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Diquark fragmentation

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Since the diquark system is both composite and colored, it is rapidly becoming an interesting topic of research with the potential to produce considerable future information on the dynamics of subhadron systems and confinement. In this paper we formulate a model with which quantitative studies of diquark (qq) fragmentation can be made. The equations of the model are based on vertices $(qq)\to$baryon $+\,\bar{q}$ and $(qq)\to$meson $+\,(qq)$ leading to recursive cascade-type Volterra integral equations. These equations take the composite nature of a diquark into account and correlate diquark fragmentation into baryons with fragmentation into mesons. Comparison with existing data is good, suggesting that our approach has utility for experimenters planning further studies. The experimenter will also find useful predictions for ud and uu (or dd) fragmentation into mesons and baryons. Our work suggests that the diquark cannot simply be treated as a single colored unit which directly forms a baryon.

I. INTRODUCTION

There exist many types of "hard" interactions, such as those listed in Table I, in which a quark gets "knocked out" of a nucleon. The system left behind, which we call a diquark, is a unique object since it is both manifestly composite (essentially having a spatially extended structure similar to a hadron) as well as colored. Effectively, diquarks are in many respects "colored hadrons" or spatially extended color sources, which presumably fragment into a jet of hadrons due to color confinement. Clearly, a study of diquark fragmentation will offer new insight into how colored objects evolve into hadron jets. For example, it is obviously of interest to ask whether the two quarks in a diquark eventually become part of a single baryon. No detailed theoretical model for diquark fragmentation has yet been studied, although some phenomenological determinations from the data have been made[1] making use of dimensional spectator counting rules.[2]

The purpose of this paper is to formulate and study the consequences of a quantitative model for diquark fragmentation which explicitly takes the composite character of a diquark into account. The model is of a recursive type and is similar in spirit to cascade models which have been successfully used in the past for describing quark and gluon jets.[3,4] The two basic vertices (breakups) of the model are diquark$\to$baryon + antiquark and diquark$\to$meson + diquark, as shown in Fig. 1. The model is mathematically formulated in terms of linear, Volterra integral equations which are solved by successive iteration. Good agreement is obtained with the small amount of currently available data on diquark fragmentation into mesons and baryons. A number of interesting predictions are made, which should be readily testable in future experiments, especially at the Fermilab Tevatron. We find that currently available diquark fragmentation data suggests that the diquark should not be simply treated as a single entity ($\bar{3}$ of color) which directly becomes part of a baryon. Before proceeding further, we wish to point out that a simple model of this type for diquark fragmentation was first discussed by Ilgenfritz *et al.*[5]

The plan of this paper is as follows. In Sec. II, we describe and discuss various experiments which can be (or have been) used to study diquark frag-

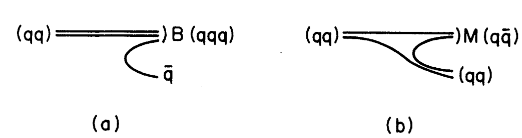

FIG. 1. Diagrams showing two ways in which a diquark can break up. (a) $(qq)\to$baryon$+\bar{q}$ and (b) $(qq)\to$meson$+(qq)$.

mentation, and a description of currently available data is given. In Sec. III, our model for the fragmentation of diquarks is first qualitatively motivated and then quantitatively formulated in terms of integral equations. The iterative solution of our model and the treatment of quark flavors is given in Sec. IV. A comparison of model predictions and available data is made in Sec. V. Our expectations for diquark fragmentation functions into various mesons and baryons are also presented. Finally, Sec. VI contains some critical comments about our work and a summary of results.

II. EXPERIMENTS ON DIQUARK FRAGMENTATION

A number of experiments in which diquark fragmentation can be studied are given in Table I. We shall now discuss the relative merits of these different approaches.

(a) Charged-current interactions with neutrino and antineutrino beams have an advantage, because the flavor content of the fragmenting diquark is known with reasonable certainty (neglecting $q\bar{q}$ sea effects). The first reactions listed in Table I with neutrinos incident on proton and neutron targets lead to (uu) and (ud) diquark systems. Bubble-chamber studies of (uu) fragmentation into charged hadrons have been made,[6] and the data are plotted in Figs. 2 and 3. It is especially worth noting that the ratio of (uu) fragmentation functions into positive and negative hadrons (see Fig. 2) is considerably bigger than unity and increases at large x, where x is the fraction of the diquark momentum carried by the detected hadron. A major part of

TABLE I. Diquark systems occurring in various reactions.

Type of reaction	Examples	Diquark system	Comments
ν reactions	$\nu p \to \mu^- X$	uu	The sea contribution to these reactions is neglected.
	$\nu n \to \mu^- X$	ud	
$\bar{\nu}$ reactions	$\bar{\nu} p \to \mu^+ X$	ud	
	$\bar{\nu} n \to \mu^+ X$	dd	
Electroproduction	$ep \to eX$	$\frac{8}{9} ud + \frac{1}{9} uu$	
	$\mu p \to \mu X$		
	$en \to eX$	$\frac{2}{3} dd + \frac{1}{3} ud$	
	$\mu n \to \mu X$		
Hadronic collisions with $\mu^+\mu^-$ trigger	$\pi^+ p \to (\mu^+\mu^-) X$	uu	Assumes that the Drell-Yan mechanism with valence quarks dominates.
	$\pi^- p \to (\mu^+\mu^-) X$	ud	
	$\bar{p} p \to (\mu^+\mu^-) X$	$\frac{2}{3} ud + \frac{1}{3} uu$	
Hadronic collisions with larger-p_T hadronic trigger	$pp \to \pi^+ X$	$\simeq ud$	Diquark flavor identification is very uncertain—complications due to hard-gluon scattering and flavor identification of hard scattered quark.
	$pp \to \pi^- X$	$\simeq uu$	
Hadronic collisons with low-p_T multiparticle production	p fragmentation	$\frac{2}{3} ud + \frac{1}{3} uu$	Assumes a two-chain model for the Pomeron based on the dual-topological- unitarization scheme.
	$\bar{p}$ fragmentation	$\frac{2}{3} \bar{u}\bar{d} + \frac{1}{3} \bar{u}\bar{u}$	
	n fragmentation	$\frac{2}{3} ud + \frac{1}{3} dd$	

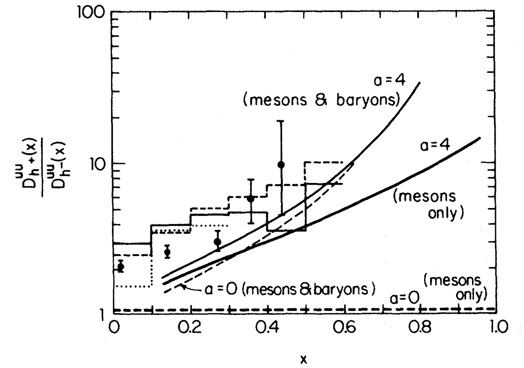

FIG. 2. Comparison of theory with the experimental ratio of (uu) diquark fragmentation into positive hadrons to fragmentation into negative hadrons versus fraction of (uu) momentum x carried by the hadron. The solid circles with error bars are from Ref. 6(a) and the histograms from Ref. 6(b). The dotted histogram gives the π^+/π^- ratio for total hadron energy W in the range $8 < W < 16$ GeV, the dashed histogram for $4 < W < 8$ GeV, and the solid histogram for $2 < W < 4$ GeV. Note that the theory curve labeled (mesons only) gives the ratio of positive to negative mesons and the curve closest to the data is calculated for h = mesons + baryons. The dashed straight line at unity indicates the $a = 0$ prediction for mesons only and the curved dashed line which joins the $a = 4$ curve at $x \simeq 0.6$ is also calculated with $a = 0$ for meson + baryon production.

this increase in the ratio $D^{uu}_{h^+}(x)/D^{uu}_{h^-}(x)$ is due to the production of protons.[6] However, one should note that only protons with $p_{\rm lab} \lesssim 1$ GeV/c are well identified in typical bubble-chamber experiments, so generally positive-meson data cannot be reliably corrected. Data[7] for (ud) fragmentation into pions obtained by using $\bar{\nu}$ beams incident on protons is also shown in Fig. 3. Furthermore, (uu), (ud), (dd) fragmentation into neutral strange particles (Λ, K) has been recently studied in bubble-chamber experiments making use of both neutrino[8] and antineutrino[9] beams. Data for the ratio of (ud) and (dd) diquark fragmentation functions into Λ's are shown in Fig. 4. (The dotted band represents our best guess for this ratio based on the preliminary results of Ref. 8.) Note that there is substantial Λ (uds) production from a (dd) diquark, in contradiction with naive expectations resulting from treating the diquark as a unit which becomes directly part of a baryon. (The treatment of the diquark as an entity which goes directly into a baryon shall be referred to as the "naive" model in this paper.) Such results provide motivation for a detailed theoretical study of diquark fragmentation.

(b) Electroproduction in the deep-inelastic region can also provide information on the fragmentation of diquarks. For ep and μp reactions, the flavor content of the fragmenting diquark is essentially (ud), whereas for en and μn reactions one gets a mixture of (ud) and (dd). Some data on $\mu^+ p$ scattering is available,[10] and it is plotted in Fig. 3.

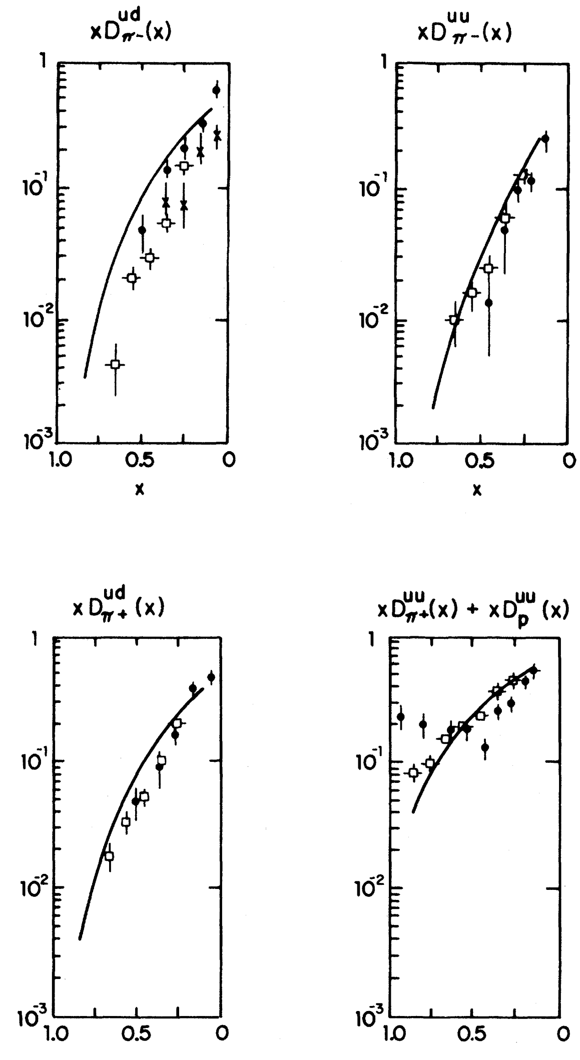

FIG. 3. Data for (uu) and (ud) diquark fragmentation into pions as collected by Hanna *et al.*, Ref. 17. The open-square data points in each plot are from a pp experiment at CERN ISR (Ref. 17). For ud fragmentation into π^- the solid circles represent data obtained in antineutrino-proton experiments, Ref. 7, and the crosses data obtained in muon production, Ref. 10. The solid circles for (uu) fragmentation into π^- are from a neutrino-proton experiment, Ref. 6. The solid circles in the $(ud) \rightarrow \pi^+$ plot come from a $\bar{\nu}p$ experiment, Ref. 7. The final plot shows the combined fragmentation of the (uu) diquark into positive pions plus protons, the data are from Refs. 5 and 17.

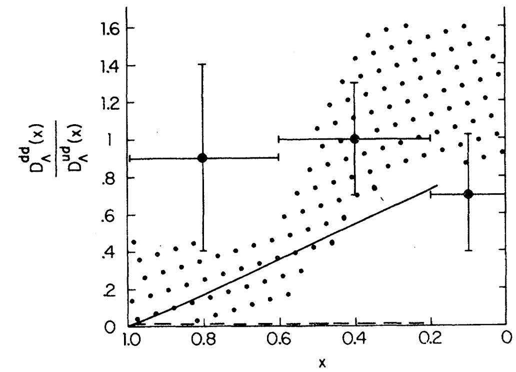

FIG. 4. Comparison of theory and data for the ratio of fragmentation functions $(dd \to \Lambda)/(ud \to \Lambda)$ as a function of x. The solid curve is calculated and gives the prediction of our model while the dashed line gives the naive $(a=0)$ result. The solid circles with error bars are data from Ref. 9. The dotted band gives *our* estimate for this ratio as gleaned from preliminary data presented at an APS meeting (Ref. 8).

(c) Hadronic collisions producing $\mu^+\mu^-$ pairs are frequently due to the Drell-Yan mechanism. If this is the case, then $\pi^{\pm}p$ collisions with a $\mu^+\mu^-$ trigger offer a good way of studying (uu), (ud) fragmentation.[11] Likewise, $\bar{p}p$ interactions triggered on $\mu^+\mu^-$ pairs will provide diquark fragmentation information.[12]

(d) Hadronic collisions (involving baryons) with a large-p_T trigger offer yet another way of studying the fragmentation of diquarks, *if* hard quark-quark scattering is the dominant mechanism for large-p_T particle production.[13] This scheme has three major drawbacks. First, the large-p_T trigger particle does not identify the jet-initiating parton with high probability[3]; consequently, the flavor content of the forward-moving diquark cannot be well determined. Second, lowest-order QCD calculations show that there are a substantial number of large-p_T gluon jets coming from hard quark-gluon or gluon-gluon scattering.[13] If this happens, the forward jet is not necessarily coming from diquark fragmentation. Third, there are indications that lowest-order QCD calculations may have non-negligible higher-order corrections[14] at present energies. Therefore, to extract diquark fragmentation functions from data on hadronic collisions with a large-p_T trigger requires a substantial number of model-dependent assumptions. Nevertheless, this has been done[15] using data on pp collisions at the CERN ISR, and the results are shown in Fig. 3 as the square data points.

(e) Low-p_T (soft) multiparticle production in hadronic collisions like pp, $\bar{p}p$, πp, etc., almost certainly contain detailed information on diquark fragmentation. However, this is a model-dependent statement, since no hard partonic interaction is involved. For example, the two-chain model based on dual topological unitarization has been shown to be remarkably successful in explaining available soft multiparticle production data.[16] In this scheme, the proton fragmentation region comes from a diquark of flavor $\frac{2}{3}(ud) + \frac{1}{3}\,(uu)$. Because of the model dependence involved, we shall not examine low-p_T data in this paper.

Figures 2-4 essentially contain all currently available data on diquark fragmentation functions. It should be pointed out that the data are not very precise and correspond to relatively small values for the energy $(W^2)^{1/2}$ of the fragmenting diquark. Nevertheless, there is reasonable agreement between the data from various sources, supporting the idea of universality of diquark fragmentation functions.[17] Clearly the quality of the data can be substantially improved and this can be done most profitably at larger values of $(W^2)^{1/2}$ which will soon become available at the Fermilab Tevatron.

III. RECURSIVE MODEL FOR DIQUARK FRAGMENTATION

It is generally thought that when a diquark fragments, it first forms a baryon, leaving behind an antiquark which fragments further,

$$(qq) \to B + \bar{q}\ . \qquad (1)$$

This breakup process is shown in Fig. 1(a). If this were the only allowed process, then it follows that the two quarks making up the initial diquark act as a unit, and both of them become part of a single baryon in the diquark jet. Intuitively, one expects this to happen when the baryon has a large momentum fraction x of the diquark momentum, but in general this need not be the case. Indeed there is experimental evidence that the two quarks in a diquark often do not become part of a single baryon:

(i) In (uu) diquark fragmentation, Fig. 3 illustrates that (uu) breaks up both into π^+ and π^- mesons, with π^+ being more abundant. If the diquark goes directly into a baryon in the first breakup, equal production of π^+ and π^- mesons would be expected in subsequent breakups. The Monte Carlo studies in Ref. 6 would suggest that in the region $x \lesssim 0.5, \pi^+$ production dominates proton production, so a model allowing for more than direct baryon production seems necessary. (More experimental study is, of course, needed.)

(ii) Measurements have been made[8,9] for $\Lambda(uds)$ production in the diquark fragmentation region of $\bar{\nu}p$ and $\bar{\nu}n$ charged-current interactions, where the fragmenting diquarks are (ud) and (dd), respectively (see Table I). If the diquark goes directly into a baryon, one would expect Λ production in $\bar{\nu}p$ interactions, but essentially none in $\bar{\nu}n$ interactions. Experimentally, the rates of Λ production are comparable in the two processes (see Fig. 4).

Both the above examples demonstrate that treating the diquark as one unit (having color $\bar{3}$) may be insufficient. This conclusion is very reasonable; a diquark does have an extended structure, and the separate momenta of its two constituent quarks must be taken into account.

Since the two quarks in a diquark do not always act as one unit, it is natural to expect that they sometimes act like two separate (or loosely bound) quarks. In that case, the natural fragmentation process is meson (M) formation [see Fig. 1(b)] with a new leftover diquark (of different flavor content) carrying reduced momentum,

$$(qq) \rightarrow M + (qq) \ . \tag{2}$$

Clearly, both the breakup mechanisms shown in Fig. 1 must contribute to diquark fragmentation. Qualitatively, if the two constitutuent quarks have nearly equal momenta, one expects them to act as a unit and form a baryon [Eq. (1), Fig. 1(a)], whereas if the momenta of the two quarks are widely separated, then meson formation [Eq. (2), Fig. 1(b)] will occur in the first breakup.

Let us recall[3,4] that quark fragmentation proceeds via the process

$$q \rightarrow M + q \ . \tag{3}$$

Successive breakups of the type given in Eqs. (1)–(3) generate quark and diquark jets in a recursive model. The diquark jet will always contain a baryon, since the breakup shown in Fig. 1(a) is the only way to remove a diquark from the fragmentation process. Thus, baryon number conservation is guaranteed.

We now proceed to formulate quantitatively a recursive cascade model which incorporates the above ideas and yields the fragmentation functions $D_B^{qq}(x)$, $D_M^{qq}(x)$ for diquarks into baryons and mesons, respectively, with x being the fraction of the (qq) momentum carried by B or M. In order to do this, it is necessary to specify three things: (i) the state of the diquark; (ii) the relative probability of fragmentation via baryon formation [Fig. 1(a)] or meson formation [Fig. 1(b)]; and (iii) the momentum-sharing functions[3] $f_B(x)$, $f_M(x)$ describing the breakups of Fig. 1(a) and 1(b). These three ingredients will allow us to give a description of diquark fragmentation.

(i) The state of the diquark. This is specified by the momenta of the constituent quarks. Consider a jet-initiating diquark of momentum P and let p_1, p_2 be the momenta of the two quarks. Their momentum fractions x_1, x_2 and the scaled momentum difference δ are given by

$$\begin{aligned} &P = p_1 + p_2 , \\ &x_1 \equiv p_1/P, \quad x_2 \equiv p_2/P = 1 - x_1 , \\ &\delta \equiv \frac{|p_1 - p_2|}{P} = |x_1 - x_2| \ . \end{aligned} \tag{4}$$

The quantities (P,δ) together specify the state of the initial diquark.

If the initial diquark (P,δ) forms a meson in the first breakup [Fig. 1(b)], we need to know the state of the leftover diquark. Let $(1-x')P$ and $x'P$ be the momenta of the meson and leftover diquark, respectively. If the constituent quarks of the initial diquark had momenta p_1 and p_2 $(p_1 > p_2)$, it is reasonable to expect that quark p_1 goes into the meson. So we shall make the simple assumption that the scaled momentum difference δ' of the leftover diquark is

$$\begin{aligned} \delta' &= \frac{|[p_1 - (1-x')P] - p_2|}{x'P} \\ &= \frac{|\delta - (1-x')|}{x'} \ . \end{aligned} \tag{5}$$

The quantities $(x'P, \delta')$ specify the leftover diquark.

(ii) Relative probability of a diquark forming a baryon or meson. We have argued qualitatively that when δ is small, the constituent quarks in a diquark have roughly equal momenta and we expect the diquark to act like one unit and form a baryon [Fig. 1(a)]. We shall take the probability $\mathscr{P}_B$ for baryon formation to be

$$\mathscr{P}_B(\delta) = e^{-a\delta^2} \ . \tag{6}$$

Then, it follows that the probability $\mathscr{P}_M$ for the diquark to act like two separate quarks and form a meson [Fig. 1(b)] is

$$\mathscr{P}_M(\delta) = 1 - \mathscr{P}_B(\delta) = 1 - e^{-a\delta^2} \ . \tag{7}$$

The parameter "a" can be roughly fixed numerically by noting (from past experience with hadronic collisions) that two particles "know" about each other (short-range correlations) if their rapidity separation $|y_1-y_2|$ is less than one unit of rapidity. Since

$$\frac{x_1}{x_2}=\exp(y_1-y_2)\ , \tag{8}$$

the criterion $|y_1-y_2|\simeq 1$ corresponds to $\delta_0=(x_1-x_2)/(x_1+x_2)\simeq\frac{1}{2}$. We shall therefore use the value $a=1/\delta_0^2=4$ for the parameter a in all subsequent numerical work.

It should be noted that if the parameter a is taken to be zero, then $\mathscr{P}_B(\delta)=1$, which corresponds to the diquark always acting like a single unit and always forming a baryon. For purposes of comparison, we shall examine $a=0$ and $a=4$ results and compare both of them with available diquark fragmentation data.

(iii) Momentum-sharing functions. Let $f_B^{qq}(x)$ and $f_M^{qq}(x)$ denote the momentum-sharing functions[3] corresponding to the breakups of Fig. 1(a) [Eq. (1)] and Fig. 1(b) [Eq. (2)], respectively. These functions specify how momentum is shared between the two products of a breakup. For example, $f_B^{qq}(x)$ is the probability that the baryon B takes momentum fraction x of the incident diquark momentum in a breakup $qq(P)\rightarrow B(xP)+\bar{q}(P-xP)$. Clearly, the momentum-sharing functions are normalized by

$$\int_0^1 f_B^{qq}(x)dx=\int_0^1 f_M^{qq}(x)dx=1\ . \tag{9}$$

In this paper, we shall take the unknown momentum-sharing functions of diquarks to be those given by dimensional spectator counting rules,[2]

$$f_B^{qq}(x)=2(1-x)\ ,$$
$$f_M^{qq}(x)=4(1-x)^3\ . \tag{10}$$

At this stage, we can write down the recursive cascade-model equations for the fragmentation functions of a diquark specified by (P,δ). Assuming these functions scale, we obtain the equations of our model,

$$D_B^{qq}(x;\delta)=\mathscr{P}_B(\delta)f_B^{qq}(x)+\mathscr{P}_M(\delta)\int_x^1\frac{dx'}{x'}f_M^{qq}(1-x')D_B^{qq}\left[\frac{x}{x'};\delta'\right]\ , \tag{11}$$

$$D_M^{qq}(x;\delta)=\mathscr{P}_M(\delta)\left[f_M^{qq}(x)+\int_x^1\frac{dx'}{x'}f_M^{qq}(1-x')D_M^{qq}\left[\frac{x}{x'};\delta'\right]\right]+\mathscr{P}_B(\delta)\int_x^1\frac{dx'}{x'}f_B^{qq}(1-x')D_M^{\bar{q}}\left[\frac{x}{x'}\right]\ . \tag{12}$$

Each term on the right-hand side of Eqs. (11) and (12) has a clear meaning. The first term in Eq. (11) corresponds to the diquark forming $B(x)$ (a baryon with momentum fraction x) immediately in the first breakup. The second term corresponds to the situation where the initial diquark first forms $M(1-x')$ [a meson with momentum fraction $(1-x')$] and a leftover diquark with momentum fraction x', and, subsequently, the diquark fragments to baryon $B(x)$.

Similarly, on the right-hand side of Eq. (12), the first term corresponds to meson $M(x)$ formed directly in the first breakup; the second term corresponds to meson $M(1-x')$ formed in the first breakup [Fig. 1(b)] along with a leftover diquark with momentum fraction x' which subsequently yields $M(x)$; the third term corresponds to baryon $B(1-x')$ formed in the first breakup [Fig. 1(a)] along with an antiquark $\bar{q}(x')$ which subsequently fragments to give $M(x)$. Note that in writing Eq. (11) we have implicitly assumed that a diquark jet produces only one baryon, since we have omitted the term

$$\int_x^1\frac{dx'}{x'}\mathscr{P}_B(\delta)f_B^{qq}(1-x')D_B^{\bar{q}}\left[\frac{x}{x'}\right]\ .$$

Such a term can be readily included.[3] However, its contribution is small, and can be neglected for simplicity.

The antiquark fragmentation function $D_M^{\bar{q}}(x)$ appearing in Eq. (12) is known experimentally from quark fragmentation data in e^+e^- collisions,[18] electroproduction, etc. A simple, analytic form[16] which describes that data well is

$$D_M^q(x)=D_M^{\bar{q}}(x)=3(1-x)^2/x\ . \tag{13}$$

This form has the behavior $(1-x)^2$ as $x \to +1$ and a $1/x$ dependence as $x \to 0$ corresponding to logarithmically increasing multiplicities $n(E)=3(\ln E)$ at large energies E.[3] The normalization is consistent with the momentum sum rule

$$\int_0^1 x D_M^{q}(x)dx = 1 . \tag{14}$$

Equation (13) is also a solution to the cascade-model integral equation corresponding to repeated breakups $\bar{q} \to M + \bar{q}$ (or $q \to M + q$),[3,4]

$$D_M^{\bar{q}}(x) = f_M^{\bar{q}}(x) + \int_x^1 \frac{dx'}{x'} f_M^{q}(1-x') D_M^{\bar{q}}\left[\frac{x}{x'}\right]; \quad f_M^{\bar{q}}(x) = 3(1-x)^2 . \tag{15}$$

In solving Eq. (12), we shall henceforth use the antiquark fragmentation function of Eq. (13). From examination of the behavior of Eq. (12) near $x=0$, it is clear that diquark jets will also have asymptotic logarithmic multiplicities $n_{qq}(E)=3(\ln E)$.

Let us now rewrite Eqs. (11) and (12) after substituting explicit expressions for the probabilities $\mathscr{P}_B$ and $\mathscr{P}_M$ and making a change of integration variable from x' to $u = x/x'$. The basic integral equations of our model for diquark fragmentation then become

$$D_B^{qq}(x;\delta) = e^{-a\delta^2} f_B^{qq}(x) + (1-e^{-a\delta^2}) \int_x^1 \frac{du}{u} f_M^{qq}\left[1-\frac{x}{u}\right] D_B^{qq}(u;\delta''), \tag{16}$$

$$D_M^{qq}(x;\delta) = \left[(1-e^{-a\delta^2}) f_M^{qq}(x) + e^{-a\delta^2} \int_x^1 \frac{du}{u} f_B^{qq}\left[1-\frac{x}{u}\right] D_M^{\bar{q}}(u)\right] + (1-e^{-a\delta^2}) \int_x^1 \frac{du}{u} f_M^{qq}\left[1-\frac{x}{u}\right] D_M^{qq}(u;\delta''), \tag{17}$$

where

$$\delta'' = \frac{u}{x}(\delta-1)+1 . \tag{18}$$

IV. MODEL DIQUARK FRAGMENTATION FUNCTIONS

Equations (16) and (17) are linear Volterra integral equations of the second kind. Both of them have the same basic structure,

$$D(x;\delta) = H(x;\delta) + \mathscr{P}_M(\delta) \int_x^1 \frac{du}{u} f_M^{qq}\left[1-\frac{x}{u}\right] D(u;\delta''), \tag{19}$$

where $H(x;\delta)$, $f_M^{qq}(x)$, and $\mathscr{P}_M(\delta)$ are known functions. A convergent series solution to Eq. (19) can always be found by successive iteration, or, alternatively, the method of moments can also be used.[4] The iterative solution is given by

$$D^{(1)}(x;\delta) = H(x;\delta) ,$$
$$D^{(j+1)}(x;\delta) = \mathscr{P}_M(\delta) \int_x^1 \frac{du}{u} f_M^{qq}\left[1-\frac{x}{u}\right] D^{(j)}\left[u;\frac{x}{u}(\delta-1)+1\right] \quad (j=1,2,\ldots) , \tag{20}$$
$$D(x;\delta) = \sum_{j=1}^{\infty} D^{(j)}(x;\delta) .$$

Clearly, the full solution is the sum of contributions coming from all breakups of various types [Eqs. (1)–(3)]. For later use, let us explicitly isolate the contributions from the first and second breakups.

The contribution of the first breakup to the diquark fragmentation function into baryons and mesons is (see Fig. 1)

$$D_B(x;\delta)=\mathscr{P}_B(\delta)f_B^{qq}(x)=2e^{-a\delta^2}(1-x)\ , \tag{21}$$

$$D_M(x;\delta)=\mathscr{P}_M(\delta)f_M^{qq}(x)=4(1-e^{-a\delta^2})(1-x)^3\ . \tag{22}$$

The contribution of the second breakup to the diquark fragmentation function into baryons $D_B^{qq}(x;\delta)$ is given by

$$D_{B(M)}(x;\delta)=\int_x^1 \frac{dx'}{x'}[\mathscr{P}_M(\delta)f_M^{qq}(1-x')]D_B\left[\frac{x}{x'};\delta'\right]\ . \tag{23}$$

The notation $D_{B(M)}$ is meant to specify that the baryon $B(x)$ was formed in the second breakup, after meson M was produced in the first breakup [see Fig. 5(a)].

The contribution of the second breakup to the diquark fragmentation function into a meson $M(x)$ is slightly more complicated, since the first breakup could have produced a meson [Fig. 5(b)] or a baryon [Fig. 5(c)]. We shall denote these contributions by $D_{M(M)}(x;\delta)$ and $D_{M(B)}(x;\delta)$, respectively. They are given by

$$D_{M(M)}(x;\delta)=\int_x^1 \frac{dx'}{x'}[\mathscr{P}_M(\delta)f_M^{qq}(1-x')]D_M\left[\frac{x}{x'};\delta'\right], \tag{24}$$

$$D_{M(B)}(x;\delta)=\int_x^1 \frac{dx'}{x'}[\mathscr{P}_B(\delta)f_B^{qq}(1-x')]f_M^{\bar{q}}\left[\frac{x}{x'}\right]. \tag{25}$$

Thus Eqs. (21)–(25) allow us to evaluate the contributions of the first and second breakups to the full fragmentation functions.

Each iteration picks up an additional suppression factor of $(1-x)$, as is clear from Eq. (19). Therefore, if we are interested in fragmentation functions for $x \gtrsim 0.2$, only four or five iterations suffice to give a good result. We have used the above-described iterative method for numerical evaluation of $D_B^{qq}(x;\delta)$ and $D_M^{qq}(x;\delta)$, keeping five iterations. However, before a comparison with experimental data can be made, we need to consider two additional complications: (i) the inclusion of quark flavors in our model; and (ii) the probability that a physical diquark is in the state (P,δ).

(i) Inclusion of quark flavors. So far, we have considered quarks of only one flavor. A more proper physical treatment requires that flavor indices be included in the cascade-model integral equations. (For quark fragmentation, this has been done in Refs. 3 and 4). For diquark fragmentation, an analogous complete inclusion of flavor is possible, but it makes the model integral equations quite complicated. So, we shall use a much simpler method of including flavor effects which is approximate, but quite adequate for most purposes. This method is based on the well-known fact[3] that the flavor of the jet-initiating quark (or diquark) is rapidly diluted with each successive breakup. For the first breakup, the initial flavor is of course important (leading particle effect); for the second breakup, the initial flavor is less important and typically for the third and subsequent breakups, jet evolution is essentially independent of the initial flavor content.

Let us consider only three flavors (u,d,s) and use experiment[19] for the probabilities of $u\bar{u}$, $d\bar{d}$, $s\bar{s}$ creation during a breakup,

$$p_u=p_d=\tfrac{4}{9},\quad p_s=\tfrac{1}{9}\ . \tag{26}$$

The general procedure will be made obvious by working out two explicit examples illustrating how we incorporate flavor. These are (ud) diquark fragmentation into a baryon with quark content (uds) and into a meson with content $(u\bar{d})$. The (ud) diquark specified by (P,δ) in the first breakup will either form a baryon $B(x)$ with probability $D_B(x;\delta)$ [Eq. (21)] or a meson $M(x)$ with probability $D_M(x;\delta)$ [Eq. (22)]. The full flavor content of mesons and baryons formed in the first breakup and their relative probabilities of formation are given in Table II. Clearly, the contribution to the fragmentation function for (ud) breaking into (uds) and $(u\bar{d})$ coming just from the first breakup is $\frac{1}{9}D_B(x;\delta)$ and $\frac{4}{18}D_M(x;\delta)$, respectively.

In the second breakup, the baryon (uds) can be formed from any of the diquarks (q_1q_2) left over after the first breakup. Thus we need to compute a conditional probability. For example, the probability of (uds) formation from a leftover (sd) diquark=[probability of having an (sd) diquark]

$\times$ [probability that a $u\bar{u}$ pair will be created in the second breakup] $=p_u/18=\frac{1}{18}\times\frac{4}{9}=\frac{2}{81}$. Adding up all intermediate (q_1q_2) diquark channels yields the contribution of the second breakup to the fragmentation function into (uds) to be $\frac{8}{81}D_{B(M)}(x;\delta)$ [Eq. (23)]. Similar reasoning shows that the contribution of the second breakup to the (ud) fragmentation function into meson $(u\bar{d})$ is $\frac{17}{81}D_{M(M)}(x;\delta)+\frac{16}{81}D_{M(B)}(x;\delta)$.

We have argued above that the third and subsequent breakups are essentially independent of the jet-initiating flavor. Thus, the probability that a baryon is $(uds)=p_up_dp_s\times$(number of permutations of three distinct quarks) $=\frac{4}{9}\times\frac{4}{9}\times\frac{1}{9}\times 6=\frac{96}{729}$, and consequently, the contribution of the third and subsequent breakups to a diquark fragmentation function into (uds) is

$$\frac{96}{729}[D_B^{(ud)}(x;\delta)-D_B(x;\delta)-D_{B(M)}(x;\delta)] \ .$$

The full fragmentation function into (uds) is

TABLE II. Possible products from the first breakup of a (ud) diquark and their relative probabilities of formation.

Products	Probability
$(udu)+\bar{u}$	$p_uD_B(x;\delta)$
$(udd)+\bar{d}$	$p_dD_B(x;\delta)$
$(uds)+\bar{s}$	$p_sD_B(x;\delta)$
$(u\bar{u})+(ud)$	$\frac{p_u}{2}D_M(x;\delta)$
$(u\bar{d})+(dd)$	$\frac{p_d}{2}D_M(x;\delta)$
$(u\bar{s})+(sd)$	$\frac{p_s}{2}D_M(x;\delta)$
$(d\bar{u})+(uu)$	$\frac{p_u}{2}D_M(x;\delta)$
$(d\bar{d})+(ud)$	$\frac{p_d}{2}D_M(x;\delta)$
$(d\bar{s})+(us)$	$\frac{p_s}{2}D_M(x;\delta)$

$$D_{(uds)}^{(ud)}(x;\delta)=\frac{1}{9}D_B(x;\delta)+\frac{8}{81}D_{B(M)}(x;\delta)+\frac{96}{729}[D_B^{(ud)}(x;\delta)-D_B(x;\delta)-D_{B(M)}(x;\delta)] \ . \tag{27}$$

The complete result for (ud) diquark fragmentation into meson $q_1\bar{q}_2$ is

$$D_{(q_1\bar{q}_2)}^{(ud)}(x;\delta)=a_1D_M(x;\delta)+a_{2M}D_{M(M)}(x;\delta)+a_{2B}D_{M(B)}(x;\delta)$$
$$+a_R[D_M^{qq}(x;\delta)-D_M(x;\delta)-D_{M(M)}(x;\delta)-D_{M(B)}(x;\delta)] \ , \tag{28}$$

where, $D_M, D_{M(M)}$, and $D_{M(B)}$ are given by Eqs. (22), (24), and (25), and $D_M^{qq}(x;\delta)$ is the solution of Eq. (17), i.e., the full diquark fragmentation function into meson M when flavor is ignored. The calculation of the coefficients a_1, a_{2M}, a_{2B}, and a_R is similar to that in the above illustration. A complete tabulation for various $(q_1\bar{q}_2)$ meson states is given in Table III. The coefficients "a" denote the relative probabilities for a meson $(q_1\bar{q}_2)$ to be formed in a particular process. For example, a_{2B} is the relative probability for the process in which a baryon is formed in the first breakup and meson $(q_1\bar{q}_2)$ is formed in the second breakup. Note that the sum of all relative probabilities in any process is unity, as indeed it must be:

$$\sum_{q_1\bar{q}_2} a_1=\sum_{q_1\bar{q}_2} a_{2M}=\sum_{q_1\bar{q}_2} a_{2B}=\sum_{q_1\bar{q}_2} a_R=1 \ . \tag{29}$$

Furthermore, the similarity of the values of the coefficients a_{2M} and a_R is an indication that even in the second breakup the jet-initiating flavor has been essentially lost, and we were therefore quite justified in using our simple technique for incorporating flavor effects.

In this paper, we shall ignore resonance production and take the $3q$ and $(q\bar{q})$ states to be the ground-state baryons and pseudoscalar mesons. We have taken this simple approach since the amount of resonance production is uncertain. However, it can also be included easily, as discussed in Ref. 4.

(ii) State of a physical diquark. We have thus far computed the fragmentation functions for a diquark of momentum P and momentum difference $\delta=|p_1-p_2|/P$. However, in an experiment, the physical diquark can have all values of δ between 0 and 1 with a weight distribution $d\mathscr{P}/d\delta$ normalized by

$$\int_0^1 \frac{d\mathscr{P}}{d\delta}d\delta=1 \ . \tag{30}$$

The distribution $d\mathscr{P}/d\delta$ can be calculated from $v_q^{qq}(x)$, the structure function of a quark in a diquark. Again, guided by dimensional counting

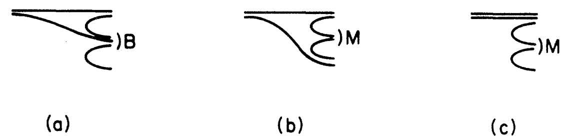

FIG. 5. Diagrams depicting diquark fragmentation through two stages. (a) $(qq)\to M+(qq)\to M+B+\bar{q}$, (b) $(qq)\to M+(qq)\to M+M+(qq)$, (c) $(qq)\to B+\bar{q}\to B+M+q$.

rules, Regge behavior, and our knowledge of measured parton structure functions in nucleons and pions, we take

$$v_q^{qq}(x)\propto x^{-1/2}(1-x)+x(1-x)^{-1/2}\ . \qquad (31)$$

Since $d\mathscr{P}/d\delta$ is proportional to v_q^{qq} and normalized by Eq. (30), we get

$$\frac{d\mathscr{P}}{d\delta}=\tfrac{3}{10}(1+\delta)(1-\delta)^{-1/2}\ . \qquad (32)$$

This function is peaked near $\delta=1$, and the average value of δ is

$$\bar{\delta}=\int_0^1 \delta\frac{d\mathscr{P}}{d\delta}d\delta=0.72\ . \qquad (33)$$

TABLE III. (ud) fragmentation functions into mesons.

$$D^{(ud)}_{q_1\bar{q}_2}(x;\delta)=a_1D_M(x;\delta)+a_{2M}D_{M(M)}(x;\delta)+a_{2B}D_{M(B)}(x;\delta)+a_R[D_M^{qq}(x;\delta)-D_M(x;\delta)-D_{M(M)}(x;\delta)-D_{M(B)}(x;\delta)]\ .$$

$q_1\bar{q}_2$	a_1	a_{2M}	$a_{2B}=a_R$
$\pi^+=u\bar{d}$	$\frac{4}{18}$	$\frac{17}{81}$	$\frac{16}{81}$
$\pi^-=d\bar{u}$	$\frac{4}{18}$	$\frac{17}{81}$	$\frac{16}{81}$
$K^+=u\bar{s}$	$\frac{1}{18}$	$\frac{4.25}{81}$	$\frac{4}{81}$
$K^-=s\bar{u}$	0	$\frac{2}{81}$	$\frac{4}{81}$
$K^0=d\bar{s}$	$\frac{1}{18}$	$\frac{4.25}{81}$	$\frac{4}{81}$
$\bar{K}^0=s\bar{d}$	0	$\frac{2}{81}$	$\frac{4}{18}$
$u\bar{u}$	$\frac{4}{18}$	$\frac{17}{81}$	$\frac{16}{81}$
$d\bar{d}$	$\frac{4}{18}$	$\frac{17}{81}$	$\frac{16}{81}$
$s\bar{s}$	0	$\frac{0.5}{81}$	$\frac{1}{81}$

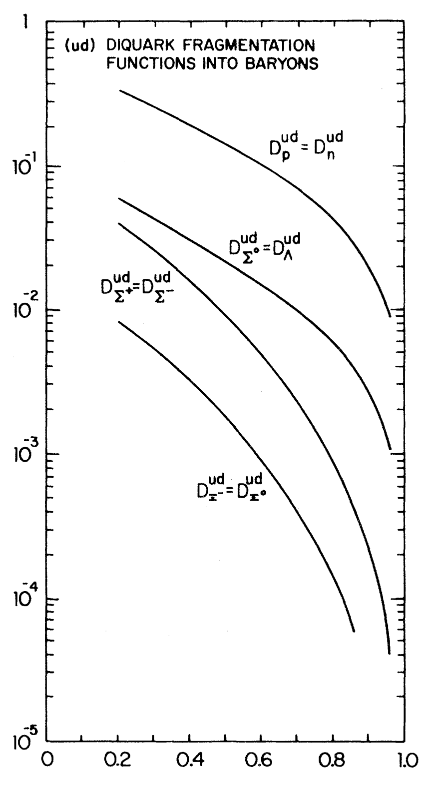

FIG. 6. Diquark fragmentation functions for $(ud)\to$ baryons as a function of momentum fraction x.

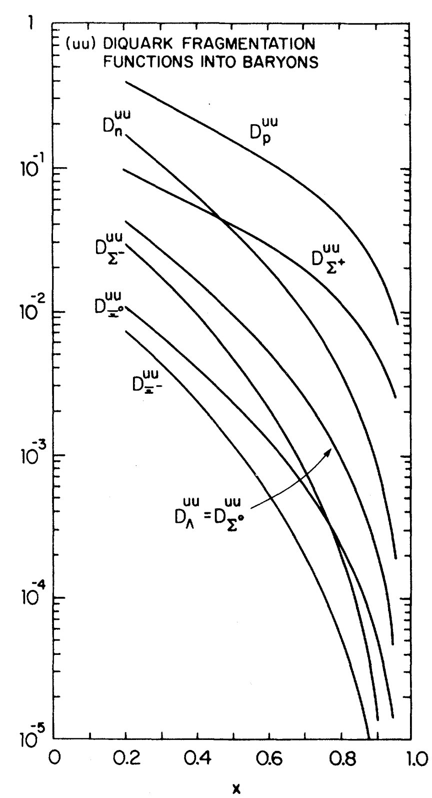

FIG. 7. Diquark fragmentation functions for $(uu)\to$ baryons as a function of momentum fraction x.

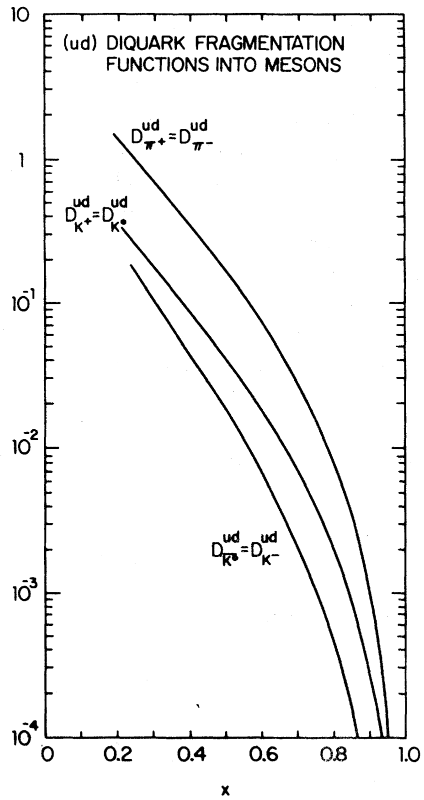

FIG. 8. Diquark fragmentation functions for (ud) $\rightarrow$ mesons as a function of momentum fraction x.

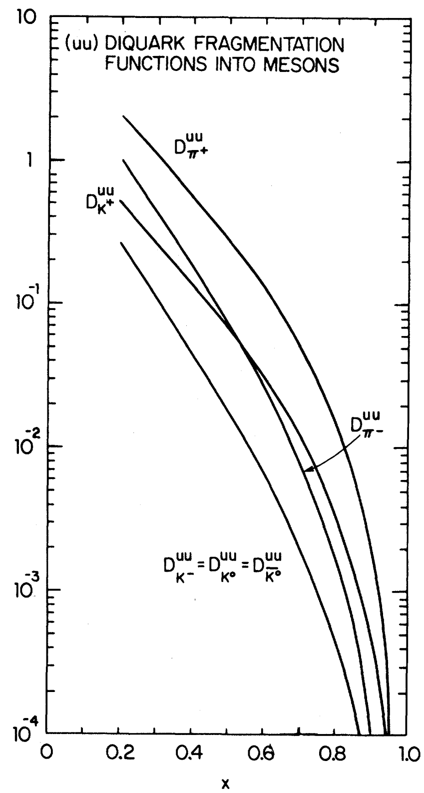

FIG. 9. Diquark fragmentation functions for (uu) $\rightarrow$ mesons as a function of momentum fraction x.

The value of $\bar{\delta}$ does not depend appreciably on the choice of the Regge power in Eqs. (31) and (32). It is easy to check that if the powers are taken to be -0.25 and -0.75 instead of $-\frac{1}{2}$, then $\bar{\delta}=0.63$ and 0.84, respectively. This shows that large momentum separations of the constituent quarks occur frequently, and consequently, in our model, the diquark often acts like two separate quarks and fragments as in Fig. 1(b). Thus, in order to get the fragmentation functions of a physical diquark, it is necessary to integrate on δ with weight $d\mathcal{P}/d\delta$,

$$D^{qq}(x)=\int_0^1 D^{qq}(x;\delta)\frac{d\mathcal{P}}{d\delta}d\delta \; . \tag{34}$$

V. COMPARISON WITH EXPERIMENTAL DATA

As discussed in Sec. II, the currently available data on diquark fragmentation is summarized in Figs. 2–4. The curves predicted by our model are also shown in these figures. In general, the comparison is satisfactory[20] when the parameter a has a value 4, which we argued was a physically plausible value (see Sec. III). The simpler approach with the diquark always acting as one entity to form a baryon via the process shown in Fig. 1(a) corresponds to $a=0$ which gives a charged-particle ratio (Fig. 2) somewhat poorer than the $a=4$ result at $x \lesssim 0.5$. The Λ production data, Fig. 4, favors $a=4$. Thus, our results suggest that the diquark does not always act as a single unit during fragmentation. Our model also predicts that the K^+/K^- ratio in (uu) fragmentation will be large similar to the π^+/π^- ratio, whereas the naive model ($a=0$) would give a ratio of unity.

It should be pointed out that we have chosen to make plausible physical assumptions and fully specify our model (including absolute normalization) rather than allow adjustable free parameters. Since reasonable agreement with available data was obtained, we feel that predictions for as yet unmeasured diquark fragmentation functions should prove useful and fairly reliable. Our model expectations for (ud),(uu),(dd) fragmentation functions are shown in Figs. 6 and 7 (for baryons) and Figs. 8 and 9 (for mesons).[21] Although the cascade-model equations which we have written down [Eqs.

(16) and (17)] are valid for all values of x, the iterative procedure (with five iterations) which we have used for obtaining the solution is reliable only for $x \gtrsim 0.2$. Also, because of sea effects, the flavor content of the fragmenting diquark cannot be reliably established for small x values.

VI. COMMENTS AND SUMMARY

In this paper we have presented a quantitative model for diquark fragmentation into mesons and baryons.[22] The model emphasizes the composite structure of diquarks by taking into account the momenta of the individual constituent quarks. Existing data suggest that fragmentation of the diquark into unfavored final states provides the strongest support for our model.[23] In our model the diquark acts like one unit only if the two constituent quarks are close in momentum. The model is based on a recursion of fundamental breakups $(qq)\to B+\bar{q}$ [Fig. 1(a)], $(qq)\to M+(qq)$ [Fig. 1(b)], and $q\to M+q$, and makes full predictions for diquark fragmentation functions. The model is in good agreement with currently available data.

Since this is a relatively new subject, we have used plausible physical assumptions and dimensional counting rules to fully specify our model, leaving no arbitrary parameters. Some of these assumptions may well require modifications when more and better data become available [for example, the powers of the momentum-sharing functions may not be exactly those from dimensional counting, the values of p_u, p_d, p_s may not be exactly those in Eq. (26), resonances may have to be included, etc.]. However, we feel that the basic ideas put forward and the general structure of the model integral equations will remain unaltered, and our work will help to stimulate further experimental and theoretical investigations of diquark fragmentation.

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[11]A proposal for such an experiment at the Fermilab Tevatron has recently been submitted by a Fermilab, Florida State, George Mason, Illinois—Chicago Circle, Indiana, Maryland, Rutgers collaboration. Also, running of a BEBC experiment on this reaction triggered by $\mu^+\mu^-$ pairs has been done by a group at CERN headed by J. Lemonne. A similar experiment by R. Barloutaud *et al.* [Nucl. Phys. B172, 25 (1980)] detected high-mass e^+e^- pairs in π^-p collisions but did not have sufficient statistics to be able to say anything about the target fragmentation region.

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[17]This observation was first made by D. Hanna *et al.*, CERN report, 1981 (unpublished) by making a compilation of diquark data. All of the data in the diquark fragmentation region from this compilation are shown

in Fig. 3. We note that the π^--triggered data are quite model dependent since 40% of such events are expected to be initiated by hard gluons.

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[19]V. V. Ammosov *et al.*, Phys. Lett. 93B, 210 (1980). (This experiment actually gives the ratio p_s/p_u to be 0.27 ± 0.04. Our use of $\frac{1}{4}$ is quite convenient for our calculations and within the experimental limits.) This small value of p_s/p_u can also be theoretically understood in a color flux tube model for jet formation. In such models, the probability of creating a $q\bar{q}$ pair is a strongly decreasing function $\exp(-Cm^2)$ of the quark mass m. For more details, see, e.g., A. Casher, H. Neuberger, and S. Nussinov, Phys. Rev. D 20, 179 (1979).

[20]It should be kept in mind that available data are not very good and restricted to low values of W, the hadron total energy. Furthermore, at these low-W values, there is a substantial uncertainty in the distributions of positively charged hadrons. This arises because the correct value of x depends on the Lorentz transformation to go to the hadronic c.m. system, which is strongly dependent on correct identification of all final-state hadrons. However, pions and fast protons are hard to distinguish in typical bubble-chamber experiments. See, for example, R. Orava, in the Proceedings of the Arctic School of Physics, Akaslompolo, Finland, 1980 [Phys. Scr. 25, 159 (1982)].

[21]Our model can also be used to discuss the fragmentation of diquarks containing strangeness, which would lead to predictions for future experiments with hyperon beams.

[22]We have only treated the longitudinal x dependence of diquark fragmentation functions. The p_T dependence of diquark jets should also be different from quark jets, since the diquark is a spatially extended object, typically 1 fm in size.

[23]On the other hand, data involving "favored" transitions of the diquark, such as the ratio in Fig. 2, do not dramatically differentiate between the $a=0$ and $a=4$ cases. The $a=0$ model was also studied recently by D. Beavis and B. Desai [Phys. Rev. D 23, 1967 (1981)]. These authors make use of the data in Ref. 15, which indeed are consistent with our parameter $a=0$ if all the many assumptions involved in large-p_T hadronic collisions are to be believed (see discussion in Sec. II).