Neutrinoless double-β decay in SU(2)$\times$U(1) theories

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It is shown that gauge theories give contributions to neutrinoless double-β decay [$(\beta\beta)_{0\nu}$] which are not covered by the standard parametrizations. While probably small, their existence raises the question of whether the observation of $(\beta\beta)_{0\nu}$ implies the existence of a Majorana mass term for the neutrino. For a "natural" gauge theory we argue that this is indeed the case.

The associated questions of neutrinoless double-β decay [denoted $(\beta\beta)_{0\nu}$] and neutrino mass have again become of general interest. A comprehensive recent discussion is given by Doi *et al.*[1] and a concise summary by Rosen.[2]

The classical analysis of the $(\beta\beta)_{0\nu}$ process, which of course predated gauge theories by many years, assumed that it arose as a result of neutrino exchange between two effective four-fermion vertices. This is shown in modern language in Fig. 1(a), which illustrates the process at the quark level. Thus, the parametrization of the $(\beta\beta)_{0\nu}$ process was considered to be given fundamentally by the parametrization of the four-fermion single-β-decay interaction.

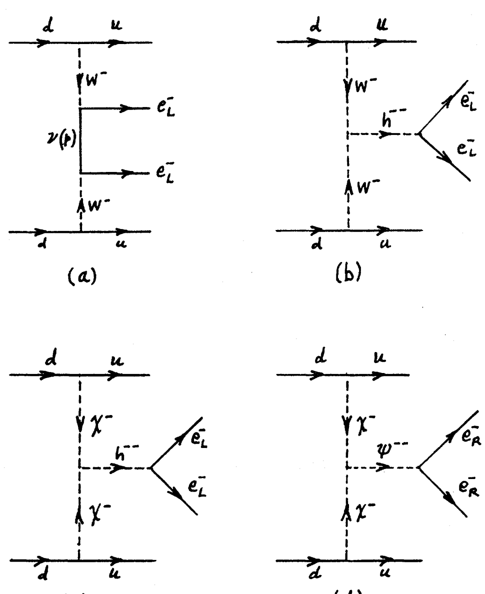

FIG. 1. Diagrams for neutrinoless double-β decay in an SU(2)$\times$U(1) gauge theory. The standard diagram is Fig. 1(a). It is the only one which contains a virtual neutrino (of four-momentum p). d and u are the down and up quarks.

We point out here that the situation is more complicated[3] if the weak interactions are described by a gauge theory. For definiteness we consider $(\beta\beta)_{0\nu}$ in the framework of the standard SU(2)$\times$U(1) gauge theory. Larger gauge groups usually contain SU(2)$\times$U(1) as a subgroup and composite models are usually contrived to also display this symmetry. Thus there is not much loss of generality in doing so.

A rather minimal way[4] to naturally include lepton-number violation in the theory is to add to the complex $Y=1$, $I=\frac{1}{2}$ Higgs doublet a complex $Y=2$, $I=1$ isotriplet H:

$$\phi=\begin{bmatrix}\phi^+\\ \phi^0\end{bmatrix},\quad H=\begin{bmatrix}\dfrac{H^+}{\sqrt{2}} & h^{++}\\ h^0 & \dfrac{-h^+}{\sqrt{2}}\end{bmatrix}. \tag{1}$$

The theory need not contain any more than one two-component Weyl neutrino field for each generation.[5] In a natural theory (no special adjustment of parameters) both ϕ^0 and h^0 will develop nonzero vacuum expectation values giving the relations[4]

$$\begin{aligned}&\langle\phi^0\rangle\equiv\lambda,\ \langle h^0\rangle\equiv y\ ,\\ &m^2(W)=\frac{g^2}{4}(\lambda^2+2y^2)\ ,\\ &m^2(Z)=\frac{g^2}{4\cos^2\theta_W}(\lambda^2+4y^2)\ ,\\ &\frac{G_F}{\sqrt{2}}=\frac{g^2}{8m^2(W)}\ ,\end{aligned} \tag{2}$$

in standard notation. The vacuum value y is expected to be small compared to λ, but the experimental constraints are not very tight.[6] For our purposes it is important to note that the singly charged field χ^- which is *not* absorbed by the gauge field is the linear combination

$$\chi^- \approx h^- + \omega\phi^- , \quad \omega = -\sqrt{2}\frac{y}{\lambda} , \tag{3}$$

where we have assumed ω to be small. Notice also that the "Yukawa" interaction of H allows a virtual h^{--} to annihilate into two left-handed electron fields (e_L^-) with strength m_ν/y (m_ν is a characteristic neutrino mass). Then there are a variety of diagrams for $(\beta\beta)_{0\nu}$ which involve no virtual neutrino line and hence cannot be parametrized by the standard β-decay interaction vertex. These diagrams will require a six-fermion amplitude for their description: The effective Lagrangian will be characterized by a parameter of dimension $(\text{mass})^{-5}$.

A basic new diagram is shown in Fig. 1(b). The trilinear $W^- W^- h^{++}$ vertex is due to a term in the fundamental Lagrangian:

$$\tfrac{1}{2} g^2 \langle \bar{h}^0 \rangle W_\mu^- W_\mu^- h^{++} + \text{H.c.} \tag{4}$$

derived from the kinetic term $-\frac{1}{2}\,\text{Tr}[(D_\mu H)^\dagger D_\mu H]$. The strength of the amplitude for $(\beta\beta)_{0\nu}$ from Fig. 1(b) is the product of the four trilinear coupling constants and the three propagators:

$$\frac{g^4 m_\nu}{m^2(h^{--}) m^4(W)} . \tag{5}$$

Note that the triplet vacuum value has canceled out of Eq. (5). For comparison the "standard" diagram given in Fig. 1(a) is characterized by a strength[7]

$$\frac{g^4 m_\nu}{m^4(W)\langle p^2 \rangle} . \tag{6}$$

Here $\langle p^2 \rangle$ represents a suitable average of the squared four-momentum carried by the virtual neutrino, say about $(10\text{ MeV})^2$. Of course the amplitude for the underlying quark process must be suitably[1,2] folded into the description of the real nucleus. The ratio

$$\frac{\text{amp for 1(b)}}{\text{amp for 1(a)}} = \frac{\langle p^2 \rangle}{m^2(h^{--})} \tag{7}$$

is thus expected to be of order of 10^{-8}. Thus the contribution of Fig. 1(b) seems negligible.

Additional new diagrams involve a trilinear Higgs interaction [see Fig. 1(c)]. The relevant terms in the Higgs Lagrangian are

$$c_1 \phi^\dagger (HH^\dagger - H^\dagger H)\phi + c_2\,\text{Tr}(HH)\,\text{Tr}(H^\dagger H^\dagger) + (d\phi^\dagger H \phi_c + \text{H.c.}) , \tag{8}$$

where

$$\phi_c \equiv \begin{bmatrix} \bar{\phi}^0 \\ -\phi^- \end{bmatrix} .$$

Taking into account the mixing between the charged components of the Higgs doublet and the triplet, given by Eq. (3), results in the trilinear vertex

$$g_3 \chi^- \chi^- h^{++} + \text{H.c.} , \qquad g_3 \approx \sqrt{2}\omega c_1 \langle \phi^0 \rangle + 2c_2 \langle h^0 \rangle - \omega^2 d . \tag{9}$$

The strength parameter associated with Fig. 1(c) is then

$$\frac{\omega^2 m_q{}^2 m_\nu g_3}{y\lambda^2 m^2(h^{--}) m^4(\chi^-)} \tag{10}$$

($m_q \approx 10$ MeV is a light-quark mass). Equation (10) involves unknown parameters characterizing the effective Higgs sector of the theory. Let us first estimate the contribution to Eq. (10) coming from the c_1 and c_2 terms. These are dimensionless parameters and can be reasonably expected to be of order unity, at most. Further estimating $m^2(h^{--}) \approx m^2(\chi^-) \approx \lambda^2$ the ratio of this amplitude to the standard one in Eq. (6) is roughly

$$\frac{y^2}{\lambda^2}\frac{m_q{}^2\langle p^2 \rangle}{\lambda^4} \approx 10^{-13}\omega^2 , \tag{11}$$

which is quite negligible. The contribution to Eq. (10) from the d term (d has dimensions of mass) is potentially more interesting. If one imposes lepton-number conservation on the theory d will be zero and there will be no contribution. Then lepton number breaks spontaneously and one has a massless "Majoron."[8] In such a case ω will also be very small. However, since d does carry a dimension one might regard it as an indicator of a new mass scale[9] and let it remain. Then the ratio of the d contribution to Eq. (10) to the standard amplitude Eq. (6) is roughly

$$\frac{\omega^3 m_q{}^2 \langle p^2 \rangle}{\lambda^4}\frac{d}{\lambda} \approx 10^{-13}\omega^3 \frac{d}{\lambda} . \tag{12}$$

This indicates that (even neglecting the suppression due to the ω^3 factor) d would have to be of the order of the grand unification mass scale for the diagram of Fig. 1(c) to play an important role.

The presence of the Higgs field χ^- in the theory also results in new diagrams of the standard form 1(a) in which one or both of the W's is replaced by a χ^-. These diagrams, which also modify the $V-A$ structure of the single-β-decay interaction, are quite small.

Thus we reach the conclusion that for the SU(2)×U(1) theory defined by the Higgs content (1), the effect of the new diagrams is quite small if one considers only mass scales lower than that of grand unification.

One type of neutrinoless diagram which may conceivably be relatively strong without superheavy masses is shown in Fig. 1(d). Here a new $Y=-4$ isosinglet Higgs field ψ^{--} is introduced in addition to the doublet and the triplet. The virtual ψ^{--} decays into two e_R's rather than two e_L's as in the previous cases. In this case the $\psi^{++}e_R^- e_R^-$ Yukawa interaction is not proportional to the neutrino mass (as was required previously since the $h^{++}e_L^- e_L^-$ Yukawa term is related by an isospin transformation to the $h^0 \nu\nu$ term which generates neutrino mass) and thus may be of order unity.[10] The term in the Higgs Lagrangian which generates the trilinear $\chi^-\chi^-\psi^{++}$ coupling in Fig. 1(d) is

$$\phi^\dagger H^\dagger \phi_c \psi^{++} + \text{H.c.} \tag{13}$$

The amplitude for Fig. 1(d) would then roughly be of order $\omega^2 m_q{}^2 y/\lambda^8$. The ratio of this to the usual amplitude, which is suppressed by a factor of m_ν, is about

$$\frac{\omega^2 y m_q{}^2 \langle p^2 \rangle}{m_\nu \lambda^4} . \tag{14}$$

This could be comparable to one if m_ν is exceptionally small.

Other models with extra Higgs fields can also boost the new contribution. For example, suppose that we add to (1) another complex doublet ϕ', as one might have in an axion scheme. Then there will be two physical singly charged fields and there is in general no need to have a suppression[10] of their Yukawa couplings to the quarks for small y. The ratio of the d contribution to the standard one [see Eq. (12)] is now roughly

$$10^{-13} \frac{\langle p^2 \rangle^{1/2}}{y} \frac{d}{\lambda} \approx 10^{-6} \frac{d}{\lambda} ,$$

where we have taken $y \approx 1$ eV. Thus an intermediate scale $d \approx 10^8$ GeV could make the new diagrams important.

To sum up we can say that while neutrinoless diagrams might not be dominant, a careful analysis of $(\beta\beta)_{0\nu}$ decays should really take into account

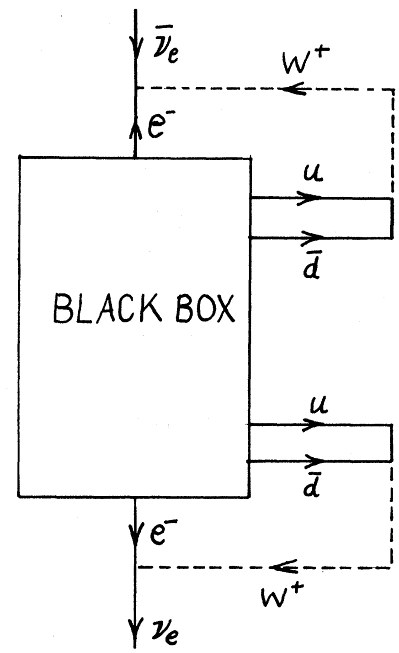

FIG. 2. Diagram showing how any neutrinoless double-β decay process induces a $\bar{\nu}_e$-to-ν_e transition, that is, an effective Majorana mass term.

their possible existence. This is because the general structure (as opposed to detailed predictions) of gauge theories seems to be the safest guide to the parametrization of weak-interaction amplitudes. It would be desirable to develop criteria[11] based on angular distributions of the decay products for distinguishing these diagrams from the usual ones.

We will conclude this paper with a brief discussion of the relation between the $(\beta\beta)_{0\nu}$ process and nonzero neutrino mass. After noticing the existence of neutrinoless diagrams one might be tempted to try to construct models without massive neutrinos and which would still give $(\beta\beta)_{0\nu}$. However, such a search would be in vain. For the model based on the Higgs content (1) this result is obvious since Eqs. (5) and (10) are proportional to m_ν. It is also true for the model with ψ^{--}: Although this model gives an amplitude with no m_ν factor there is an overall factor of $y=\langle h^0 \rangle$. Now in a natural theory H will couple to the basic lepton doublet so that a nonzero value of y will generate a neutrino mass.

Still one might think that a yet more clever choice of the Higgs-representation content could do the job. Rather than attempt an enumeration of all possible Higgs structures we will give a general and yet very simple proof that the existence of $(\beta\beta)_{0\nu}$ implies that the electron neutrino has nonzero mass. Essentially all one needs is to assume that the weak interactions are described by a local gauge theory. In this framework crossing symmetry will hold so the existence of $(\beta\beta)_{0\nu}$ implies a nonzero amplitude for the virtual process $0 \to u u \bar{d} \bar{d} \bar{e} \bar{e}$. This is shown in Fig. 2, where the "black box" may contain any mechanism whatsoever for generating $(\beta\beta)_{0\nu}$. Now any realistic gauge theory will include the ordinary W-gauge-field interaction with the left-handed electron and neutrino and with the u and d quarks. Using four of these vertices and connecting the lines together as in Fig. 2 shows that we develop an amplitude which gives a nonzero Majorana mass for the electron neutrino.

One might object that some other diagram might precisely cancel[12] Fig. 2, but this would clearly involve fine tuning of parameters and would be unnatural.

The converse question is whether a nonzero neutrino mass implies the existence of $(\beta\beta)_{0\nu}$. If the massive neutrino is of Majorana type, Fig. 1(a) shows that $(\beta\beta)_{0\nu}$ will occur. If the neutrino is of Dirac type the $(\beta\beta)_{0\nu}$ will not occur.[13] One may, however, exclude the possibility of neutrinos being of Dirac type if one postulates a "strong naturality" in which no global conservation laws are assumed *a priori*. In such a case massive neutrinos will be[14] of Majorana type.

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[1]M. Doi, T. Kotani, H. Nishiura, K. Okuda, and E. Takasugi, Prog. Theor. Phys. 66, 1739 (1981); 66, 1765 (1981); see also H. Nishiura, Kyoto University Report No. RIFP-453, 1981 (unpublished).

[2]S. P. Rosen, in *Neutrino 81,* proceedings of the International Conference on Neutrino Physics and Astrophysics, Maui, Hawaii, edited by R. J. Cence, E. Ma, and A. Roberts (University of Hawaii High Energy Physics Group, Honolulu, 1981).

[3]After the work described in this paper was completed we received a paper [Phys. Rev. Lett. 47, 1713 (1981)] by R. N. Mohapatra and J. D. Vergados which also points out the need for the intrinsic six-fermion interactions. However, the discussions of the two papers are somewhat different.

[4]See, for example, J. Schechter and J. W. F. Valle, Phys. Rev. D 22, 2227 (1980); T. P. Cheng and L. F. Li, *ibid.* 22, 2860 (1980); G. B. Gelmini and M. Roncadelli, Phys. Lett. 99B, 411 (1981); H. M. Georgi, S. L. Glashow, and S. Nussinov, Harvard Report No. HUTP-81/A026 (unpublished); P. B. Pal and L. Wolfenstein, Phys. Rev. D 25, 766 (1982).

[5]For simplicity we will consider the theory to contain only one generation.

[6]See Cheng and Li, Ref. 4 above.

[7]The lepton-number-violating Majorana-neutrino propagator in Fig. 1(a) is responsible for the factor $m_\nu/\langle p^2\rangle$.

[8]See, for example, Y. Chikashige, R. N. Mohapatra, and R. D. Peccei, Phys. Lett. 98B, 265 (1981); Gelmini and Roncadelli and Georgi *et al.*, Ref. 4 above; J. Schechter and J. W. F. Valle, Phys. Rev. D 25, 774 (1982); V. Barger, W. Y. Keung, and S. Pakvasa, *ibid.* 25, 907 (1982).

[9]See, for example, R. N. Mohapatra and G. Senjanovic, Phys. Rev. D 21, 165 (1981).

[10]If the leptonic Yukawa coupling constant in Fig. 1(c) is of order unity, y will be approximately m_ν and one will have enormous suppression of this diagram due to the factor ω^2 in Eq. (10).

[11]See Refs. 1 and 2, for example.

[12]Also one would have to cancel all variations on the basic Fig. 2, which is extremely unlikely. For example, a photon may be exchanged between each u and $\bar{d}$ line.

[13]A Dirac neutrino can be conceived of as a pair of mass-degenerate Majorana neutrinos with opposite *CP* assignments. As pointed out by L. Wolfenstein [Carnegie-Mellon Report No. COO-3066-180 (unpublished)], their contributions to the $(\beta\beta)_{0\nu}$ rate cancel due to their relative *CP* phase. With several Majorana neutrinos with different *CP* eigenvalues the neutrinoless diagrams discussed here may not be entirely negligible.

[14]See, for example, the discussion in Sec. 1 of the first of Refs. 4 above.