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Partially conserved axial-vector current condition in the quark model: G_A/G_V for vector mesons and the $\omega\rho\pi$ coupling constant

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The PCAC (partially conserved axial-vector current) condition is implemented at the quark level for hadrons considered as bound states of constituent quarks. Using the Goldberger-Treiman relation and the $\omega\rho\pi$ coupling constant, it is shown that G_A/G_V for the $\omega\to\rho$ Gamow-Teller transition is lower than the quark-model prediction by 20%. This supports the current view that G_A/G_V for bound quarks is reduced by confinement effects. With this reduction, the decay rates $\Gamma(D^*\to D\pi^0)$ are calculated and found to be more or less in agreement with measurements.

I. INTRODUCTION

It is a well-known fact that SU(3)$\times$SU(3) is an approximate symmetry of strong interactions.[1,2] This can be realized in a most simple way[3] by giving a small mass for the current quarks which appear in the QCD Lagrangian. In the limit of zero current quark masses ($m_{0q}\to 0$, $q=u,d,s$) the axial-vector currents associated with the symmetry group SU(3)$\times$SU(3) are conserved, but the appearance of an octet of massless pseudoscalar Goldstone bosons implies that the symmetry is spontaneously broken.[4] In particular, for $m_{0u}=m_{0d} << m_{0s}$, SU(2)$\times$SU(2) is an almost exact symmetry. This seems to represent the physical situation where the pion has a very small mass compared to other pseudoscalar-meson masses. Thus the isovector axial-vector current is almost conserved and for all practical purposes we have, to a good approximation,

$$\partial_\mu A_{i\mu}=0,\quad i=1,2,3\ . \tag{1}$$

Since Eq. (1) is an operator relation one should get a relation between the form factors (at $q^2=0$) for the axial-vector current and the pion-hadron coupling constant for any hadron. This is the generalized Goldberger-Treiman (GT) relation; the most familiar such relation is naturally the GT relation[5] for G_A/G_V in neutron β decay. This in turn imposes strong constraints on the coupling constants between the Goldstone bosons and the bound states. In this paper we shall first implement the PCAC (partially conserved axial-vector current) condition at the quark level for confined quarks in hadrons and then apply the GT relation to low-lying vector-meson states. Using a nonrelativistic quark model to calculate the Gamow-Teller transition matrix elements, we obtain a value for the $\omega\rho\pi$ coupling constant in agreement with experiment. We discuss also the case of transitions between vector and pseudoscalar mesons ($1^-\to 0^-$ transitions).

II. GOLDBERGER-TREIMAN RELATION FOR HADRONS IN THE QUARK MODEL

The success of the nonrelativistic quark model in describing hadron spectroscopy shows that quarks confined in hadrons acquire an effective mass about $\frac{1}{3}$ of the nucleon mass (the so-called constituent quark mass) and that their motion is nonrelatiavistic to a good approximation.[6] Starting from this approximation, static quantities[6] such as magnetic moments, radiative transitions, β-decay matrix elements for low-lying states as well as finer details of hadron mass splitting[7] (i.e., hyperfine splitting) can be calculated from the QCD interactions and are found to be in good agreement with experiments. In particular, the axial-vector currents can be approximated by the Gamow-Teller transition operator ($\sum_i \frac{1}{2}\tau_i^+\vec{\sigma}_i$). In this case the PCAC condition can be implemented at the quark level. Consider now the isovector axial-vector current. In the limit of m_{0u}, $m_{od}\to 0$ together with the absence of QCD anomalies for the isovector currents $A_{i\mu}$ ($A_{i\mu}=\bar{q}\frac{1}{2}\tau_i\gamma_\mu\gamma_5 q$, $i=1,2,3$) are conserved and Eq. (1) is a strong operator relation which must be satisfied by all the matrix elements of (1) between any hadron. The existence of massless pseudoscalar Goldstone bosons follows from this condition, which gives also the generalized GT relation applied to any hadron considered as a bound state of constitutent quarks. This constraint is automatically met if it is satisfied by quarks confined in a QCD potential. Let $g_{\pi qq}$ be the coupling constant of pion with quarks. Since the motion of quarks in the QCD potential is assumed to be nonrelativistic, we have in the limit $q^2 \to 0$ and for the charged pion,

$$2m_q\langle\gamma_5\rangle + \frac{g_{\pi^+ qq}}{f}\langle\gamma_5\rangle = 0 \,, \qquad (2)$$

where f is the inverse of the pion decay constant[8] f_π ($f^{-1}=f_\pi=m_\pi$), m_q is the constituent quark mass, and $\langle\gamma_5\rangle$ is the expectation value of the single-particle quark field operator $\bar{q}\gamma_5 q$ in this potential. From (2) we have the GT relation at the quark level,

$$g_{\pi^+ qq} = -2m_q f \,. \qquad (3)$$

Note that $g_{\pi^+ qq}$ is the $\pi^+ qq$ vertex function when the quark momenta are on the mass shell ($\not{p}_1=\not{p}=m_q$). Thus if condition (3) is met and a nonrelativistic description of the bound states is valid, then the GT relation for hadrons can be obtained easily.

In the presence of chiral-symmetry breaking, the pseudoscalar mesons acquire masses which are related to the current quark masses. The axial-vector currents are no longer conserved, but Eq. (3) is still valid if one makes the usual PCAC assumption:

$$\partial_\mu A_{i\mu} = \frac{m_\pi^2 f_\pi}{\sqrt{2}}\phi_i, \quad i=1,2,3 \,,$$

$$\partial_\mu A_{i\mu} = \frac{m_K^2 f_K}{\sqrt{2}}\phi_i, \quad i=4,5,6,7 \,.$$

We then have for $q^2 \to 0$,

$$f_\pi g_{\pi^+ ud} + (m_u+m_d) = 0 \,,$$
$$f_K g_{K^+ us} + (m_u+m_s) = 0 \,. \qquad (3')$$

Furthermore, if one assumes the SU(3)-symmetry relation between the dimensionless meson-quark coupling constants, i.e.,

$$g_{\pi^+ ud} = g_{K^+ us} \,,$$

one gets

$$\frac{f_K}{f_\pi} = \frac{m_u+m_s}{m_u+m_d} \qquad (4)$$

for $m_u=m_d=350$ MeV, $m_s=500$ MeV, Eq. (4) gives

$$\frac{f_K}{f_\pi} = 1.2 \,, \qquad (5)$$

which agrees well with the experimental value of 1.28. A probable explanation for the success of this prediction is that SU(3) relation can be used for the dimensionless meson-quark coupling constants even in the presence of SU(3) breaking.

It is easy to see that Eq. (2), when applied to nucleon matrix elements, reproduces the usual GT relation for G_A/G_V. In fact, the pion-nucleon coupling constant is defined by the following relation:

$$g_{\pi^+ pn}\left\langle \frac{\tau^+ \vec{q}\cdot\vec{\sigma}}{2m_N}\right\rangle = g_{\pi^+ ud}\left\langle \sum_i \frac{\tau_i^+ \vec{q}\cdot\vec{\sigma}_i}{2m_q}\right\rangle , \qquad (6)$$

where $\frac{1}{2}\tau^+\vec{\sigma}$ and $\frac{1}{2}\sum_i \tau_i^+\vec{\sigma}_i$ are the nucleon and quark Gamow-Teller transition operators, respectively. Since

$$\left\langle \sum_i \tau_i^+\vec{\sigma}_i \right\rangle = \left[\frac{G_A}{G_V}\right]\langle\tau^+\vec{\sigma}\rangle \,, \qquad (7)$$

we have

$$g_{\pi^+ pn} = -\left[\frac{G_A}{G_V}\right]\frac{2m_N}{f_\pi} \,,$$

which is the GT relation for G_A/G_V in terms of the $\pi^+ pn$ coupling constant. Using the nonrelativistic quark model [SU(6)] value for G_A/G_V ($G_A/G_V=-\frac{5}{3}$), we get

$$\frac{g_{\pi^+ pn}^2}{4\pi} = 42 \,,$$

which is bigger than the measured value by 40% ($g_{\pi NN}^2/4\pi=15$). This discrepancy could be attributed to a renormalization due to QCD interactions for G_A/G_V of confined quarks and/or possible configuration mixing of the nucleon wave function as discussed in the literature.[6,9]

Similar relations can be obtained for any bound states with nonvanishing Gamow-Teller matrix elements. In particular, the matrix elements of the axial-vector current between vector-meson states is another example in which the dynamics can be described by a nonrelativistic constituent quark model which is successful to some extent in describing radiative transitions between low-lying meson states.[6]

The covariant matrix element of the axial-vector currents between $\omega(p,\epsilon^{\omega})$ and $\rho^0(p',\epsilon^{\rho})$ is defined as

$$\langle \omega(p) | A_{3\mu}(0) | \rho^0(p') \rangle = i\epsilon_{\mu\nu\rho\sigma}(p_\nu + p'_\nu)\epsilon_\rho^\omega \epsilon_\sigma{}^{\rho *} F_A(q^2) . \quad (8)$$

The $\omega\rho^0\pi^0$ vertex is defined similarly as

$$\langle \omega(p) | j_3(0) \rho^0(p') \rangle = i\epsilon_{\mu\nu\rho\sigma} q_\mu p_\nu \epsilon_\rho^\omega \epsilon_\sigma{}^{\rho *} g_{\omega\rho^0\pi^0}(q^2)$$

(ϵ^{ω} and ϵ^{ρ} are the polarization vector for vector mesons and $q=p-p'$ the momentum carried by the neutral pion). $j_3(0)$ is the source of the pion field defined as

$$(\Box + m_\pi{}^2)\phi_3(0) = j_3(0) .$$

The GT relation for vector mesons can be obtained either from the PCAC condition or from the analog of Eq. (6) applied to 1^- states. For $q^2 \rightarrow 0$, we have then

$$2F_A(0) + \frac{f_\pi}{\sqrt{2}} g_{\omega\rho^0\pi^0} = 0 . \quad (9)$$

On the other hand, in a nonrelativistic quark model, $F_A(0)$ is given by the Gamow-Teller transition matrix element for a $1^- \rightarrow 1^-$ transition,

$$\langle \omega(p) | A_{3i}(0) | \rho^0(p') \rangle = 2p_0 \left\langle \chi_\omega^\dagger \left| \sum_a \frac{\tau_a^3}{2} \sigma_{ai} \right| \chi_\rho \right\rangle . \quad (10)$$

χ_ω and χ_ρ are the spin-coordinate wave functions which are identified with the polarization vectors in the nonrelativistic limit (we have assumed that the vector mesons are in a triplet S state according to the $q\bar{q}$ bound-state picture of low-lying mesons). In terms of the quark field operator, A_{3i} is given by

$$A_{3\mu}(0) = \tfrac{1}{2}(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d) .$$

Since the axial-vector current is *even* under charge conjugation, the Gamow-Teller matrix element for an anti-quark is *the same* as for a quark,

$$\langle \bar{q} | \vec{\sigma} | \bar{q} \rangle = \langle q | \vec{\sigma} | q \rangle .$$

Evaluating (10) for ω and ρ^0 in an $M=1$ state (spin parallel) and comparing (8) with (10), we get

$$F_A(0) = 1 ,$$

which gives [using (9)]

$$g_{\omega\rho^0\pi^0} = -\left[\frac{2\sqrt{2}}{f_\pi}\right] . \quad (11)$$

This corresponds to a value of $0.6/m_\pi{}^2$ for $g_{\omega\rho^0\pi^0}{}^2/4\pi$, which is only 30% larger than the experimental value of $(0.41 \pm 0.09)/m_\pi{}^2$ obtained from $\omega \rightarrow 3\pi$ decay.[10]

The agreement with experiment is rather good considering the crude approximation for $F_A(0)$. To produce a $\omega\rho\pi$ coupling constant close to the measured value, as with the case of G_A/G_V for the nucleon, one needs an effective $F_A(0)$ lower than the nonrelativistic-quark-model value by 20%. Configuration mixing of the $q\bar{q}$ wave function and renormalization of G_A/G_V for bound quarks may be responsible for this reduction. Since G_A/G_V for current quarks are found not to be renormalized (i.e., $G_A/G_V = 1$) in deep-inelastic neutrino hadron interactions, renormalization of G_A/G_V for constituent quarks would probably come from confinement effects associated with transverse motion of quarks in a QCD potential. Standard quark-model and MIT-bag-model calculations seem to obtain such a reduction of the right order of magnitude.[6,9] We believe that this reduction of the single-quark matrix element $\langle \bar{q}\gamma_\mu\gamma_5 q \rangle$ is more important than configuration mixing since it reduces also the SU(6) value for the nucleon G_A/G_V by the same amount. Assuming that $\langle \bar{q}\gamma_\mu\gamma_5 q \rangle$ for a single bound quark is renormalized by g_A, we have instead of (11)

$$g_{\omega\rho^0\pi^0} = -\left[\frac{2\sqrt{2}}{f_\pi}\right] g_A . \quad (12)$$

From the nucleon G_A/G_V, we have

$$g_A = -\left[\frac{G_A}{G_V}\right] \times \frac{3}{5} = \frac{3}{4} .$$

Hence

$$\frac{g_{\omega\rho^0\pi^0}{}^2}{4\pi} = \frac{0.36}{m_\pi{}^2}$$

is in good agreement with the measured value mentioned above.

Note that provided that g_A for $\bar{u}\gamma_\mu\gamma_5 d$ and $\bar{u}\gamma_\mu\gamma_5 s$ currents are the same the ratio f_K/f_π is unaffected by the renormalization effects and is still given by Eq. (4).

We now turn to the transitions between vector meson and pseudoscalar meson ($1^- \rightarrow 0^-$ transition). For bound states of unequal masses, the GT relation in general involves more than one form factor. For the matrix elements between bound states with a small mass difference, the soft-pion limit ($p' \to p$, $q \to 0$) can be used to eliminate terms proportional to $(p^2 - p'^2)$ in the GT relation. The main contribution is given by the Gamow-Teller transition matrix element. As an example, consider the matrix element of $A_{3\mu}(0)$ between D^{*+} and D^+. These states are known to be well described by $S=1$ and $S=0$ S states of the $c\bar{d}$ system. The matrix element is given by

$$\langle D^{*+}(p) | A_{3\mu}(0) | D^+(p') \rangle = \epsilon_\mu G_1 + \epsilon \cdot q [(p+p')_\mu G_+ + q_\mu G_-] \, .$$

Using PCAC, we have as $q^2 \to 0$

$$f_\pi g_{D^{*+}D^+\pi^0} = -\frac{\sqrt{2}}{2}[G_1(0) + (p^2 - p'^2) G_+(0)] \, , \tag{13}$$

where the $D^{*+} \to D^+ \pi^0$ transition matrix element is defined as

$$\mathcal{M}(D^{*+} \to D^+ \pi^0) = 2i \epsilon \cdot q g_{D^{*+}D^+\pi^0} \, .$$

In the soft-pion limit ($p' \to p$, $q \to 0$), the coupling constant $g_{D^{*+}D^+\pi^0}$ is determined completely by $G_1(0)$. The nonrelativistic quark model gives

$$G_1(0) = p_0 g_A \, .$$

Hence

$$g_{D^{*+}D^+\pi^0} = -\frac{\sqrt{2}}{2} \left[\frac{M_{D^{*+}}}{f_\pi} \right] g_A \, . \tag{14}$$

With $g_A = \frac{3}{4}$, we get

$$\Gamma(D^{*+} \to D^+ \pi^0) = 42 \text{ keV} \, .$$

Similarly, for $D^{*0} \to D^0 \pi^0$,

$$\Gamma(D^{*0} \to D^0 \pi^0) = 86 \text{ keV} \, .$$

For the radiative decays of D^*, we have after a straightforward calculation,

$$\frac{\Gamma(D^{*+} \to D^+ \gamma)}{\Gamma(D^{*+} \to D^+ \pi^0)} = 20\%$$

and

$$\frac{\Gamma(D^{*0} \to D^+ \gamma)}{\Gamma(D^{*0} \to D^+ \pi^0)} = 40\% \, .$$

Both these calculated branching ratios are more or less in agreement with measurements[11] on D^*. Further measurements on $D^* \to D\pi^0$ would provide a direct check on our essentially model-independent prediction for $\Gamma(D^* \to D\pi^0)$ obtained from (14). Note, however, the calculated values of Eichten *et al.*[11] for $\Gamma(D^* \to D\pi^0)$ are smaller than our rates by a factor of 2.

It is doubtful whether Eq. (14) can be applied to $K^* \to K\pi^0$ and $\rho \to \pi\pi$ transitions since the contribution to $g_{K^*K\pi^0}$ (or $g_{\rho\pi\pi}$) from $G_+(q^2)$ in Eq. (13) cannot in general be neglected. However, if one ignores $G_+(q^2)$ as usually done in nonrelativistic quark-model calculations, then Eq. (14) can be used to obtain $g_{K^*k\pi^0}$. This gives a decay rate

$$\Gamma(K^{*+} \to K^+ \pi^0) = \frac{g_A{}^2}{12\pi} \left[\frac{p_{\text{c.m.}}}{f_\pi} \right]^2 p_{\text{c.m.}} = 15 \text{ MeV} \, ,$$

in good agreement with the experimental value of 17 MeV (the measured width of K^{*+} is 50 MeV). It appears that $G_+(q^2)$ gives a negligible contribution to $g_{K^*K\pi^0}$. It follows than that $G_+(q^2)$ for $\rho^+ \to \pi^+$ transitions is also small [using SU(3)] and $g_{\rho\pi\pi}$ can now be obtained in terms of $G_1(0)$ only. Applying (14) to the $\rho \to \pi\pi$ transition, we get

$$\frac{g_{\rho\pi\pi}{}^2}{4\pi} = \frac{g_A{}^2}{4\pi} \left[\frac{2m_\rho{}^2}{f_\pi{}^2} \right] = 2.68 \, ,$$

in good agreement with the value 2.86 obtained from the ρ-meson width. The expressions for $g_{\rho\pi\pi}$ and other coupling constants discussed above are identical with those obtained[6] with the nonrelativistic quark model in which the pion-hadron coupling constant is defined as in Eq. (6) and $g_{\pi qq}$ can be obtained from $g_{\pi NN}$.

Despite the apparent success of the above calculation, it is not clear to what extent these results can be trusted. The main problem is how to reconcile the constituent $q\bar{q}$-bound-state picture for the pseudoscalar-meson octet (π, K, η) with the special role that these mesons play in the spontaneous breakdown of SU(3)×SU(3). Because of this special role, the coupling constants $g_{\rho\pi\pi}$ and $g_{K^*K\pi}$ should be treated by an effective-Lagrangian approach which incorporates the chiral symmetry in a simple manner.[4,12] Thus Eq. (14), when applied to $g_{\rho\pi\pi}$ and $g_{K^*K\pi}$, serves only as a qualitative indication on how these coupling constants are related to f_π, the fundamental constant which characterizes the spontaneous breakdown of chiral symmetry.

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