Magnetic dipole transitions of narrow resonances

Howard Grotch
Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802

Kunnat J. Sebastian
Department of Physics, University of Lowell, Lowell, Massachusetts 01854
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We examine the relativistic corrections to the $M1$ decays of charmonium and find that additional terms are present which have not yet been calculated. These arise as a consequence of the recoil of the composite system and are of the same order of magnitude as terms already calculated, contrary to what was previously believed.

In recent years the narrow resonances of the $c\bar{c}$ system have been investigated extensively both theoretically and experimentally.[1] The interaction potential must consist of a Coulombic term $1/r$ due to gluon exchange at short distance as well as a confining term which can be chosen to be linear in r. Thus the potential $ar-4\alpha_S/3r+b$ has become popular for use in fitting the gross features of the spectrum while additional relativistic interactions provide calculable corrections. The parameters are fit to the spectrum, but additional information, such as the widths of the states, is also known and can provide further tests of the theory.

Several years ago Feinberg and Sucher,[2] Sucher,[3] and others[4] studied $M1$ transitions of charmonium. We have reexamined these decays by another method, which in itself would not reveal anything new. In the process of the reexamination, however, we have evaluated an important contribution, previously considered negligible, due to recoil of the final-state composite system. Contrary to what was previously believed, this correction is of the same order of magnitude as the leading contributions to $M1$ decay.

In this short paper, which precedes a more extensive analysis of these decays, we first discuss our approach to $M1$ transitions based upon appropriate coupling to the electromagnetic field. Next we examine and evaluate the recoil effect mentioned above using two methods. The first employs relativistic center-of-mass operators while the second utilizes Lorentz boosts of the internal wave function. We then show that except for this recoil term our expression for the $M1$ decay agrees with that of Sucher.[3] Finally, we briefly examine the numerical contribution of this new correction to decay rates of charmonium.

We shall assume as a working model that at the relativistic level the quark-quark interaction is simulated by the exchange of vector particles and scalar particles. Thus, in the presence of the quantized radiation field the Hamiltonian is

$$H=\vec{\alpha}_1\cdot(\vec{p}_1-e_1\vec{A}_1)+\beta_1 m_1+\vec{\alpha}_2\cdot(\vec{p}_2-e_2\vec{A}_2)+\beta_2 m_2+\sum \omega a^\dagger a$$
$$+(1-\tfrac{1}{2}\vec{\alpha}_1\cdot\vec{\alpha}_2)V_V+\tfrac{1}{2}\vec{\alpha}_1\cdot\vec{r}\,\vec{\alpha}_2\cdot\vec{r}\frac{V_V'}{r}+\beta_1\beta_2 V_S\ . \quad (1)$$

We wish to stress at the outset that this is not the Hamiltonian we shall use, rather it is the reduction of it to the two-component space of each particle which will be utilized. This procedure has been standard in the literature for many years and it gives correct results even though the exact Hamiltonian of Eq. (1) (in the absence of the A field) is known to have serious defects.[5] These defects have led Sucher[3] to separate the field-theoretic Hamiltonian into no-pair and pair terms, treating the latter perturbatively. We show that to the desired accuracy, our approach, based on the v^2/c^2 expansion of (1), agrees with this method for the $M1$ decay (i.e., ignoring recoil).

From (1) we can carry out a Barker-Glover-Chraplyvy[6] reduction to a transformed approximate Hamiltonian,

$$H_{\rm tr}=m_1+m_2+\mathcal{O}_{EE}+\frac{1}{2m_1}(\mathcal{O}_{OE})^2+\frac{1}{2m_2}(\mathcal{O}_{EO})^2-\frac{(\mathcal{O}_{EO})^4}{8m_2^3}-\frac{(\mathcal{O}_{OE})^4}{8m_1^3}+\frac{1}{8m_1^2}[[\mathcal{O}_{OE},\mathcal{O}_{EE}],\mathcal{O}_{OE}]$$
$$+\frac{1}{8m_2^2}[[\mathcal{O}_{EO},\mathcal{O}_{EE}],\mathcal{O}_{EO}]+\frac{1}{4m_1m_2}[[\mathcal{O}_{OE},\mathcal{O}_{OO}]_+,\mathcal{O}_{EO}]_++\cdots \; . \tag{2}$$

In this expression $\mathcal{O}_{EE}$, $\mathcal{O}_{OE}$, $\mathcal{O}_{EO}$, and $\mathcal{O}_{OO}$ are, respectively, even-even, odd-even, even-odd, and odd-odd Dirac operators obtained from Eq. (1). The procedure leading to Eq. (2) is analogous to the Foldy-Wouthuysen (FW) transformation for the one-particle system.[7] The resulting Hamiltonian $H_{\rm tr}$ may also be arrived at by an alternative procedure which avoids the objection to Eq. (1). This involves carrying out a FW reduction at the level of field theory, as originally suggested by Lin,[8] and then deriving a two-particle Hamiltonian. We shall discuss this in greater detail in a more complete article.

Whichever approach is used, we find that we obtain

$$H_{\rm tr}-m_1-m_2=\frac{\vec{\pi}_1^{\,2}}{2m_1}+\frac{\vec{\pi}_2^{\,2}}{2m_2}+V_V+V_S-\frac{e_1}{2m_1}\vec{\sigma}_1\cdot\vec{\rm B}_1-\frac{e_2}{2m_2}\vec{\sigma}_2\cdot\vec{\rm B}_2+\sum\omega a^\dagger a$$
$$-\frac{\vec{\pi}_1^{\,4}}{8m_1^3}-\frac{\vec{\pi}_2^{\,4}}{8m_2^3}+\frac{e_1}{8m_1^3}[\vec{\sigma}_1\cdot\vec{\rm B}_1,\vec{\rm p}_1^{\,2}]_++\frac{e_2}{8m_2^3}[\vec{\sigma}_2\cdot\vec{\rm B}_2,\vec{\rm p}_2^{\,2}]_+$$
$$+\frac{1}{4m_1m_2}(\vec{\sigma}_1\cdot\vec{\sigma}_2\nabla^2V_V-\vec{\sigma}_1\cdot\vec{\nabla}\vec{\sigma}_2\cdot\vec{\nabla}V_V)$$
$$+\frac{1}{4m_1^2}\vec{\sigma}_1\cdot(\vec{\nabla}V_V+e_1\dot{\vec{\rm A}}_1)\times\vec{\pi}_1+\frac{1}{4m_2^2}\vec{\sigma}_2\cdot(-\vec{\nabla}V_V+e_2\dot{\vec{\rm A}}_2)\times\vec{\pi}_2$$
$$+\frac{ie_1}{8m_1^2}\vec{\sigma}_1\cdot\dot{\vec{\rm B}}_1+\frac{ie_2}{8m_2^2}\vec{\sigma}_2\cdot\dot{\vec{\rm B}}_2-\frac{1}{2m_1m_2}\vec{\sigma}_1\cdot\vec{\nabla}V_V\times\vec{\pi}_2-\frac{1}{2m_1m_2}\vec{\sigma}_2\cdot(-\vec{\nabla}V_V)\times\vec{\pi}_1$$
$$-\frac{1}{8m_1m_2}[\vec{\pi}_2\cdot,\vec{\pi}_1V_V+V_V\vec{\pi}_1-(\vec{\pi}_1\cdot\vec{\rm r})\vec{\nabla}V_V-\vec{\nabla}V_V(\vec{\rm r}\cdot\vec{\pi}_1)]_++\left[\frac{1}{8m_1^2}+\frac{1}{8m_2^2}\right]\nabla^2V_V$$
$$-\frac{1}{4m_1^2}\vec{\sigma}_1\cdot\vec{\nabla}V_S\times\vec{\pi}_1-\frac{1}{4m_2^2}\vec{\sigma}_2\cdot(-\vec{\nabla}V_S)\times\vec{\pi}_2+\frac{e_1}{2m_1^2}V_S\vec{\sigma}_1\cdot\vec{\rm B}_1+\frac{e_2}{2m_2^2}V_S\vec{\sigma}_2\cdot\vec{\rm B}_2$$
$$-\frac{1}{8m_1^2}([V_S,\vec{\pi}_1^{\,2}]_++2\vec{\pi}_1\cdot V_S\vec{\pi}_1)-\frac{1}{8m_2^2}([V_S,\vec{\pi}_2^{\,2}]_++2\vec{\pi}_2\cdot V_S\vec{\pi}_2)\;, \tag{3}$$

where

$$\vec{\pi}_i=\vec{\rm p}_i-e_i\vec{\rm A}_i\;.$$

There are four $M1$ decays of interest. These are

(a) $2\,{}^1S_0\to 1\,{}^3S_1+\gamma$ or $\eta_c'\to\psi+\gamma$,

(b) $2\,{}^3S_1\to 1\,{}^1S_0+\gamma$ or $\psi'\to\eta_c+\gamma$,

(c) $2\,{}^3S_1\to 2\,{}^1S_0+\gamma$ or $\psi'\to\eta_c'+\gamma$,

(d) $1\,{}^3S_1\to 1\,{}^1S_0+\gamma$ or $\psi\to\eta_c+\gamma$.

The first two, (a) and (b), can only occur due to relativistic terms in the Hamiltonian, because the leading $M1$ operator will have zero matrix elements between nonrelativistic radial wave functions of the $n=1$ and $n=2$ states. The last two decays, (c) and (d), do occur nonrelativistically.

We shall separate the transition matrix element into two terms. The first of these arises when we ignore recoil for the final-state composite system. When this is done only spin-dependent operators in Eq. (3) can lead to transitions between the triplet and singlet spin states. These states are characterized by a conventional direct product of two-component spinors for each of the particles. The second type of correction, which to our knowledge has not been evaluated, arises from the Lorentz boost of the composite wave function. When this boost is taken into account, the interaction terms

$$-\frac{e_1}{m_1}\vec{\rm p}_1\cdot\vec{\rm A}_1-\frac{e_2}{m_2}\vec{\rm p}_2\cdot\vec{\rm A}_2$$

lead to $M1$ contributions, as will be demonstrated later.

Consider now the explicitly spin-dependent terms of Eq. (3) which are linear in the radiation field. When the matrix element is taken between the initial photon vacuum and a final one-photon state of momentum $\vec{k}$ what remains is an interaction Hamiltonian acting between composite states presumed to be at rest. The spin-dependent part of this Hamiltonian, for the case of particles of equal mass m and opposite charge (let $e_1=q$) is

$$\begin{aligned}
&-\frac{q}{2m}\vec{\sigma}_1\cdot(-i)\vec{k}\times\vec{\epsilon}^{\,*}e^{-i\vec{k}\cdot\vec{r}/2}+\frac{q}{2m}\vec{\sigma}_2\cdot(-i)\vec{k}\times\vec{\epsilon}^{\,*}e^{i\vec{k}\cdot\vec{r}/2}\\
&\quad+\frac{q}{8m^3}\vec{\sigma}_1\cdot(-i)\vec{k}\times\vec{\epsilon}^{\,*}e^{-i\vec{k}\cdot\vec{r}/2}[\vec{p}^{\,2}+(\vec{p}-\vec{k})^2]-\frac{q}{8m^3}\vec{\sigma}_2\cdot(-i)\vec{k}\times\vec{\epsilon}^{\,*}e^{i\vec{k}\cdot\vec{r}/2}[\vec{p}^{\,2}+(\vec{p}+\vec{k})^2]\\
&\quad+\frac{1}{4m^2}\vec{\sigma}_1\cdot\vec{\nabla}V_V\times(-q)\vec{\epsilon}^{\,*}e^{-i\vec{k}\cdot\vec{r}/2}+\frac{1}{4m^2}\vec{\sigma}_2\cdot(-\vec{\nabla}V_V)\times q\vec{\epsilon}^{\,*}e^{i\vec{k}\cdot\vec{r}/2}\\
&\quad+\frac{qi\omega}{4m^2}\vec{\sigma}_1\cdot\vec{\epsilon}^{\,*}e^{-i\vec{k}\cdot\vec{r}/2}\times\vec{p}-\frac{qi\omega}{4m^2}\vec{\sigma}_2\cdot\vec{\epsilon}^{\,*}e^{i\vec{k}\cdot\vec{r}/2}\times(-\vec{p})\\
&\quad+\frac{iq}{8m^2}(i\omega)(-i)\vec{\sigma}_1\cdot\vec{k}\times\vec{\epsilon}^{\,*}e^{-i\vec{k}\cdot\vec{r}/2}-\frac{iq}{8m^2}(i\omega)(-i)\vec{\sigma}_2\cdot\vec{k}\times\vec{\epsilon}^{\,*}e^{i\vec{k}\cdot\vec{r}/2}\\
&\quad-\frac{q}{2m^2}\vec{\sigma}_1\cdot\vec{\nabla}V_V\times\vec{\epsilon}^{\,*}e^{i\vec{k}\cdot\vec{r}/2}-\frac{q}{2m^2}\vec{\sigma}_2\cdot\vec{\nabla}V_V\times\vec{\epsilon}^{\,*}e^{-i\vec{k}\cdot\vec{r}/2}\\
&\quad+\frac{q}{4m^2}\vec{\sigma}_1\cdot\vec{\nabla}V_S\times\vec{\epsilon}^{\,*}e^{-i\vec{k}\cdot\vec{r}/2}+\frac{q}{4m^2}\vec{\sigma}_2\cdot\vec{\nabla}V_S\times\vec{\epsilon}^{\,*}e^{i\vec{k}\cdot\vec{r}/2}\\
&\quad+\frac{q}{2m^2}V_S(-i)\vec{\sigma}_1\cdot\vec{k}\times\vec{\epsilon}^{\,*}e^{-i\vec{k}\cdot\vec{r}/2}-\frac{q}{2m^2}V_S(-i)\vec{\sigma}_2\cdot\vec{k}\times\vec{\epsilon}^{\,*}e^{i\vec{k}\cdot\vec{r}/2}\,. \qquad (4)
\end{aligned}$$

Since the initial and final states have the same parity, only the even-parity part of Eq. (4) can contribute. We expand exponentials keeping at most the quadratic terms in $\vec{k}\cdot\vec{r}$. This yields

$$\begin{aligned}
&\frac{qi}{2m}(\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{k}\times\vec{\epsilon}^{\,*}[1-\tfrac{1}{8}(\vec{k}\cdot\vec{r})^2]-\frac{qi}{8m^3}(\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{k}\times\vec{\epsilon}^{\,*}(2\vec{p}^{\,2}+\vec{k}^2+i\vec{k}\cdot\vec{r}\vec{k}\cdot\vec{p})\\
&\quad+\frac{qiV_V'}{8m^2r}(\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{r}\times\vec{\epsilon}^{\,*}(\vec{k}\cdot\vec{r})+\frac{q\omega}{8m^2}(\vec{k}\cdot\vec{r})(\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{\epsilon}^{\,*}\times\vec{p}+\frac{qi\omega}{8m^2}(\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{k}\times\vec{\epsilon}^{\,*}\\
&\quad-\frac{qiV_V'}{4m^2r}(\vec{\sigma}_1-\vec{\sigma}_2)\cdot(\vec{r}\times\vec{\epsilon}^{\,*})(\vec{k}\cdot\vec{r})-\frac{qiV_S'}{8m^2r}(\vec{\sigma}_1-\vec{\sigma}_2)\cdot(\vec{r}\times\vec{\epsilon}^{\,*})(\vec{k}\cdot\vec{r})-\frac{qiV_S}{2m^2}(\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{k}\times\vec{\epsilon}^{\,*}\,. \qquad (5)
\end{aligned}$$

To proceed further we now carry out an angular average utilizing the fact that initial and final states are S states. Correction terms of order $\vec{k}^2/m^2$ are dropped since they are too small. In the fourth term we replace ω multiplied by the operator by a commutator with the nonrelativistic Hamiltonian. This can be done since ω is the difference between the initial- and final-composite-energies. We then find

$$\begin{aligned}
&\frac{qi}{2m}(\vec{\sigma}_1-\vec{\sigma}_2)\cdot(\vec{k}\times\vec{\epsilon}^{\,*})\left[1-\frac{k^2r^2}{24}-\frac{\vec{p}^{\,2}}{2m^2}+\frac{rV_V'}{12m^2}+\frac{\omega}{4m}-\frac{rV_V'}{6m^2}-\frac{rV_S'}{12m^2}-\frac{V_S}{m}\right]\\
&\qquad\qquad-\frac{q}{24m^2}(\vec{\sigma}_1-\vec{\sigma}_2)\cdot(\vec{k}\times\vec{\epsilon}^{\,*})[\vec{r}\cdot\vec{p},H_0]\,. \qquad (6)
\end{aligned}$$

We now use

$$[\vec{r}\cdot\vec{p},H_0]=\left[\vec{r}\cdot\vec{p},\frac{\vec{p}^{\,2}}{m}+V_V+V_S\right]=\frac{2i}{m}\vec{p}^{\,2}-ir(V_V'+V_S')\,. \qquad (7)$$

Substituting Eq. (7) into (6) we find

$$\frac{qi}{2m}(\vec{\sigma}_1-\vec{\sigma}_2)\cdot(\vec{k}\times\vec{\epsilon}^*)\left[1-\frac{k^2r^2}{24}-\frac{\vec{p}^2}{2m^2}+\frac{rV_V'}{12m^2}+\frac{\omega}{4m}-\frac{rV_V'}{6m^2}-\frac{rV_S'}{12m^2}-\frac{V_S}{m}-\frac{\vec{p}^2}{6m^2}+\frac{r}{12m^2}(V_V'+V_S')\right]$$
$$=\frac{qi}{2m}(\vec{\sigma}_1-\vec{\sigma}_2)\cdot(\vec{k}\times\vec{\epsilon}^*)\left[1+\frac{\omega}{4m}-\frac{V_S}{m}-\frac{k^2r^2}{24}-\frac{2}{3}\frac{\vec{p}^2}{m^2}\right]. \quad (8)$$

Except for the small factor of $\omega/4m$ which is of the order of terms deliberately dropped (e.g., in Ref. 3), this expression will agree with previous results (2)–(4). It should be noted that the leading term in Eq. (8) will give I_{spin} of (6.73) of Ref. 3 when spin-dependent interactions are incorporated into the unperturbed wave functions. This $\omega/4m$ term is negligible when nonrelativistic initial and final radial wave functions are orthogonal. This occurs for transitions (a) and (b). On the other hand for transitions (c) and (d) the frequency ω is quite small and ω/m is less significant than the other corrections.

As mentioned earlier Eq. (8) is not the whole story and we must now examine the effect of recoil of the final state. This can be accomplished in a variety of ways. When the isolated Hamiltonian of Eq. (3) (with $\vec{A}=0$) is written in terms of conventional nonrelativistic relative and center-of-mass coordinates, it is found that internal coordinates and center-of-mass coordinates do not separate, and therefore knowledge of the internal wave function does not immediately provide the total wave function for the moving composite system. Many authors[9] have discussed this problem and have found that constituent particle variables can be expressed in terms of relativistic relative and center-of-mass variables. After this is done the total wave function is easily determined from the internal wave function.

To extract the corrections, consider the terms

$$-\frac{q}{m}\vec{p}_1\cdot\vec{\epsilon}^*e^{-i\vec{k}\cdot\vec{x}_1}+\frac{q}{m}\vec{p}_2\cdot\vec{\epsilon}^*e^{-i\vec{k}\cdot\vec{x}_2} \quad (9)$$

and now use the relations between constituent and center-of-mass variables as given by Kracjik and Foldy.[9] Let us at first ignore any dynamical effects contained in the Lorentz boost and concentrate only on kinematics. We can then write $\vec{x}_1=\vec{\rho}_1+\vec{R}+\delta\vec{x}_1$ and $\vec{x}_2=\vec{\rho}_2+\vec{R}+\delta\vec{x}_2$ where $\vec{\rho}_i$ are the center-of-mass relative variables and $\vec{R}$ is the position operator of the center of mass. We readily find that $\delta\vec{x}_1$ and $\delta\vec{x}_2$ contain spin-dependent terms $(\delta\vec{x}_1)_{\text{sp}}$ and $(\delta\vec{x}_2)_{\text{sp}}$,

$$(\delta\vec{x}_1)_{\text{sp}}=\frac{(\vec{\sigma}_1-\vec{\sigma}_2)\times\vec{p}}{8m^2}-\frac{(\vec{\sigma}_1-\vec{\sigma}_2)\times\vec{P}}{16m^2}, \quad (10)$$

$$(\delta\vec{x}_2)_{\text{sp}}=\frac{(\vec{\sigma}_1-\vec{\sigma}_2)\times\vec{p}}{8m^2}+\frac{(\vec{\sigma}_1-\vec{\sigma}_2)\times\vec{P}}{16m^2}. \quad (11)$$

The constituent momenta do not have spin-dependent kinematical corrections. Expanding the exponentials in Eq. (9) and setting $\vec{P}=0$ in Eqs. (10) and (11) we now find

$$\frac{iq}{m}(\vec{p}\cdot\vec{\epsilon}^*)\vec{k}\cdot\frac{[(\vec{\sigma}_1-\vec{\sigma}_2)\times\vec{p}]}{8m^2}\times 2, \quad (12)$$

which upon angular averaging gives

$$\frac{iq}{2m}(\vec{\sigma}_1-\vec{\sigma}_2)\cdot(\vec{k}\times\vec{\epsilon}^*)\left[\frac{-\vec{p}^2}{6m^2}\right]. \quad (13)$$

Hence we find a new correction.

Let us now arrive at this correction from a different point of view. Again consider Eq. (3) with the radiation field turned off and imagine expressing the individual-particle position and momentum operators in terms of conventional nonrelativistic variables, that is $\vec{x}_1=\vec{R}+\vec{r}/2$, $\vec{x}_2=\vec{R}-\vec{r}/2$, $\vec{p}_1=\vec{p}+\vec{P}/2$, and $\vec{p}_2=-\vec{p}+\vec{P}/2$. After this is done the Hamiltonian contains an internal part as well as a part which depends in some manner on $\vec{P}$, call it H_P. If H_P is treated as a perturbation it will lead to important wave-function corrections needed to evaluate transitions from a composite state at rest to one with final momentum $-\vec{k}$. Another way of accomplishing the same end is to perform a Lorentz boost on the final wave function. Recall that this wave function is a two-component spinor for each particle.

The easiest way to do this is to go back to the Dirac representation and use approximate kinematic boost operators to get approximate two-component wave functions. Suppose we write the boosted wave function as

$$\psi=\left[1-\frac{\vec{\alpha}_1\cdot\vec{k}}{2M}\right]\left[1-\frac{\vec{\alpha}_2\cdot\vec{k}}{2M}\right]e^{-i\vec{k}\cdot\vec{R}}\psi_{\text{rel}}, \quad (14)$$

where $M\cong 2m$. The upper components of ψ are now modified by the presence of boost operators. If we write ψ_{rel} in terms of upper and lower components, and if we express the lower components of $\psi_{\rm rel}$ in terms of the upper components we will find that

$$\psi=e^{-i\vec{k}\cdot\vec{R}}\psi_{\rm rel}+\delta\psi\ . \tag{15}$$

The leading terms of $\delta\psi$ have spin-dependent terms,

$$\left[-\frac{i}{4mM}\vec{\sigma}_1\cdot(\vec{k}\times\vec{p})+\frac{i}{4mM}\vec{\sigma}_2\cdot(\vec{k}\times\vec{p})\right]\times\psi_{\rm rel}^{uu}e^{-i\vec{k}\cdot\vec{R}}\ , \tag{16}$$

where uu stands for the upper components of the wave function. Returning to the matrix element of Eq. (9), we find that the inclusion of the above correction to the upper components of the final-state wave function implies the additional term

$$\frac{i}{8m^2}(\vec{\sigma}_1-\vec{\sigma}_2)\cdot(\vec{k}\times\vec{p})\left[-\frac{2q}{m}\vec{p}\cdot\vec{\epsilon}^{\,*}\right]\ , \tag{17}$$

which upon angular averaging yields

$$\frac{iq}{2m}(\vec{\sigma}_1-\vec{\sigma}_2)\cdot(\vec{k}\times\vec{\epsilon}^{\,*})\left[-\frac{\vec{p}^{\,2}}{6m^2}\right]\ , \tag{18}$$

in agreement with the result obtained earlier [Eq. (13)] by an alternative method.

Adding Eq. (18) to Eq. (8) gives an $M1$ decay operator

$$\frac{qi}{2m}(\vec{\sigma}_1-\vec{\sigma}_2)\cdot(\vec{k}\times\vec{\epsilon}^{\,*})\times\left[1+\frac{\omega}{4m}-\frac{V_S}{m}-\frac{k^2r^2}{24}-\frac{5}{6}\frac{\vec{p}^{\,2}}{m^2}\right]\ . \tag{19}$$

We might now wonder whether dynamical modifications of the boost operator can also lead to additional corrections of the same order as those already given. We have looked at this question and have convinced ourselves that such terms would be too small.

The easiest way to see this is to start from the expression for the $M1$ amplitude given by one of us earlier.[10] Those expressions[10] included the effects of the interaction-dependent part of the boost operator. We find that we can reproduce Eq. (19) in this way too, provided the spin-dependent part of the operator $\vec{W}^{(1)}$ of Ref. 10 commutes with the nonrelativisitc internal Hamiltonian. For the Hamiltonian under consideration we can show that this condition is definitely satisfied.

As mentioned in the introduction our approach is somewhat different than that given in Ref. 3. We would now like to show that they are equivalent. Let us first consider the spin-dependent terms linear in the radiation field arising from all terms of Eq. (2) except the last term involving $\mathcal{O}_{OO}$ and the double-commutator terms in which $\mathcal{O}_{EE}=V_V-\beta_1\beta_2V_S$. From Eq. (3) we get

$$-\frac{q}{2m}\vec{\sigma}_1\cdot\vec{B}_1+\frac{q}{8m^3}[\vec{\sigma}_1\cdot\vec{B}_1,\vec{p}_1^{\,2}]_+-\frac{q\omega}{8m^2}\vec{\sigma}_1\cdot\vec{B}_1+\frac{qi\omega}{4m^2}\vec{\sigma}_1\cdot\vec{A}_1\times\vec{p}_1+(1\leftrightarrow2)\ . \tag{20}$$

Between the initial and final states we may replace this expression by

$$\begin{aligned}
-\frac{q}{2m}\vec{\sigma}_1\cdot\vec{B}_1&+\frac{q}{8m^3}[\vec{\sigma}_1\cdot\vec{B}_1,\vec{p}_1^{\,2}]_+-\frac{q}{8m^2}[\vec{\sigma}_1\cdot\vec{B}_1,H_0]+\frac{qi}{4m^2}[\vec{\sigma}_1\cdot(\vec{A}_1\times\vec{p}_1),H_0]+(1\leftrightarrow2)\\
&=\frac{qi}{2m}\vec{\sigma}_1\cdot\vec{k}\times\vec{\epsilon}^{\,*}e^{-i\vec{k}\cdot\vec{x}_1}-\frac{qi}{8m^3}\vec{\sigma}_1\cdot\vec{k}\times\vec{\epsilon}^{\,*}e^{-i\vec{k}\cdot\vec{x}_1}(2\vec{p}_1^{\,2}-2\vec{p}_1\cdot\vec{k}+\vec{k}^2)\\
&\quad+\frac{qi}{8m^2}\vec{\sigma}_1\cdot\vec{k}\times\vec{\epsilon}^{\,*}\frac{1}{2m}\left[\vec{k}\cdot\vec{p}_1e^{-i\vec{k}\cdot\vec{x}_1}+e^{-i\vec{k}\cdot\vec{x}_1}\vec{k}\cdot\vec{p}_1\right]-\frac{q}{4m^2}\vec{\sigma}_1\cdot\vec{\nabla}(V_V+V_S)\times\vec{A}_1\\
&\quad+\frac{qi}{8m^3}\left[\vec{k}\cdot\vec{p}_1e^{-i\vec{k}\cdot\vec{x}_1}+e^{-i\vec{k}\cdot\vec{x}_1}\vec{k}\cdot\vec{p}_1\right](\vec{\sigma}_1\times\vec{\epsilon}^{\,*})\cdot\vec{p}_1+(1\leftrightarrow2)\ .
\end{aligned} \tag{21}$$

We now drop those terms which would constitute relative corrections of order $\vec{k}^2/m^2$. The present analysis shows that our previous contribution in Eq. (19) involving $\omega/4m$ is much smaller when matrix elements are actually evaluated. We find that Eq. (21) may be replaced by

$$\frac{qi}{2m}\vec{\sigma}_1\cdot\vec{k}\times\vec{\epsilon}^{\,*}\left[1-\frac{(\vec{k}\cdot\vec{x}_1)^2}{2}\right]-\frac{qi}{4m^3}\vec{p}_1^{\,2}\vec{\sigma}_1\cdot\vec{k}\times\vec{\epsilon}^{\,*}-\frac{qi}{4m^3}\vec{\sigma}_1\cdot\vec{p}_1\times\vec{\epsilon}^{\,*}(\vec{p}_1\cdot\vec{k})-\frac{q}{4m^2}\vec{\sigma}_1\cdot\vec{\nabla}(V_V+V_S)\times\vec{A}_1+(1\leftrightarrow2)\ . \tag{22}$$

This expression should be compared to the no-pair contribution of Sucher,[3] given by

$$\frac{qi}{2m}\vec{\sigma}_1\cdot\vec{k}\times\vec{\epsilon}^*\left[1-\frac{(\vec{k}\cdot\vec{x}_1)^2}{2}\right]-\frac{qi}{4m^3}\vec{p}_1{}^2\vec{\sigma}_1\cdot\vec{k}\times\vec{\epsilon}^*-\frac{qi}{4m^3}\vec{\sigma}_1\cdot\vec{p}_1\times\vec{\epsilon}^*(\vec{p}_1\cdot\vec{k})\ . \tag{23}$$

To complete the comparison we must now obtain the previously discarded terms. It is straightforward to show that neglected double commutators can be written as

$$-\frac{q}{4m^2}[\alpha_1\cdot\vec{A}_1,[V_V+\beta_1\beta_2V_S,\vec{\alpha}_1\cdot\vec{p}_1]]+\frac{q}{4m^2}\vec{\sigma}_1\cdot\vec{\nabla}(V_V+V_S)\times\vec{A}_1+(1\leftrightarrow 2)\ . \tag{24}$$

The corresponding term of Sucher coming from pair terms is

$$-\frac{q}{4m^2}[\vec{\alpha}_1\cdot\vec{A}_1,[V_V+\beta_1\beta_2V_S,\vec{\alpha}_1\cdot\vec{p}_1]]+(1\leftrightarrow 2)\ . \tag{25}$$

Finally, our anticommutator term of Eq. (2) agrees exactly with the pair anticommutator term, Eq. (6.64) of Sucher.[3] Hence when we add our contributions they agree with those of Sucher.[2]

In this paper we have shown that our approach is equivalent to that of Ref. 3 but that a new correction of relative order $-\vec{p}^2/6m^2$ emerges as a consequence of recoil of the final composite system. To what extent does this correction modify the decay widths previously calculated? The answer is model dependent. An examination of Table II of Ref. 3 indicated that $M1$ decays for which the initial and final radial wave function are the same ($\psi'\to\eta_c'+\gamma$ and $\psi\to\eta_c+\gamma$) tend to be dominated by the leading $M1$ operator. These are nonrelativistic $M1$ transitions and the correction we found above lowers the rate by 8 or 9 % for $\psi'\to\eta_c'+\gamma$ and 4 or 5 % for $\psi\to\eta_c+\gamma$. A more dramatic effect occurs for the relativistic $M1$ transitions $\eta_c'\to\psi+\gamma$ and $\psi'\to\eta_c+\gamma$. For the first of these, corrections of the order of 50% can occur while for the second, which has a very low rate, the effect is much larger. For example, on the basis of model (i) of Sucher[3] (see Table II) we find that the rate 0.07 would be replaced by 0.45.

In a more extensive article, now in preparation, we will provide numerical rates which contain the correction we now find as well as a comparison with existing data.

In conclusion we should emphasize that the correction term we have found for the $M1$ decays of charmonium is independent of any model we are using and will play an important role in the "relativistic" $M1$ decays of any bound system whose constituent particles have comparable masses.

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[1]For a review of the relevant literature see V. A. Novikov *et al.*, Phys. Rep. C41, 1 (1978); M. Krammer and H. Krasemann, Acta Phys. Austriaca, Suppl. XXI, 259 (1979); K. Gottfried, in *Proceedings of the International Symposium on Lepton and Photon Interactions at High Energies, Hamburg, 1977,* edited by F. Gutbrod (DESY, Hamburg, 1977); T. Appelquist, R. M. Barnett, and K. D. Lane, Annu. Rev. Nucl. Sci. 28, 387 (1978); E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T. M. Yan, Phys. Rev. D 21, 203 (1980).

[2]G. Feinberg and J. Sucher, Phys. Rev. Lett. 35, 1740 (1975).

[3]J. Sucher, Rep. Prog. Phys. 41, 1781 (1978).

[4]J. S. Kang and J. Sucher, Phys. Rev. D 18, 2698 (1978); M. S. Chanowitz and F. J. Gilman, Phys. Lett. B 63, 178 (1976); K. Gottfried, in *Proceedings of the International Symposium on Lepton and Photon Interactions at High Energies, Hamburg, 1977* (Ref. 1); J. D. Jackson, in *Proceedings of the 1977 European Conference on Particle Physics, Budapest,* edited by L. Jenik and I. Montvay (CRIP, Budapest, 1978); J. Borenstein and R. Shankar, Phys. Rev. Lett. 34, 619 (1976).

[5]G. E. Brown and D. G. Ravenhall, Proc. R. Soc. London A208, 552 (1951); see also J. Sucher (Ref. 3), p. 1795.

[6]Z. V. Chraplyvy, Phys. Rev. 91, 388 (1953); W. A. Barker and F. N. Glover, *ibid.* 99, 317 (1955).

[7]L. L. Foldy and S. A. Wouthuysen, Phys. Rev. 78, 29 (1950).

[8]D. L. Lin, Phys. Rev. A 15, 2324 (1977).

[9]H. Osborn, Phys. Rev. 176, 1514 (1968); 176, 1523 (1968); F. E. Close and L. A. Copley, Nucl. Phys. B19, 477 (1970); F. E. Close and H. Osborn, Phys. Rev. D 2, 2127 (1970); R. A. Krajcik and L. L. Foldy, Phys. Rev. D 10, 1777 (1974); K. J. Sebastian and D. Yun, *ibid.* 19, 2509 (1979).

[10]K. J. Sebastian, Phys. Rev. A 23, 2810 (1981); K. J. Sebastian, Phys. Lett. 80A, 109 (1980).