Maximal grand unification, gauge hierarchies, and baryon nonconservation

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We study the constraints on the hierarchy of gauge-boson mass scales from low-energy physics in the maximal grand unification model based on the group SU(16). We show that for certain values of $\sin^2\theta_W(m_W)$ and $\alpha_{\rm strong}(m_W)$ allowed by low-energy data, the intermediate mass scales associated with local $B-L$ symmetry and right-handed gauge interactions may be as low as 10^3 GeV. We use a previously suggested method to study the Higgs-boson effects on these mass scales. We also discuss the implications for baryon nonconservation in this model and point out that the most likely $\Delta B\neq 0$ processes in simple versions of the model are the ones obeying $\Delta(B-L)=0$ selection rule.

I. INTRODUCTION

The maximal symmetry associated with the eight fermions and eight antifermions of each generation $(\nu,e^-,u_i,d_i;i=1,2,3)$ is SU(16). An obvious and perhaps natural candidate for grand unification of electroweak and strong interactions is therefore the broken local SU(16) group[1] associated with this symmetry. In contrast to other unification groups such as SU(5) and SO(10), SU(16) has the feature that the baryon and lepton numbers are exact symmetries of the Lagrangian, to be spontaneously broken. Furthermore, a new possibility in maximal gauging schemes is to allow for a richer variety of selection rules associated with baryon- and lepton-number violation, which are not present in other more economical grand unification schemes. Observability of these different selection rules at low energies, however, depends on the pattern of the breakdown of local SU(16) to $SU(3)_c\times U(1)_{EM}$ as well as on the hierarchy of gauge-boson masses. Several patterns of symmetry breakdown in this model have been discussed in Ref. 1, in order to extract the various selection rules associated with $\Delta B\neq 0$ and $\Delta L\neq 0$ processes. Particular scenarios have been isolated where proton decay[2] and n-$\bar{n}$ oscillations[3,4] may coexist. It is the purpose of this paper to study the constraints imposed on the hierarchy of gauge-boson masses in the SU(16) model allowed by the present values of low-energy parameters such as $\sin^2\theta_W$ and $\alpha_{\rm strong}$ and their implications for baryon- and lepton-number nonconservation. For this purpose, we write down the equations for the evolution of the various gauge coupling constants[5] in different chains of symmetry breaking of the SU(16) model. We present our analysis with and without the inclusion of Higgs-boson effects. We then discuss how various $\Delta B\neq 0$ and $\Delta L\neq 0$ processes could arise in such models.

The paper is organized as follows. In Sec. II, we introduce the SU(16) model to fix conventions and notations and note the various patterns of symmetry breaking. In Secs. III and IV we write down the equations for the evolution of various coupling constants for two patterns of symmetry breaking and write down the formula for $\sin^2\theta_W$ and $\alpha_s(m_W)$ and extract the constraints on intermediate mass scales of the model ignoring the effect of Higgs bosons for each chain of symmetry breaking. In Sec. V we introduce the Higgs multiplets necessary for breakdown of SU(16) symmetry and in Sec. VI, outline the criteria for inclusion of Higgs multiplets in the evolution equations and study their effect on the mass hierarchies. In Sec. VII, the implications of the model for baryon nonconservation are discussed.

II. THE SU(16) MODEL

The grand unification symmetry per generation is assumed to be the SU(16) group[1] with left-handed particles and antiparticles belonging to the fundamental representation of the group as follows:

$$\psi_A=\begin{bmatrix} u_i\\ \nu\\ d_i\\ e^-\\ u_i^c\\ \nu^c\\ d_i^c\\ e^+\end{bmatrix}_L \quad i=1,2,3\ , \tag{1}$$

where the ψ_A stands for entry in the Ath row: $A=1,\cdots 16$.

In order to cancel the anomalies, we will introduce[1] a set of mirror fermions with right-handed chirality transforming also as the 16-dimensional representation of SU(16). These mirror fermions will be assumed to be in the mass range of 100 to 200 GeV. We consider the following chains of breaking for SU(16) down to $SU(3)_c \times U(1)_{EM}$:

$$\text{(I)}\ \ SU(16) \to SU(12)_q \times SU(4)_l \times U(1)_{|B|-|L|} \to SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \to SU(3)_c \times SU(2)_L \times U(1)_Y\ ,$$

$$\text{(II)}\ \ SU(16) \to SU(8)_L \times SU(8)_R \times U(1)_F \to SU(4) \times SU(2)_L \times SU(2)_R \to SU(3)_c \times SU(2)_L \times U(1)_Y\ ,$$

$$\text{(III)}\ \ SU(16) \to SO(10) \to SU(5) \to SU(3)_c \times SU(2)_L \times U(1)_Y\ .$$

The Higgs multiplets necessary for the various breaking chains will be displayed later. In the rest of the paper, the chain (III) will not be discussed since that leads to the results already familiar for SO(10) and SU(5) models.

The fermion masses will arise by introducing a 136-dimensional symmetric Higgs multiplet $\Phi_{\{AB\}}$, which couples to Ψ as $h\psi_A^T C^{-1}\psi_B \Phi^{\{AB\}}$, and assigning vacuum expectation values to appropriate components of $\Phi^{\{AB\}}$. We will discuss the detailed nature of symmetry breakdown in a subsequent section.

III. SU(16) BREAKING TO $SU(12)_q \times SU(4)_l \times U(1)_{|B|-|L|}$ AND EVOLUTION OF COUPLING CONSTANTS

In this section, we propose to study the relation between the $SU(3)_c$, $SU(2)_L$, U(1) coupling constants and the grand unified coupling g_U via the Gell-Mann—Low equations. We wish to investigate the symmetry-breaking pattern (I) of the previous section, where SU(16) first breaks down to $SU(12)_q \times SU(4)_l \times U(1)_{|B|-|L|}$. The $SU(12)_q$ is the maximal symmetry of quarks and antiquarks and $SU(4)_l$ operates on the leptonic space. At the next stage, we may have two possibilities:

$$SU(12)_q \times SU(4)_l \times U(1)_{|B|-|L|} \longrightarrow SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}\ ,$$

$$SU(12)_q \times SU(4)_l \times U(1)_{|B|-|L|} \searrow SU(3)_c \times SU(4)_q \times SU(4)_l \times U(1)_{|B|-|L|} \nearrow SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

where $SU(2)_{L,R}$ operates both on quarks and leptons. The left-right-symmetric group[7] is broken down in the usual manner[8] to $SU(3)_c \times SU(2)_L \times U(1)$. We will use the method of Georgi, Quinn, and Weinberg[5] as appropriately extended to include models with intermediate mass scales. To write down the general formula relating the gauge coupling constants at two different energies, we simply have to integrate the Gell-Mann—Low equation between successive mass scales with appropiate values for the β function and appropriate normalization factor for generators. To state the general formula, let us assume breaking of a group as follows:

$$G_N \to G_{N-1} \to G_{N-2} \to \cdots$$

with associated mass scales $\mu_N, \mu_{N-1}, \mu_{N-2} \cdots$ and where $G^x = \prod_\alpha \otimes G_\alpha^x$; i.e., it may be a direct product of simple groups, with associated coupling constants denoted by g_α^x corresponding to group G_α^x. The relevant formula relating the coupling constants at two mass scales is then given by

$$\frac{1}{[g_\alpha^x(\mu_x)]^2} = \sum_\beta \frac{P_{\alpha\beta}^{x,x+1}}{[g_\beta^{x+1}(\mu_{x-1})]^2} + 2b_\alpha^x \ln\frac{\mu_{x+1}}{\mu_x}\ . \qquad (2)$$

This formula is a somewhat modified form of the formula given by Dawson and Georgi,[6] the difference being that in Eq. (2) $P_{\alpha,\beta}^{x,x+1}$ are not normalized to add up to 1. This is because all our coupling constants g_α^x will represent physical couplings and therefore, the generators at each stage defined in the 16-dimensional space are not necessarily normalized, i.e., $\mathrm{Tr}(T^x_\alpha T^x_\beta)\neq\frac{1}{2}\delta_{\alpha\beta}$. The simplest way to construct the $P^{xy}_{\alpha\beta}$ is to choose a convenient basis for the diagonal generators for G and find the diagonal generators for the subgroups G^y_β in the same basis and express it in terms of those for G^x_α. We will give the explicit construction for $P^{xy}_{\alpha\beta}$ for the SU(16) breaking. We normalize all our SU(16) generators by

$$\mathrm{Tr}(T_a T_b)=\tfrac{1}{2}\delta_{ab}\ . \tag{3}$$

T_a will be 16×16 matrices in the space of Ψ. We first give the SU(12) generators ($\underline{0}$ stands for the null 8×8 matrix):

$$T_1^{W\times c}=\frac{1}{2}\begin{pmatrix} 1 &&&&&&&& \\ & 0 &&&&&&& \\ && 0 &&&&&& \\ &&& 0 &&&&& \\ &&&& -1 &&&& \\ &&&&& 0 &&& \\ &&&&&& 0 && \\ &&&&&&& 0 & \\ &&&&&&&& \underline{0} \end{pmatrix},$$

$$T_2^{W\times c}=\frac{1}{2}\begin{pmatrix} 0 &&&&&&&& \\ & 1 &&&&&&& \\ && 0 &&&&&& \\ &&& 0 &&&&& \\ &&&& 0 &&&& \\ &&&&& -1 &&& \\ &&&&&& 0 && \\ &&&&&&& 0 & \\ &&&&&&&& \underline{0} \end{pmatrix}, \tag{4}$$

$$T_3^{W\times c}=\frac{1}{2}\begin{pmatrix} 0 &&&&&&&& \\ & 0 &&&&&&& \\ && 1 &&&&&& \\ &&& 0 &&&&& \\ &&&& 0 &&&& \\ &&&&& 0 &&& \\ &&&&&& -1 && \\ &&&&&&& 0 & \\ &&&&&&&& \underline{0} \end{pmatrix}$$

These are the $\mathrm{SU}(2)_{W_l}$ generators for particles. The corresponding generators for antiparticles are obtained by interchanging the upper and lower 8×8 block matrices:

$$\bar{T}_1^{W\times c}=\frac{1}{2}\begin{pmatrix} \underline{0} &&&&&&&& \\ & -1 &&&&&&& \\ && 0 &&&&&& \\ &&& 0 &&&&& \\ &&&& 0 &&&& \\ &&&&& 1 &&& \\ &&&&&& 0 && \\ &&&&&&& 0 & \\ &&&&&&&& 0 \end{pmatrix} \tag{5}$$

and similarly $\bar{T}_2$ and $\bar{T}_3$. Next, we define the diagonal generators for chiral $SU(3)_c$ color:

$$
\begin{aligned}
T_4^c &= \frac{1}{2\sqrt{2}} \begin{bmatrix} 1 &&&&&&&& \\ & -1 &&&&&&& \\ && 0 &&&&&& \\ &&& 0 &&&&& \\ &&&& 1 &&&& \\ &&&&& -1 &&& \\ &&&&&& 0 && \\ &&&&&&& 0 & \\ &&&&&&&& \underline{0} \end{bmatrix} , \\
\bar{T}_5^c &= \frac{1}{2\sqrt{6}} \begin{bmatrix} 1 &&&&&&&& \\ & 1 &&&&&&& \\ && -2 &&&&&& \\ &&& 0 &&&&& \\ &&&& 1 &&&& \\ &&&&& 1 &&& \\ &&&&&& -2 && \\ &&&&&&& 0 & \\ &&&&&&&& \underline{0} \end{bmatrix} , \\
\bar{T}_4^c &= \frac{1}{2\sqrt{2}} \begin{bmatrix} \underline{0} &&&&&&&& \\ & -1 &&&&&&& \\ && 1 &&&&&& \\ &&& 0 &&&&& \\ &&&& 0 &&&& \\ &&&&& -1 &&& \\ &&&&&& 1 && \\ &&&&&&& 0 & \\ &&&&&&&& 0 \end{bmatrix} , \\
\bar{T}_5^c &= \frac{1}{2\sqrt{6}} \begin{bmatrix} \underline{0} &&&&&&&& \\ & -1 &&&&&&& \\ && -1 &&&&&& \\ &&& 2 &&&&& \\ &&&& 0 &&&& \\ &&&&& -1 &&& \\ &&&&&& -1 && \\ &&&&&&& 2 & \\ &&&&&&&& 0 \end{bmatrix} .
\end{aligned}
\tag{6}
$$

The color- and flavor-singlet diagonal generator— the last diagonal generator of SU(12)—is

$$T_{11}=\frac{1}{2\sqrt{6}}\begin{bmatrix}\operatorname{diag}(1,\ 1,\ 1,\ 0,\ 1,\ 1,\ 1,\ 0,\ -1,\ -1,\ -1,\ 0,\ -1,\ -1,\ -1,\ 0)\end{bmatrix}. \tag{7}$$

The $SU(4)_l$ generators are given by

$$T_l^W=\frac{1}{2}\begin{bmatrix}\operatorname{diag}(0,\ 0,\ 0,\ 1,\ 0,\ 0,\ 0,\ -1,\ \underline{0})\end{bmatrix},$$

$$\bar{T}_l^W=\frac{1}{2}\begin{bmatrix}\operatorname{diag}(\underline{0},\ 0,\ 0,\ 0,\ -1,\ 0,\ 0,\ 0,\ 1)\end{bmatrix}, \tag{8}$$

$$T_{14}=\frac{1}{2\sqrt{2}}\begin{bmatrix}\operatorname{diag}(0,\ 0,\ 0,\ 1,\ 0,\ 0,\ 0,\ 1,\ 0,\ 0,\ 0,\ -1,\ 0,\ 0,\ 0,\ -1)\end{bmatrix}.$$

The $B-L$ generator is given by

$$T_{|B|-|L|}=\frac{1}{2\sqrt{24}}\,\mathrm{diag}(1,1,1,-3,1,1,1,-3,1,1,1,-3,1,1,1,-3)\,. \tag{9}$$

The above generators are part of the Lie algebra of both $G_0=\mathrm{SU}(16)$ as well as $G_1=\mathrm{SU}(12)_q\times\mathrm{SU}(4)_l\times\mathrm{U}(1)_{|B|-|L|}$. Therefore, if the grand unification mass is taken as M_U and the scale at which G_l breaks down is taken as M_c, the relation between coupling constants in this region is simply given by

$$\begin{aligned}\frac{1}{g_{12}{}^2(M_c)}&=\frac{1}{g_U{}^2(M_U)}+2b_{12}\ln\frac{M_U}{M_c}\,,\\ \frac{1}{g_4{}^2(M_c)}&=\frac{1}{g_U{}^2(M_U)}+2b_4\ln\frac{M_U}{M_c}\,,\end{aligned} \tag{10}$$

where b_{12} and b_4 stand for the one-loop contributions to the β function for SU(12) and SU(4).

To consider the breaking of $\mathrm{SU}(12)_q\times\mathrm{SU}(4)_L\times U(1)_{|B|-|L|}$ to $\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}_{B-L}(1)$, we must express the diagonal generators of G_2, in terms of those of G_1:

$$Y_{B-L}=(T_{11}-\sqrt{3}T_{14}) \tag{11}$$

which leads to the following values for relevant "probabilities,"

$$P_{B-L,\mathrm{SU}(12)_q}=1,\quad P_{B-L,\mathrm{SU}(4)_l}=3 \tag{12}$$

coming to the case of $\mathrm{SU}(2)_{L,R}$, note that, at the SU(2) level, the physical $T_{3L,R}$ generator is given by

$$T_{3L}=T_1^{W\times c}+T_2^{W\times c}+T_3^{W\times c}+T_l^{W}\,. \tag{13}$$

This implies that

$$\frac{1}{g_{L,R}{}^2(M_c)}=\frac{3}{g_{12}{}^2(M_c)}+\frac{1}{g_4{}^2(M_c)}$$

or, in the language of "probabilities,"

$$P_{\mathrm{SU}(2)_L;\mathrm{SU}(12)_q}=3,\quad P_{\mathrm{SU}(2)_L;\mathrm{SU}(4)_l}=1\,. \tag{14}$$

The same relation holds for the $\mathrm{SU}(2)_R$ group also. Similar considerations for $\mathrm{SU}(3)_c$ lead to

$$P_{\mathrm{SU}(3)_c;\mathrm{SU}(12)_q}=4\,. \tag{15}$$

Using Eqs. (11)–(15), we can write down the expression for the gauge coupling constants at the mass scale M_R, whose G_2 breaks down:

$$\begin{aligned}\frac{1}{g_3{}^2(M_R)}&=\frac{4}{g^2(M_U)}+2\left[4b_{12}\ln\frac{M_U}{M_c}+b_3\ln\frac{M_c}{M_R}\right],\\ \frac{1}{g_{2L/R}{}^2(M_R)}&=\frac{4}{g^2(M_U)}+2\left[(3b_{12}+b_4)\ln\frac{M_U}{M_c}+b_{2L/R}\ln\frac{M_c}{M_R}\right],\\ \frac{1}{g_{B-L}{}^2(M_R)}&=\frac{4}{g^2(M_U)}+2\left[(b_{12}+3b_4)\ln\frac{M_U}{M_c}+b_{B-L}\ln\frac{M_c}{M_R}\right].\end{aligned} \tag{16}$$

Using a similar procedure and noting that for $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$,

$$Y=\sqrt{3/5}T_{3R}+\sqrt{2/5}Y_{B=L} \tag{17}$$

we can write down the expression for gauge coupling constants at $\mu=M_L$ where the symmetry $SU(2)_L\times U(1)_Y$ is broken down to $U(1)_{EM}$. They are

$$\begin{aligned}
\frac{1}{g_3{}^2(M_L)}&=\frac{4}{g^2(M_U)}+2\left[4b_{12}\ln\frac{M_U}{M_c}+b_3\ln\frac{M_c}{M_L}\right],\\
\frac{1}{g_{2L}{}^2(M_L)}&=\frac{4}{g^2(M_U)}+2\left[(3b_{12}+b_4)\ln\frac{M_U}{M_c}+b_{2L}\ln\frac{M_c}{M_L}\right],\\
\frac{1}{g_Y{}^2(M_L)}&=\frac{4}{g^2(M_U)}+2\left[\tfrac{1}{5}(11b_{12}+9b_4)\ln\frac{M_U}{M_c}+\tfrac{1}{5}(3b_{2R}+2b_B)\ln\frac{M_c}{M_R}+b_Y\ln\frac{M_R}{M_L}\right].
\end{aligned} \tag{18}$$

Let us recall that the value of b_N for a group $SU(N)$ is

$$b_N=-\frac{1}{16\pi^2}\left[\tfrac{11}{3}N-\tfrac{4}{3}f-\tfrac{1}{3}T_N\right], \tag{19}$$

where f is the number of fermion multiplets transforming as the fundamental representation of the group $SU(N)$, and T_N is the value of the second-order Casimir operator on the representation of Higgs mesons. Using this, we obtain the following formulas for the $\sin^2\theta_W(M_L)$ and $\alpha_s(M_L)$ for chain I of symmetry breaking ($M_L=M_W$, the conventional W-boson mass):

$$\begin{aligned}
\sin^2\theta_W(M_L)=\frac{3}{8}-\frac{11\alpha(M_c)}{24\pi}\Bigg\{&4[4-\tfrac{1}{22}(T_{12}-T_4)]\ln\frac{M_U}{M_c}\\
&+[2-\tfrac{1}{22}(5T_{2L}-3T_{2R}-2T_B)]\ln\frac{M_c}{M_R}+5[1-\tfrac{1}{22}(T_{2L}-T_Y)]\ln\frac{M_{12}}{M_L}\Bigg\},
\end{aligned} \tag{20}$$

TABLE I. Allowed intermediate mass scales (in GeV) for the symmetry-breaking chain $SU(16)\rightarrow SU(12)_q\times SU(4)_l\times U(1)_{|B|-|L|}$ without the effect of Higgs bosons.

Mass W_R	Mass M_C	Unification mass	$\sin^2\theta_W$
	$\alpha_s=0.11$		
1.5×10^{12}	6.7×10^{14}	10^{15}	0.22
1.8×10^{10}	4.9×10^{14}	10^{15}	0.23
1.8×10^{10}	9.5×10^{16}	10^{17}	0.23
2.8×10^{6}	5.1×10^{8}	10^{10}	0.25
2.8×10^{6}	2.6×10^{14}	10^{15}	0.25
2.8×10^{6}	5.1×10^{16}	10^{17}	0.25
4.1×10^{2}	5.2×10^{2}	10^{5}	0.27
4.1×10^{2}	2.7×10^{8}	10^{10}	0.27
4.1×10^{2}	1.4×10^{14}	10^{15}	0.27
4.1×10^{2}	2.7×10^{16}	10^{17}	0.27
	$\alpha_s=0.12$		
6.3×10^{11}	5.6×10^{14}	10^{15}	0.22
7.7×10^{9}	4.1×10^{14}	10^{15}	0.23
7.7×10^{9}	7.9×10^{16}	10^{17}	0.23
1.2×10^{6}	4.2×10^{8}	10^{10}	0.25
1.2×10^{6}	2.2×10^{14}	10^{15}	0.25
1.2×10^{6}	4.2×10^{16}	10^{17}	0.25
1.7×10^{2}	4.3×10^{2}	10^{5}	0.27
1.7×10^{2}	2.2×10^{8}	10^{10}	0.27
1.7×10^{2}	1.2×10^{14}	10^{15}	0.27
1.7×10^{2}	2.2×10^{16}	10^{17}	0.27

$$\frac{1}{\alpha_s(M_L)}=\frac{3}{8\alpha(M_L)}-\frac{11}{24\pi}\left\{4[12-\tfrac{3}{22}(T_{12}-T_4)]\ln\frac{M_U}{M_c}+[6-\tfrac{1}{22}(8T_3-3T_{2L}-3T_{2R}-2T_B)]\ln\frac{M_c}{M_R}\right.$$
$$\left.+[9-\tfrac{1}{22}(8T_3-3T_{2L}-5T_Y)]\ln\frac{M_R}{M_L}\right\}. \quad (21)$$

Using a range of values of $\sin^2\theta_W(M_L)$ from 0.22 to 0.27, in Table I, we give the allowed values for the various intermediate mass scales without taking into account the effect of Higgs bosons. Interestingly enough, we find that, there do exist solutions with rather low mass parity restoration,[9] if we accept $\sin^2\theta_W=0.26-0.27$. As has been shown, by Rizzo and Senjanovic,[9] such "large" values of $\sin^2\theta_W$ are allowed once the right-handed current effects are taken into account. Thus, SU(16) grand unified theory provides another class of models in which a low-W_R $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ electroweak theory[7,8] can be embedded. Phenomenological implications for such models have been discussed in Refs. 7–9. We note that the value of the unification mass $M_U=M_X$ in this case can be as low as 10^5. One therefore has to be careful in introducing baryon-number violation in this situation.

IV. SU(16) BREAKING VIA $SU(8)_L\times SU(8)_R$ SYMMETRY

In this section, we consider the breaking of SU(16) via the chain $SU(8)_L\times SU(8)_R\times U(1)_F$. This subgroup has already been discussed in literature as a candidate for the grand unification group.[10] It is clear, that the physics above the $SU(8)_L\times SU(8)_R$ breaking scale is not of importance to us here. We therefore concentrate on the breaking pattern below the mass scale M_8 where $SU(8)_L\times SU(8)_R$ is broken down to $SU(2)_L\times SU(2)_R\times SU(4)_c\times U(1)_F$, etc., i.e., chain II of Sec. II.

For completeness, we present the basis and the diagonal generators to obtain the evolution equations for the gauge couplings:

$$T_1^{W\times c}=\frac{1}{2}\begin{pmatrix}1&&&&&&&&\\&0&&&&&&&\\&&0&&&&&&\\&&&0&&&&&\\&&&&-1&&&&\\&&&&&0&&&\\&&&&&&0&&\\&&&&&&&0&\\&&&&&&&&\underline{0}\end{pmatrix},\quad T_3^{W\times c}=\frac{1}{2}\begin{pmatrix}0&&&&&&&&\\&0&&&&&&&\\&&1&&&&&&\\&&&0&&&&&\\&&&&0&&&&\\&&&&&0&&&\\&&&&&&-1&&\\&&&&&&&0&\\&&&&&&&&\underline{0}\end{pmatrix},$$

$$T_2^{W\times c}=\frac{1}{2}\begin{pmatrix}0&&&&&&&&\\&1&&&&&&&\\&&0&&&&&&\\&&&0&&&&&\\&&&&0&&&&\\&&&&&-1&&&\\&&&&&&0&&\\&&&&&&&0&\\&&&&&&&&\underline{0}\end{pmatrix},\quad T_4^{W\times c}=\frac{1}{2}\begin{pmatrix}0&&&&&&&&\\&0&&&&&&&\\&&0&&&&&&\\&&&1&&&&&\\&&&&0&&&&\\&&&&&0&&&\\&&&&&&0&&\\&&&&&&&-1&\\&&&&&&&&\underline{0}\end{pmatrix}. \quad (22)$$

$$T_6^c=\frac{1}{2\sqrt{2}}\begin{bmatrix}1 &&&&&&&& \\ & -1 &&&&&&& \\ && 0 &&&&&& \\ &&& 0 &&&&& \\ &&&& 1 &&&& \\ &&&&& -1 &&& \\ &&&&&& 0 && \\ &&&&&&& 0 & \\ &&&&&&&& \underline{0}\end{bmatrix},$$

$$T_6^c=\frac{1}{2\sqrt{6}}\begin{bmatrix}1 &&&&&&&& \\ & 1 &&&&&&& \\ && -2 &&&&&& \\ &&& 0 &&&&& \\ &&&& 1 &&&& \\ &&&&& 1 &&& \\ &&&&&& -2 && \\ &&&&&&& 0 & \\ &&&&&&&& \underline{0}\end{bmatrix},$$

$$T_7^l=\frac{1}{4\sqrt{3}}\begin{bmatrix}1 &&&&&&&& \\ & 1 &&&&&&& \\ && 1 &&&&&& \\ &&& -3 &&&&& \\ &&&& 1 &&&& \\ &&&&& 1 &&& \\ &&&&&& 1 && \\ &&&&&&& -3 & \\ &&&&&&&& \underline{0}\end{bmatrix},\qquad (23)$$

$$T_{15}=\frac{1}{4\sqrt{2}}\begin{bmatrix}1 &&&&&&&& \\ & 1 &&&&&&& \\ && 1 &&&&&& \\ &&& 1 &&&&& \\ &&&& 1 &&&& \\ &&&&& 1 &&& \\ &&&&&& 1 && \\ &&&&&&& 1 & \\ &&&&&&&& -\mathrm{I}\end{bmatrix}.$$

For the antiparticle sector the null 8×8 matrix and the first 8×8 matrix in T_i are interchanged with a negative sign.

For the next stage of $SU(2)_L\times SU(2)_R\times SU(4)_c$, we can write down the "probability" functions as

$$P_{SU(2)_L,SU(8)_L}=4,\ \ P_{SU(2)_R,SU(8)_R}=4,\ \ P_{SU(4)_c,SU(8)_L}=2,\ \ P_{SU(4)_c,SU(8)_R}=2\ . \qquad (24)$$

At the next stage, we break the group down to $SU(3)_c\times SU(2)_L\times U(1)_Y$ so that, only two nontrivial probability functions arise, i.e.,

$$P_{Y,SU(2)_R}=\tfrac{3}{5},\ \ P_{Y,SU(4)_c}=\tfrac{2}{5}\ . \qquad (25)$$

Using all these results and following exactly the same procedure as in Sec. III, we obtain the following formula for $\sin^2\theta_W(M_L)$ and $\alpha_s(M_L)$:

$$\sin^2\theta_W=\tfrac{3}{8}-\frac{11\alpha(M_L)}{48\pi}\left\{[-4-\tfrac{1}{11}(5T_{2L}-3T_{2R}-2T_4)]\ln\frac{M_8}{M_R}+5[2-\tfrac{1}{11}(T_{2L}-T_Y)]\ln\frac{M_R}{M_L}\right\},$$
$$\frac{1}{\alpha_s(M_L)}=\frac{3}{8\alpha(M_L)}-\frac{11}{48\pi}\left\{[12-\tfrac{1}{11}(6T_4-3T_{2R}-3T_{2L})]\ln\frac{M_8}{M_R}+[18-\tfrac{1}{11}(8T_3+5T_{2L}-13T_Y)]\ln\frac{M_R}{M_L}\right\}. \tag{26}$$

Again as in Sec. III, we will defer the inclusion of Higgs contributions to a subsequent section and present in Table II the mass scales without their effect. In this case, we do not find any solutions with low-energy parity restoration even for "large" values of $\sin^2\theta_W$.

V. HIGGS BOSONS IN SU(16)

We will assume that the symmetry breaking of the SU(16) model is implemented by including explicit Higgs scalar multiplets into the theory and giving nonzero vacuum expectation values to appropriate components. We first discuss the Higgs multiplets necessary for breakdown of SU(16) symmetry to $SU(3)_c\times U(1)_{EM}$. We discuss their implications for neutrino masses and in the next section study their impact on the gauge-boson mass hierarchy.

We will consider the following Higgs multiplets: one belonging to the 255-dimensional adjoint representation denoted by Φ_B^A; a second one belonging to the 18 240-dimensional representaiton denoted by $\Phi\{^{A,B}_{C,D}\}$ where the curly bracket stands for symmetrization with respect to the indices within the brackets; a third one belonging to the 16-dimensional representation Φ_A and finally the 136-dimensional symmetric Higgs field $\Phi_{\{A,B\}}$. Symmetry breaking discussed in Ref. 1 is different from ours. We, therefore, describe it below. Φ_B^A will be used to implement the first stage of the symmetry breaking in the chains II. Note that it cannot be used to implement[11] the first stage of symmetry breaking in chain I, i.e., SU(16) $\rightarrow SU(12)_q\times SU(4)_l\times U(1)_{|B|-|L|}$. We will use the $\Phi\{^{A,B}_{C,D}\}$ multiplet to implement this stage of the symmetry breaking. Let us first concentrate on chain I. For this purpose, we need the representation contents of the Higgs multiplets under the various symmetry groups involved at different stages in this chain. We use two $\Phi\{^{A,B}_{C,D}\}$ multiplets to implement the first and second stages of the symmetry breaking. We first display the representation content of $\Phi\{^{A,B}_{C,D}\}$ under $SU(12)_q\times SU(4)_l\times U(1)_{B-L}$:

$$\underline{18\,240}=(924;4)+(\overline{78};10)+(\overline{12};4)+(12,36)+\text{H.c.}+(5940;1)+(1;84)+(143;1)+(143;15)+(1;15)+(1;1)\,, \tag{27}$$

where H.c. stands for conjugate representations to the ones preceding it. It is, therefore, clear that by giving a nonzero vacuum expectation value (VEV) to the (1;1) component of 18 240, we break the group down to $SU(12)_q\times SU(4)_l\times U(1)_{|B|-|L|}$. We note that in Eq. (27), the representation $(\overline{78},10)$ and (143;15) representations contain singlets under $SU(3)_c\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$. We assume that these components develop VEV's which break the group down to $SU(3)_c\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$.

To achieve the final stages of the breaking, we use the fundamental and the symmetric Higgs multiplet 136 which has the following representation content under $SU(3)_c\times SU(2)_L\times SU(2)_R U(1)_{B-L}$:

$$\underline{16}=(3,2,1)_1+(\overline{3},1,2)_{-1}+(1,2,1)_{+3}+(1,1,2)_{-3}\,, \tag{28}$$

$$\underline{136}=(3,1,1)_2+(6,3,1)_2+(1+8,2,2)_0+(3,1,1)_{-2}+(\overline{6},1,3)_{-2}+(1,3,1)_{-6}+(1,2,2)_0+(1,1,3)_6\,. \tag{29}$$

We may use 16 to break the $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ symmetry down to $SU(2)_L\times U(1)$ and 136 to break the group down to $SU(3)_c\times U(1)_{EM}$.

It is necessary to point out the neutrino mass is sensitive to the multiplets used in breaking left-right symmetry. If we use 16 to break $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$, at the tree level the neutrino has only a Dirac mass. However, it can have a Majorana mass arising in higher order by a mechanism originally discussed by Witten.[12] We

TABLE II. Allowed intermediate mass scale (in GeV) for the symmetry-breaking chain $SU(16)\to SU(8)_L \times SU(8)_R \times U(1)_F$ without including the effect of Higgs bosons. $\alpha_s = 0.1$.

Mass M_R	Mass M_C	$\sin^2\theta_W$
1.2×10^{14}	6.3×10^{14}	0.22
4.0×10^{13}	1.1×10^{15}	0.23
1.3×10^{13}	1.9×10^{15}	0.24
4.4×10^{12}	3.3×10^{15}	0.25
1.5×10^{12}	5.7×10^{15}	0.26
4.9×10^{11}	9.9×10^{15}	0.27

show the relevant graph in Fig. 1. Noting that $\underline{136}$ gives mass to the light fermions, we can estimate the Majorana mass of the right-handed neutrino to be

$$m_{\nu_R} \cong \frac{\alpha^2}{16\pi^2}\frac{m_U}{M_L}\left[\frac{M_R}{M_U}\right]^2 \lambda M , \quad (30)$$

where λM stands for the dimensional $\underline{16}\times\underline{16}\times\underline{136}$ Higgs coupling. A natural choice for M is M_U since $\underline{16}\times\underline{16}\times\underline{136}$ coupling is likely to be a low-energy remnant of the $\Phi_B \Phi_A \Phi_C^A \Phi^{\{BC\}}$ coupling on setting $\langle \Phi_C^A \rangle \neq 0$. Thus, we expect

$$m_{\nu_R} \cong \frac{\alpha^2}{16\pi^2}\lambda \left[\frac{m_U}{M_L}\right]\left[\frac{M_R}{M_U}\right] M_R . \quad (31)$$

For $M_R \simeq 10^8$GeV, $M_U \simeq 10^{15}$ GeV (a choice allowed by $\sin^2\theta_W$ and α_s) we expect $m_{\nu_R} \simeq 1$ eV. In this case, the Dirac mass is much bigger than the Majorana mass. However if we have $M_U \simeq M_R \simeq 10^{14}$ GeV, this leads to $m_{\nu_R} \simeq 10^4$ GeV which is quite an interesting prediction. This predicts the light left-handed Majorana neutrino mass to be around $\simeq 1-10$ eV.

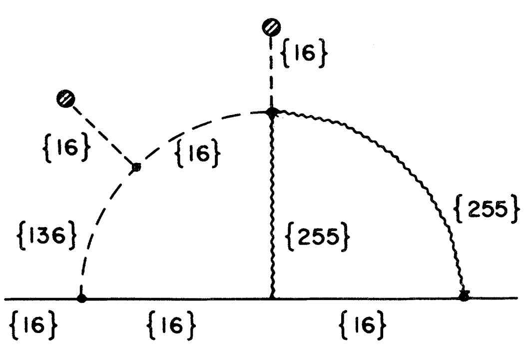

FIG. 1. The two-loop graph that generates the Majorana mass for ν_R.

Thus, if we accept the low-mass-W_R solutions, the left-right symmetry as well as the $SU(2)_L \times U(1)$ symmetry must be broken by $\underline{136}$ dimensional representations. The Majorana mass for ν_R is then given by $\sim M_R$ and $m_{\nu_L} \cong m_U{}^2/M_R$. For the sake of completeness, we mention that the chain II of symmetry breaking via the $SU(8)_L \times SU(8)_R$ route is affected by using at the first stage the adjoint representation Φ_A^B. It has the following representation content under $SU(8)_L \times SU(8)_R \times U(1)_F$:

$$\underline{255} = (1,1)_0 + (63,1)_0 + (1,63)_0 + (8,8)_2 + (\bar{8},\bar{8})_{-2} . \quad (32)$$

Giving $(1,1)_0$ a nonzero VEV breaks SU(16) to $SU(8)_L \times SU(8)_R$. The rest of the breakdown is achieved by the $\Phi\{^{AB}_{CD}\}$, Φ_A, and $\Phi_{\{AB\}}$ and the discussion is similar to that just given.

VI. EFFECT OF HIGGS BOSONS ON HIERARCHY OF GAUGE-BOSON MASSES

In this section, we will describe how to include the Higgs-boson effects consistently in the renormalization-group equations in grand unified theories.[13] The main problem is to find what the masses of the various components of each Higgs multiplet are likely to be. Then, it is straightforward to include their effect in the equations for the evolution of the various gauge coupling constants.

In Ref. 13, a set of rules has been given to isolate which components of a given Higgs multiplet are important at a given mass scale. We summarize them here:

(i) Minimal fine tuning. In the Higgs potential, we will do no more fine tuning than is required to obtain the hierarchy of gauge-boson masses. Also, we will assume all Higgs self-couplings to be of order unity.

(ii) Spontaneous symmetry breakdown and intramultiplet mass splitting. Let the grand unification group G_0 breakdown be as follows:

$$G_0 \supset G_1 \supset G_2 \supset \cdots . \quad (33)$$

Let the associated mass scales be $\mu_0, \mu_1, \mu_2, \ldots$. Let Φ belong to an irreducible representation of G_0 and be used to break the group G_m to G_{m+1}. Let Φ have the following representation content under G_m:

$$\Phi = \sum_j \Phi_j^m . \quad (34)$$

If $\langle \Phi_k^m \rangle = \mu_m$ is responsible for the breaking of G_m to G_{m+1}, we postulate that[13] the whole submultiplet Φ_j^m has mass of order μ_m and will contribute only above $Q > \mu_m$.

(iii) Survival hypothesis. To discuss this, let us discuss the above example. The question is what is the mass of Φ_j^m; $j \neq k$. We postulate that, of Φ_j^m, any set of multiplets which constitutes a full irreducible representation under any of the groups G_{m-n}, $n=0,1,2,\ldots\ m-1$, will acquire a mass corresponding to the mass scale breaking the group G_{m-n-1} to G_{m-n}, i.e., μ_{m-n-1}, unless any submultiplet happens to be a pseudo-Goldstone boson. This is similar to the survival hypothesis discussed by Georgi[14] for the case of fermions.

We will now apply these criteria to the SU(16) Higgs multiplets. Let us take each multiplet one by one for case I. It is obvious that, since $\Phi\left\{\begin{smallmatrix}AB\\CD\end{smallmatrix}\right\}$ is involved in the first stage, all its submultiplets have mass of order M_U and, therefore, do not contribute to coupling-constant evolution. Let us next consider the multiplet $\Phi\left\{\begin{smallmatrix}AB\\CD\end{smallmatrix}\right\}$ that breaks $SU(12)_q \times SU(4)_c \times U(1)_{|B|-|L|}$. Since it is the $(78;\overline{10})$ component which acquires a nonzero VEV, its mass will be of order M_c and will therefore contribute to b_{12} and b_4 above M_c. All the remaining multiplets in Eq. (26) are superheavy and are not relevant to us. Now, let us assume that $\underline{16}$ breaks $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Under $SU(12)_q \times SU(4)_l$,

$$\underline{16} = (12;1) + (1;4)\ , \tag{35}$$

where, under $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$,

$$(1,4) = (1,2,1) + (1,1,2)\ . \tag{36}$$

It is the (1,1,2) which breaks $SU(2)_R \times U(1)_{B-L}$, therefore, (1,1,2) has mass of order M_R and will contribute b_{2R} and $b_{(B-L)}$. On the other hand, the multiplet (1,2,1) will also have a mass of order M_R by left-right symmetry. On the other hand, the (12,1) part of the multiplet will be superheavy by the survival hypothesis (iii) above.

Let us, finally, consider the multiplet $\underline{136}$ that breaks $SU(2)_L \times U(1)$ to $U(1)_{EM}$. Under $SU(12)_q \times SU(4)_l$,

$$\underline{136} = (78,1) + (12,4) + (1,10)\ . \tag{37}$$

To break $SU(2)_L \times U(1)$, we use (1,10) and (78,1). Therefore, by the survival hypothesis, (12,4) will acquire mass of order M_U. Now, under $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$,

$$(78,1) = (\overline{3},1,1)_2 + (3,1,1)_{-2} + (6,3,1)_2 + (\overline{6},1,3)_{-2} + (1+8,2,2)_0\ , \tag{38}$$

TABLE III. Allowed intermediate mass scale (in GeV) for the symmetry-breaking chain $SU(16) \to SU(12)_q \times SU(4)_l \times U(1)_{|B|-|L|}$ with the Higgs-boson effect taken into account.

Mass W_R	Mass M_C	Unification mass	$\sin^2\theta_W$
	$\alpha_s = 0.11$		
4.1×10^{12}	9.6×10^{14}	10^{15}	0.23
6.6×10^{7}	4.3×10^{9}	10^{10}	0.25
5.3×10^{8}	7.9×10^{14}	10^{15}	0.25
1.2×10^{9}	1.0×10^{17}	10^{17}	0.25
7.0×10^{2}	1.7×10^{3}	10^{4}	0.27
1.1×10^{3}	1.9×10^{4}	10^{5}	0.27
8.6×10^{3}	3.5×10^{9}	10^{10}	0.27
7.0×10^{4}	6.4×10^{14}	10^{15}	0.27
1.6×10^{5}	8.2×10^{16}	10^{17}	0.27
	$\alpha_s = 0.12$		
1.7×10^{12}	8.1×10^{14}	10^{15}	0.23
2.7×10^{7}	4.1×10^{9}	10^{10}	0.25
2.2×10^{8}	7.4×10^{14}	10^{15}	0.25
5.1×10^{8}	9.4×10^{16}	10^{17}	0.25
2.9×10^{2}	1.6×10^{3}	10^{4}	0.27
4.4×10^{2}	1.8×10^{4}	10^{5}	0.27
3.6×10^{3}	3.3×10^{9}	10^{10}	0.27
2.9×10^{4}	6.0×10^{14}	10^{15}	0.27
6.6×10^{4}	7.7×10^{16}	10^{17}	0.27

$$(1,10)=(1,3,1)_{-6}+(1,1,3)_{6}+(1,2,2) \ . \quad (39)$$

The $(1,2,2)_0$ parts in both Eqs. (38) and (39) will acquire VEV's of order m_{W_L} and will give Dirac masses to quarks and leptons. It is clear that the multiplets $(\bar{3},1,1)$, $(3,1,1)$, $(6,3,1)$, $(\bar{6},1,3)$, $(8,2,2)$, $(1,3,1)$, and $(1,1,3)$ will acquire mass of order M_c and will therefore contribute to b_{12} and b_4. There are two left-handed doublets with mass of order M_R and two left-handed doublets with mass of order M_L. Thus, above M_R, two (1,2,2) multiplets contribute to b_{2L} and b_{2R} whereas above M_L, two left-handed doublets contribute to b_{2L}.

We illustrate their effect only for the symmetry-breaking chain I and the resulting mass scales are given in Table III. We have presented this simply as an illustration and in subsequent discussions, we will not use the particular numbers reported. We will in the subsequent section discuss phenomenological implications of this model including different kinds of Higgs multiplets. The main point, we wish to make, however, is that the Higgs-boson effects on mass scales are not completely negligible.

VII. SPONTANEOUS SYMMETRY BREAKING AND PREDICTIONS FOR BARYON NONCONSERVATION

In this section, we wish to include some comments on the detailed mechanism for spontaneous breakdown via the $SU(12)_q \times SU(4)_l \times U(1)_{|B|-|L|}$ route and its phenomenological implications. We remind the reader that, in this chain, we will temporarily use two symmetric-adjoint multiplets $\Phi\left\{\begin{smallmatrix}AB\\CD\end{smallmatrix}\right\}$ to break the group from SU(16) to $SU(12)_q \times SU(4)_l \times U(1)_{|B|-|L|}$ and then to $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. To implement the first stage, we assign

$$\langle \Phi^{\alpha a}_{\alpha a} \rangle = \frac{M_U}{g} \ , \quad (40)$$

where α runs over the quarks and antiquarks, i.e., $\alpha = 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 13, 14, 15$ and a goes over the leptons and antileptons: $a = 4, 8, 12$, and 16.

We want to discuss which gauge bosons acquire mass at a given stage. For that purpose, we fix the following notation:

Color gluons: $V_i^j - \frac{1}{3}\sum V_i^i, \ i,j = 1 \cdots$;

Left—and right- handed weak gauge bosons: $W_{L,R,p}, \ p = 1,2,3$;

$U(1)_{B-L}$ gauge boson: B ;

Leptoquark bosons: $X^0_\alpha a = 4,8,12,16, \ \alpha = 1,2,3,5,6,7,9,10,11,13,14,15$;

Diquark gauge bosons connecting quarks and antiquarks: $Y_\alpha^\beta, \ \beta = 1,2,3,5,6,7, \ \alpha = 8,10,11,13,14,15$;

Dileptons: $Y_b'^{\,a}, \ a = 4,8, \ b = 12,16$.

Owing to Eq. (40), the 96 gauge bosons X_α^a acquire mass of order M_U. Note that since the gauge bosons connecting quarks and leptons of each flavor acquire mass of order $M_U \geq 10^{15} \sim 10^6$ GeV, it is consistent with the present data on $K_L \rightarrow \mu e$.

At the second stage of symmetry breaking, we assume that a second $\Phi\left\{\begin{smallmatrix}AB\\CD\end{smallmatrix}\right\}$ with the following VEV exists (summation over color indices i understood):

$$\langle \Phi^{i,i+8}_{12,4} \rangle = \langle \Phi^{4+i,12+i}_{16,8} \rangle = \frac{M_c}{g} \ . \quad (41)$$

This gives mass to 144 gauge bosons including Y_α^β, Y_e^a. But it does not give rise to any mixings between X, Y, and Y' type gauge bosons since Eq. (41) conserves baryon and lepton number. In fact, we find that using this multiplet, it is difficult to get any baryon nonconservation. The point is that the only interesting VEV that leads to $\Delta B \neq 0$ processes and breaks the symmetry appropriately is of type

$$\epsilon^{ijk} \langle \Phi^{u_i d_j}_{d_k^c \nu^c} \rangle = \epsilon^{ijk} \langle \Phi^{u_i d_j}_{u_k^c e^c} \rangle \neq 0 \ , \quad (42)$$

where we have used explicit fermion labels instead of numerical indices. Note that we needed a multiplet with antisymmetrical indices. If we had used symmetric indices, then $\{u_L, d_L\}$ would have given rise to an $SU(2)_L$ triplet and could not have a scale more than $M_L/10$ or so. This conflicts with our previous requirement that

$$\langle \Phi\left\{\begin{smallmatrix}AB\\CD\end{smallmatrix}\right\} \rangle = \frac{M_c}{g} \ .$$

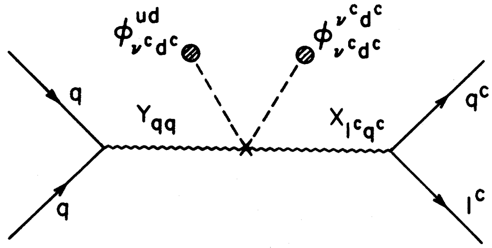

FIG. 2. $(B-L)$-conserving proton decay $p \to e^+\pi^0$.

A way to avoid this problem is to replace the symmetric-adjoint multiplet at the second stage by the antisymmetric-adjoint Higgs multiplet in addition to the symmetric one. This will induce $(B-L)$-conserving proton decay via the diagram of Fig. 2. This amplitude is given by

$$M(p \to e^+\pi^0) \cong \frac{\alpha M_c{}^2 \epsilon}{M_c{}^2 M_U{}^2} \cong \frac{\alpha\epsilon}{M_U{}^2}\ , \tag{43}$$

where ϵ is the mixing parameter between the diquark and leptoquark gauge bosons. For $M_U \cong 10^4-10^5$ GeV, this gives a proton lifetime

$$\tau_p \cong 10^{30}-10^{31}\ \text{yr}\ .$$

Unless ϵ is chosen unnaturally small, proton lifetime would constrain $M_U \geq 10^{14}$ GeV. So, from this point on, we will discuss physics for these values of M_U.

Let us now look for $\Delta B \neq 0$ processes with other selection rules. Note that up to the scale M_R, $B-L$ is an exact symmetry: therefore, $\Delta(B-L) \neq 0$ selection rules must be proportional to M_R. Once we are below the scale M_R, local $B-L$ symmetry is broken by

$$\langle \Phi_{\nu^c\nu^c} \rangle = \frac{M_R}{g} \tag{44}$$

which obeys the selection rule $\Delta(B-L)=2$. The combination of $\Delta(B-L)=2$ and $\Delta(B-L)=0$ operators produce both $\Delta B=2$ processes such as n-$\bar{n}$ oscillation and $\Delta(B+L)=0$ processes such as $n \to e^-\pi^+$ decays. It appears that the dominant contribution to n-$\bar{n}$ oscillations comes from the graph in Fig. 3. This graph is similar to the one noted in Ref. 15 and as already noted this leads to an extremely slow n-$\bar{n}$ oscillation time, $\tau_{n-\bar{n}} \cong 10^{30}$ yr. Thus in this model n-$\bar{n}$ oscillation is suppressed. In fact, it appears to use that n-$\bar{n}$ oscillation will always be suppressed in simple SU(16) models being considered for the following reasons. The only $\Delta(B-L)=2$ VEV allowed by charge conservation is that due to $\nu_R\nu_R$ condensate. But this obeys $\Delta L=2;\Delta B=0$. So, to obtain $\Delta B=2$ processes, we must insert *two* $\Delta(B-L)=0$ VEV's that break ΔB and only such VEV's allowed by color and electric charge conservation are $\langle \Phi^{ud}_{\nu^c d^c} \rangle$, which also gives rise to $p \to e^+\pi^0$ decay. So, roughly speaking, one obtains $M_{n-\bar{n}} \simeq |M_{p\to e^+\pi^0}|^2$ as an order of magnitude estimate. This means $\tau_{n-\bar{n}} \gtrsim 10^{30}$ yr. One way to avoid this, of course, is to introduce very complicated Higgs multiplets with six SU(16) indices, Φ_{ABCDEF}, and choose low unification mass. Then, there will be allowed Higgs-boson self-coupling of the form

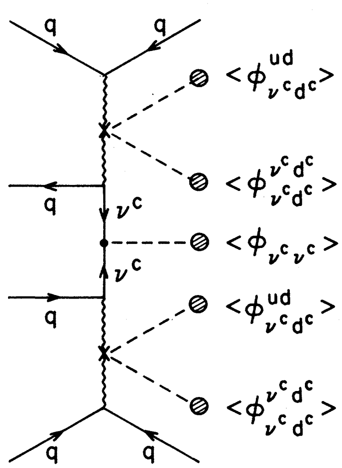

FIG. 3. Feynman graph for n-$\bar{n}$ oscillation in the SU(16) model.

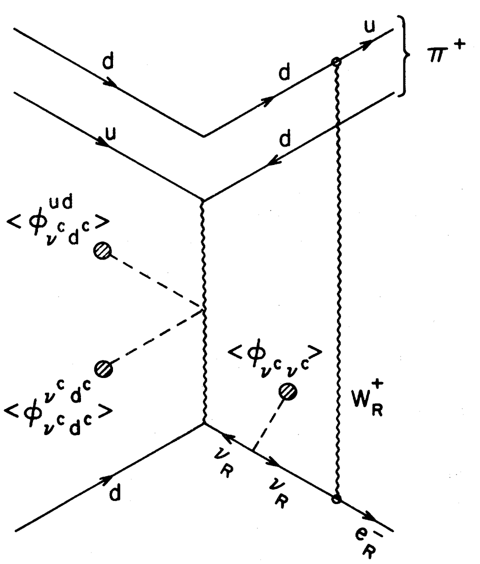

FIG. 4. The one-loop graph leading to $(B+L)$-conserving decay mode $n \to e^-\pi^+$.

$\Phi^{AB}\Phi^{CD}\Phi^{EF}\Phi_{ABCDEF}$. Then giving $\langle \Phi_{uddudd} \rangle \neq 0$ will lead to large n-$\bar{n}$ oscillation amplitude.

The other $\Delta(B-L)=2$ process which respects the $\Delta(B+L)=0$ selection rule arises from the Feynman diagram in Fig. 4, and leads to decays such as $n \to e^- \pi^+$. But these appear to have a strength

$$M_{n\to e^-\pi^+} \cong G_F \alpha \epsilon \frac{m_p{}^2}{M_U{}^2} \simeq 10^{-35}\ \mathrm{GeV}^{-2} . \quad (45)$$

It thus appears that in the model discussed here, the dominant proton decay mode is the $(B-L)$-conserving mode $p \to e^+\pi^0$ which has a lifetime of 10^{30} yr. Finally, we note that in the cases with a low-mass W_R scale, an outstanding signature will be provided by the existence of neutrinoless double-β decay transitions in a manner similar to that discussed in earlier papers.[8] Other processes in the leptonic domain will be processes like $\mu^- \to e^-\gamma$, $m \to 3e$, etc., as already discussed.[8,16]

To summarize, SU(16) has the following predictions for baryon and lepton nonconservations: For high-mass unification, the dominant $\Delta B \neq 0$ process is $\Delta(B-L)=0$ process $p \to e^+\pi^0$, where for low-mass unification (i.e., $M_U \lesssim 10^5$ GeV), the only possible $\Delta B \neq 0$ process is n-$\bar{n}$ oscillation with $\tau_{n-\bar{n}} \simeq 10^{17} - 10^8$ sec. In either case, if $\sin^2\theta_W \simeq 0.27$, and $\alpha_s \simeq 0.1-0.2$, one obtains a low value for m_{W_R}, the scale of right-handed interactions. This has many interesting implications as already discussed in the literature,[8,16] for example, neutrinoless $(\beta\beta_0)$ decay $\mu \to e\gamma$, $\mu \to 3e$, etc.

Note added. After this paper was completed, we were informed that a similar detailed analysis of mass scaled in the SU(16) model has been carried out by J. C. Pati and A. Mohanty, Maryland report (unpublished).

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