Polarization experiments and the isotropy of space

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(Received 5 May 1981)

It is shown on an example that sensitive tests of the isotropy of space (i.e., of rotation invariance) in strong-interaction particle reactions are almost identical to tests of parity conservation, and hence the two can be confused without some additional experiments which we specify.

The test of various conservation laws connected with symmetries is a central concern in nuclear and particle physics both because of cosmological implications and because theories of particles themselves depend on such conservation laws. Rotation invariance (i.e., the isotropy of space) is a symmetry that is relatively rarely studied. Our present belief, for example, that space is isotropic with respect to strong interactions is not based on experimental information of very high precision.[1] The aim of this article is therefore to explore the type of particle reaction experiments which can test rotation invariance in strong interactions. The conclusions of the investigation can be summarized in three points:

(1) One can construct tests, by using polarization quantities that lend themselves to "null experiments," which can be performed to a reasonably high degree of accuracy, such as one part in 10^7.

(2) These tests are virtually identical with experiments which test parity conservation, and hence evidence for parity nonconservation can be easily mistaken for evidence for violation of rotaion invariance.

(3) There are feasible additional experiments which can distinguish between exidence for parity nonconservation and evidence for anisotropy of space.

It would be quite feasible to discuss this problem in the framework of a general formalism of polarization phenomena. For didactic reasons, however, it might be much preferable to select instead a simple reaction as an example. The nature of the discussion will be such that it should be evident to the reader that nothing essential hinges on the specific properties of the example reaction and that therefore the generalization to any other reaction is straightforward.

The reaction we choose as an example is $0+\frac{1}{2}\rightarrow 0+\frac{1}{2}$, where the 0 and $\frac{1}{2}$ denote particles with spins 0 and $\frac{1}{2}$, respectively. A specific instance of such a reaction may be elastic pion-nucleon scattering, but there are many other instances also throughout particle and nuclear physics. We will first discuss this reaction in the case when rotation invariance holds.

In that case, the M matrix can be written in the following form:

$$M=a_0+a_1\vec{\sigma}\cdot\vec{q}_1+a_2\vec{\sigma}\cdot\vec{q}_1\times\vec{q}_2+a_3\vec{\sigma}\cdot\vec{q}_2\ , \tag{1}$$

where q_1 and q_2 are the initial and final center-of-mass momenta, the a's are the reaction amplitudes which are complex numbers depending on kinematic factors, and $\vec{\sigma}$ is the usual Pauli spin matrix. This is one of the multiply infinite number of ways of writing the M matrix. From the point of view of our discussion, it makes no difference which of the ways of writing the M matrix we consider, and hence this one is used since it may be familiar to many of the readers.

The amplitudes a_i are functions of the rank-zero tensors one can construct from the vectors that determine the kinematics. In the present case these vectors are $\vec{q}_1$ and $\vec{q}_2$, and hence the rank-zero tensors are ${q_1}^2$, ${q_2}^2$, and $\vec{q}_1\cdot\vec{q}_2$. The fact that these three are not independent of each other is of no concern to us in the present discussion. It is important to note, however, that all three of these rank-zero tensors are scalars and not pseudoscalars.

Now let us impose, in addition to rotation invariance, also parity conservation, and let us also assume that the product of the intrinsic parities of the four particles in the reaction is $+1$. This is not an essential constraint; the argument carries through in an analogous way when this quantity is -1.

With this additional constraint the M matrix reduces to

$$M=a_0+a_2\vec{\sigma}\cdot\vec{q}_1\times\vec{q}_2 \tag{2}$$

because the amplitudes in Eq. (1) are all scalars, and hence the two pseudoscalar combinations $\vec{\sigma}\cdot q_1$ and $\vec{\sigma}\cdot q_2$ in Eq. (1) must be eliminated by making $a_1=a_3=0$ in order to have an M matrix which is a scalar. This form is identical to the M matrix which is generally used, for example, in elastic pion-nucleon scattering.

This completes the discussion of the M matrix for the case when rotation invariance holds. We will now turn to the case when rotation invariance is violated.

In particular, we will assume that his violation takes the form of the existence of a unit vector $\hat{U}$ which points at a distinguished direction in space. It is possible to imagine violation of rotation invariance in such a way that such a vector does not exist but rather the violation is expressable in terms of a higher-rank tensor $U_{ij\cdots m}$ which represents a distinguished orientation. Our discussion can be carried out analogously in that case. For the sake of simplicity, however, in this paper we will consider the case when the anisotropy can be treated in terms of a unit vector $\hat{U}$ (i.e., in terms of a tensor of rank one). We do not need to specify whether $\vec{U}$ is a vector or a pseudovector.

In the presence of such a violation of rotation invariance, the M matrix becomes more complicated in three ways:

(1) The M matrix itself now need not be a rank-zero tensor, but can also have a part that is proportional to $\hat{U}$. In our discussion we will ignore this type of a modification of M because it is not connected with and does not substantially influence the type of sensitive tests we want to propose.

(2) In the expression for M of the type given in Eq. (1) we will now have additional rank-zero terms which explicitly also contain $\hat{U}$.

(3) The amplitudes now will depend on additional rank-zero kinematic tensors which will contain $\hat{U}$.

We will now discuss in detail the second and the third of these modifications.

To start with the second, we will now construct those additional rank-zero terms which have been made possible by the addition of another vector, namely $\hat{U}$. There are three such new terms, $\vec{\sigma}\cdot\hat{U}$, $\vec{\sigma}\cdot\vec{q}_1\times\hat{U}$, and $\vec{\sigma}\cdot\vec{q}_2\times\hat{U}$. The fourth term one can construct, $\vec{\sigma}[\vec{q}_1\times(\hat{U}\times\vec{q}_2)]$, can clearly be written in terms of the other three.

To deal with the three new terms, we realize that since our previous three vectors $\vec{q}_1$, $\vec{q}_2$, and $\vec{q}_1\times\vec{q}_2$ span the three-dimensional space, these new terms can be decomposed in terms of the old ones. In particular, we can write

$$\hat{U}=\alpha\vec{q}_1+\beta\vec{q}_2+\gamma\vec{q}_1\times\vec{q}_2\ , \tag{3}$$

where α, β, and γ are coefficients which depend on the *absolute* orientation of $\vec{q}_1$ and $\vec{q}_2$, and hence they are denoted by Greek letters. In contrast, our previous constants, the a_i's, do not depend on such *absolute* directions in space. We will continue to denote by Latin letters coefficients of this latter type, and by Greek letters coefficients of the former type, since the distinction between the two will be crucial in our agrument.

In order to write these new terms in terms of the old ones, we note that by Eq. (3) we have

$$\begin{aligned}
&b_1\vec{\sigma}\cdot U=b_1\alpha\vec{\sigma}\cdot\vec{q}_1+b_1\beta\vec{\sigma}\cdot\vec{q}_2+b_1\gamma\vec{\sigma}\cdot\vec{q}_1\times\vec{q}_2\ ,\\
&b_2\vec{\sigma}\cdot\vec{q}_1\times\hat{U}=b_2\beta\vec{\sigma}\cdot\vec{q}_1\times\vec{q}_2+b_2\gamma\vec{\sigma}\cdot\vec{q}_1\times(\vec{q}_1\times\vec{q}_2)\equiv b_2\gamma A\,\vec{\sigma}\cdot\vec{q}_1+b_2\gamma B\,\vec{\sigma}\cdot\vec{q}_2+b_2\beta\vec{\sigma}\cdot\vec{q}_1\times\vec{q}_2\ ,\\
&b_3\vec{\sigma}\cdot\vec{q}_2\times\hat{U}=b_3\alpha\vec{\sigma}\cdot\vec{q}_2\times\vec{q}_1+b_3\gamma\vec{\sigma}\cdot\vec{q}_2\times(\vec{q}_1\times\vec{q}_2)\equiv b_3\gamma C\,\vec{\sigma}\cdot\vec{q}_1+b_3\gamma D\,\vec{\sigma}\cdot\vec{q}_2-b_3\alpha\vec{\sigma}\cdot\vec{q}_1\times\vec{q}_2\ ,
\end{aligned} \tag{4}$$

where $A=-D=\vec{q}_1\cdot\vec{q}_2$, $B=-(\vec{q}_1)^2$, $C=(\vec{q}_2)^2$, and therefore we can write for the M matrix in the absence of parity conservation,

$$\begin{aligned}
M&=a_0+a_1\vec{\sigma}\cdot\vec{q}_1+\vec{a}_2\vec{\sigma}\cdot\vec{q}_1\times\vec{q}_2+a_3\vec{\sigma}\cdot\vec{q}_2+b_1\vec{\sigma}\cdot\hat{U}+b_2\vec{\sigma}\cdot\vec{q}_1\times\hat{U}+b_3\vec{\sigma}\cdot q_2\times\hat{U}\\
&=a_0+(a_1+b_1\alpha+b_2\gamma A+b_3\gamma C)\vec{\sigma}\cdot\vec{q}_1+(a_2+b_1\gamma+b_2\beta-b_3\alpha)\vec{\sigma}\cdot\vec{q}_1\times\vec{q}_2+(a_3+b_1\beta+b_2\gamma B+b_3\gamma D)\vec{\sigma}\cdot\vec{q}_2\\
&\equiv a_0+(a_1+\omega_1)\vec{\sigma}\cdot\vec{q}_1+(a_2+\omega_2)\vec{\sigma}\cdot\vec{q}_1\times\vec{q}_2+(a_3+\omega_3)\vec{\sigma}\cdot\vec{q}_2\ ,
\end{aligned} \tag{5}$$

where we denote

$$\omega_1 \equiv b_1\alpha + b_2\gamma A + b_3\gamma C ,$$
$$\omega_2 \equiv b_1\gamma + b_2\beta - b_3\alpha , \quad (6)$$
$$\omega_2 \equiv b_1\beta + b_2\gamma B + b_3\gamma D ,$$

and the ω_i's depend on the absolute orientation in space.

Let us store, for the moment, Eq. (5) and let us turn to the third of the modifications discussed earlier, namely the altered dependence of the amplitudes on rank-zero tensors composed of vectors describing the kinematics. We recall that in the case of rotation invariance we had only three such tensors, all of them scalars. In the presence of the U, we will now have six such tensors,

$$q_1^2,\ q_2^2,\ \vec{q}_1\cdot\vec{q}_2 ,$$
$$\vec{q}_1\cdot\hat{U},\ \vec{q}_2\cdot\hat{U},\ \vec{q}_1\cdot\vec{q}_2\times\hat{U} , \quad (7)$$

More important than the increase in the number of such tensors is the fact that whether U is a vector or a pseudovector, the six tensors now will include some scalars and some pseudoscalars. To take advantage of this, we will now explicitly decompose each such coefficient into its scalar and its pseudoscalar part, for example,

$$a_i \equiv a_i^S + a_i^P . \quad (8)$$

Now let us impose parity conservation on our M matrix, just as we did earlier when we discussed the case of rotation invariance. In contrast to the earlier case, however, this time the M matrix is *not* reduced from four terms to two terms, since that reduction hinged on all amplitudes being scalar which is no longer the case. Instead, now we obtain from Eq. (5),

$$M = a_0^S + (a_2^S + \omega_2^S)\vec{\sigma}\cdot\vec{q}_1\times\vec{q}_2 + (a_1^P + \omega_1^P)\vec{\sigma}\cdot\vec{q}_1 + (a_3^P + \omega_3^P)\vec{\sigma}\cdot\vec{q}_2 \quad (9)$$

so that the alteration now from the parity-nonconserving to the parity-conserving case consists only of the specification of the reflection properties of the amplitudes and not of the reduction in the number of terms. This situation is identical to the case or reactions with more than four particles in which the analogous effect leads to some interesting consequences[2] with respect to parity tests.

The remainder of our discussion will be based on Eq. (9). We see from that equation that the general form of the M matrix as far as the structure of the four terms is concerned (that is, apart form the reflection properties of the amplitudes) is the same for the case of a non-rotation-invariant but parity-conserving reaction as for the case of a rotation-invariant but parity-nonconserving reaction. For the latter case, when parity is slightly violated, we get

$$M = a_0 + a_2\vec{\sigma}\cdot\vec{q}_1\times\vec{q}_2 + d_1\vec{\sigma}\cdot\vec{q}_1 + d_3\vec{\sigma}\cdot\vec{q}_2 , \quad (10)$$

where a_0, a_2, d_1, and d_3 are all scalars and $d_i << a_j$.

We see therefore that at first glance, the experiments testing rotation noninvariance and parity nonconservation are identical, since they are both aimed at detecting the presence of the two additional terms [as compared to the two in Eq. (2)]. What are these experiments?

There are effects, of course, in all observables, but in some of them detection would be practically impossible. For example, let us consider the differential cross section calculated from Eq. (9) which is

$$\frac{d\sigma}{d\Omega} \propto |a_0^S|^2 + |a_2^S + \omega_2^S|^2 + |a_1^P + \omega_1^P|^2 + |a_3^P + \omega_3^P|^2 . \quad (11)$$

In this expression the effect of rotation noninvariance (if the extent of the anisotropy is small) would be a tiny effect superimposed on a huge cross section, and this cross section cannot be calculated accurately from any present theory. Hence the detection of anisotropy effects in such cross sections would be impossible.

If, on the other hand, we select some other observables, the detection of such effects can be feasible. In particular, we should select observables which vanish in the presence of only two of the four terms in the M matrix [that is, when Eq. (2) holds]. Such an observable is the polarization in any direction in the reaction plane. For example, using Eq. (9) we can calculate (using a small amount of spin arithmetic) the polarization in the $\vec{q}_1$ direction, which is

$$P_{q_1} \propto q_1^2\mathrm{Re}[a_0^S(a_1^P + \omega_1^P)^* + (\vec{q}_1\cdot\vec{q}_2)\mathrm{Re}[a_0^S(a_3^P + \omega_3^P)^*] + (\vec{q}_1\times\vec{q}_2)^2\mathrm{Im}(a_2^S + \omega_2^S)(a_3^P + \omega_3^P)^*] . \quad (12)$$

We see from this that the effect in that case is of first order in a^P and/or ω^P, and that we have to perform a "null experiment," that is, measure a small effect on top of a "background" which is zero. Such experiments have in fact been performed[3] in connection with a search for parity-violating components in strong-interaction reactions and it has been possible to attain accuracies of one part in 10^7.

To complete our discussion, we must find a way to distinguish between rotation noninvariance and parity nonconservation, both of which would give the same type of effect in the experiment suggested above. There are two ways of making such a distinction.

The first way is to note that in the case of parity nonconservation with rotation invariance, all amplitudes remain scalar and hence the polarization quantity we measure is also a scalar. In contrast, we can see from Eq. (12) that when we have parity conservation and rotation noninvariance, we have products of amplitude involving one scalar and one pseudoscalar amplitude, and hence the product will be a pseudoscalar. Therefore a comparison of the measurement with another one corresponding to space inversion will show equality or a flip of sign depending on whether we deal with parity nonconservation or with rotation noninvariance.

The second way is to perform the experiment, say, in March and then again in June, using always the same part of the day. During that time the absolute orientation of the experimental equipment changes, and hence the effect should be unequal in the two cases if it is due to rotation noninvariance. In contrast, if the effect comes from parity nonconservation, the effect should be independent of what part of the year we perform the measurement in.

To summarize, we would like to urge the performance of the type of experiment suggested in this discussion not only because we want to test rotation invariance, but because, on account of the already substantial activity in experiments testing parity nonconservation, it is important to make sure that these experiments indeed measure parity nonconservation and not rotation noninvariance.

This work was supported in part by the U.S. Department of Energy.

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[1]In fact we have not been able to find any published experiment specifically dealing with the measurement of rotation invariance in strong interactions. Discussions with various colleagues generally resulted in the statement that if rotation invariance were grossly violated, partial-wave expansions (which depend on angular momentum conservation) would not work. This however, places only a very weak limit on the extent to which rotation invariance holds. There has been one experiment [R. Newman and S. Wiesner, Phys. Rev. D 14, 1 (1976)] which tested rotation invariance in *weak* interactions to the extent of one part in 10^6-10^7, that is, slightly less accurately than the tests suggested in our paper for *strong* interactions. On the anisotropy of inertial mass, which is yet another aspect of the properties of space, there is an upper limit by V. W. Hughes, H. G. Robinson, and V. Beltram-Lopez [Phys. Rev. Lett. 4, 342 (1960)] $\Delta m/m < 10^{-20}$.

[2]P. L. Csonka, M. J. Moravcsik, and M. D. Scadron, Phys. Rev. Lett. 14, 861 (1965).

[3]For a recent summary of the status of such parity experiments, see W. Haeberli, in *Polarization Phenomena in Nuclear Physics—1980,* proceedings of the Fifth Symposium, Santa Fe, 1980, edited by G. G. Ohlsen, R. E. Brown, Nelson Jarmie, W. W. McNaughton, and G. M. Hale (AIP, New York, 1981), p. 1340.