e.g., $E_{\text{c.m.}}\lesssim 1$ TeV) are conceivable in the next twenty years.[73] This is a very small range compared with the possibility that the second *Z* we consider here could be as heavy as the grand unification mass, $\gtrsim 10^{15}$ GeV/c^2, or need not exist at all if SU(5) is the ultimate step in grand unification. In this last context it will be interesting to watch the results of forthcoming experiments on proton stability. These have the capability both of demonstrating the validity of SU(5) as an intermediate stage in grand unification, and of telling whether a symmetry beyond SU(5) is valid.[74] The first question will be answered in the affirmative if $\tau_p\lesssim 10^{31}$ yr (Ref. 48); the second, if branching ratios differ from the SU(5) predictions.[75] If proton-decay experiments show there is grand unification physics beyond SU(5), the existence of a second Z_0 is almost a certainty, and the only question is whether we can be so fortunate as to observe it.

ACKNOWLEDGMENTS

We are grateful to V. Barger, M. A. B. Bég, J. D. Bjorken, S. Dawson, G. R. Farrar, N. Fortson, H. Frisch, E. Henley, C. N. Leung, F. Schrempp, M. Shochet, R. Slansky, L. Wolfenstein, and A. Zee for useful discussions, and C. N. Leung for comments on the manuscript.

This work was supported in part by the U. S. Department of Energy under Contracts Nos. EY-76-C-02-1764 and DE-AC0276ER00881. Part of this work forms the Ph.D. thesis of one of us (R.R.) submitted to the University of Minnesota. J.L.R. wishes to thank the Aspen Center for Physics and Professor H. Fritzsche of the University of Chicago for hospitality during part of this work.

APPENDIX

In this appendix we consider the question, discussed in Sec. IV, of when the effective neutral-current interactions of gauge models with additional U(1) factors derived from the same larger group through different breakdown schemes, are equivalent. We will find that the condition that various U(1) coupling constants be equal, Eq. (4.15), can be generalized.

We consider two different breakdown schemes of a single larger group whose neutral currents are described by couplings, diagonal generators, and neutral fields g_i, D_i, $A_{\mu i}$ and g_i', D_i', $A_{\mu i}'$ respectively. The D_i, D_i' are taken to be orthonormal vectors in their group space (as in Sec. II B) so that

$$D_i\cdot D_j=D_i'\cdot D_j'=\tfrac{1}{2}\delta_{ij}\ . \tag{A1}$$

The interaction Lagrangian for the unprimed case can be written as

$$-\mathscr{L}_f=\bar f\gamma^\mu(g_iD_iA_{\mu i})f=\bar f\gamma^\mu\underline{\mathscr{D}}^T\underline{A}_\mu f\ , \tag{A2}$$

where

$$\underline{A}_\mu=(A_1,\ldots,A_N)\ , \tag{A3}$$

$$\underline{\mathscr{D}}=(g_1D_1,\ldots,g_ND_N)\ , \tag{A4}$$

with a similar expression for the primed case,

$$-\mathscr{L}_f'=\bar f\gamma^\mu\underline{\mathscr{D}}^{T'}\underline{A}_\mu' f\ . \tag{A5}$$

If the primed and unprimed fields are to be related by an orthogonal transformation,

$$\mathscr{O}\underline{A}_\mu=\underline{A}_\mu'\ , \tag{A6}$$

then demanding that the effective interactions be equivalent,

$$-\mathscr{L}_f=\bar{f}\gamma^{\mu}(\underline{\mathscr{D}}^T\mathscr{O}^T\mathscr{O}\underline{A}_{\mu})f$$
$$=\bar{f}\gamma^{\mu}(\underline{\mathscr{D}}^T\mathscr{O}^T\underline{A}'_{\mu})f$$
$$=-\mathscr{L}'_f \tag{A7}$$

requires

$$\mathscr{O}\underline{\mathscr{D}}=\underline{\mathscr{D}}' \tag{A8}$$

or

$$\mathscr{O}_{ij}g_jD_j=g_i'D_i' \ . \tag{A9}$$

Multiplication (in the group space) by D_k on both sides,

$$\mathscr{O}_{ij}g_jD_j\cdot D_k=g_i'D_i'\cdot D_k$$

or (A10)

$$\mathscr{O}_{ij}g_j\tfrac{1}{2}\delta_{jk}=g_i'D_i'\cdot D_k$$

gives

$$\mathscr{O}_{ij}=\frac{g_i'}{g_j}2D_i'\cdot D_j \ . \tag{A11}$$

The D_i' are linear combinations of the D_i, so defining the transformation A by

$$D_i'=A_{ij}D_j \ ,$$

we find that A is orthogonal and

$$A_{ij}=2D_i'\cdot D_j \ , \tag{A12}$$

so that

$$\mathscr{O}_{ij}=\frac{g_i'}{g_j}A_{ij} \tag{A13}$$

with both $\mathscr{O}$ and A orthogonal. Equation (A13) gives

$$\mathscr{O}_{ij}g_j=g_i'A_{ij}$$

or (A14)

$$\mathscr{O}G=G'A \ ,$$

where

$$(G)_{ij}=g_j\delta_{ij}$$
$$(G')_{ij}=g_i'\delta_{ij} \tag{A15}$$

Using (A14), we find that

$$AG^2=G'^2A$$

or (A16)

$$\sum_k A_{ik}{g_k}^2\delta_{kj}=\sum_k {g_i'}^2\delta_{ik}A_{kj} \ ,$$

i.e.,

$$A_{ij}{g_j}^2=A_{ij}{g_i'}^2 \ , \tag{A17}$$

so that $g_j=g_i'$ if $A_{ij}\neq 0$. Thus, any two diagonal operators in the different breakdowns that are connected, i.e., for which $A_{ij}=2D_i'\cdot D_j\neq 0$, must have their coupling constants equal in order to rotate the various U(1) factors among themselves and still have the same interactions. Furthermore, the matrix A can be made block diagonal for an appropriate ordering of the labels. We note that this discussion is just a restatement of Schur's lemma.

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