PHYSICAL REVIEW D VOLUME 25, NUMBER 1 1 JANUARY 1982

Photon and gluon radiative weak decays and the inclusive decays of the charmed and bottom mesons

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(Received 10 August 1981; revised manuscript received 26 October 1981)

We show that the processes $\pi^+ \to \gamma e^+ \nu$ and $K^+ \to \gamma e^+ \nu$ with hard photons can be calculated accurately and without free parameters in the quark model. We extend the calculation to gluon radiative decays $P \to Gq\bar{q}$ and $P \to GX$(hadron) of heavy pseudoscalar mesons and show that while the former is too weak to explain existing data, the latter could be the dominant hadronic decay mode. We test our approach by studying the inclusive decay properties of the heavy mesons.

Experimental measurements[1-3] of the decay properties of the charmed D and F mesons show conclusively that the nonleptonic decay modes are strongly enhanced relative to the semileptonic decays. More recent measurements[4] of the B meson indicates that this enhancement may persist, perhaps to a lesser extent. Two schools of thought have arisen to explain this phenomenon: one is that a gluon, which carries one unit of spin, is emitted before the weak interaction takes place so that the helicity suppression that characterizes the weak annihilation of a spin-zero meson is circumvented[5]; the other is that hadronic decay modes are enhanced through final-state interactions.[6] Here we shall not consider the latter except to note that it alone does not appear to be sufficient to explain the D^0 decays.[6] On the other hand, we shall show that the closely related structure-dependent amplitudes for $P \to \gamma l \nu$, $P \to \gamma q\bar{q}$, and $P \to Gq\bar{q}$ (P stands for a pseudoscalar meson) are calculable. We demonstrate this by verifying that our calculated results for $\pi^+ \to \gamma e^+ \nu$ and $K^+ \to \gamma e^+ \nu$ are in agreement with the experimental data. We point out that even though $P \to Gq\bar{q}$ is not helicity suppressed and is very much enhanced $[\Gamma(P \to Gq\bar{q})/\Gamma(P \to \gamma q\bar{q}) \sim 50\alpha_s/\alpha \sim 10^3]$ it is still *not* competitive[7] with the quark-decay process $Q \to 3q$, where Q is the heavy constituent quark in P. We then point out that, depending on a dimensionless quantity characterizing the hadronization process, the two-jet decay $P \to GX$(hadron) could have a width exceeding that of $Q \to 3q$ and be the dominant hadronic decay mode. This result does not rely on the D and B mesons having unrealistically large decay constants. Finally, we calculate inclusive decay properties for the heavy mesons and observe qualitative agreement with available data. We also make predictions that have a bearing on the basic weak Hamiltonian and are easily testable.

The amplitude[8] for $P \to \gamma l \nu$ (or $P \to \gamma q\bar{q}$) is

$$M(P \to \gamma l \nu) = \frac{eG}{\sqrt{2}} \{ [\,\cdots\,]_{\rm IB} - \frac{1}{M}[i v_\gamma P^\alpha k^\beta \epsilon_{\lambda\alpha\beta\mu} + a_\gamma (P\cdot k g_{\lambda\mu} - P_\lambda k_\mu)]_{\rm SD} \epsilon^\lambda J^\mu \} \ , \qquad (1)$$

where G is the weak coupling constant including the Cabibbo factors, P and M are the momentum and mass of the pseudoscalar, ϵ and k are the polarization and momentum of the photon, v_γ and a_γ are, respectively, the vector and axial-vector form factors, and J^μ is the lepton-neutrino (or $q\bar{q}$) weak current. As is well known,[8] the inner-bremsstrahlung (IB) amplitude, not given explicitly in (1), is helicity suppressed but infrared divergent, and when combined with radiative corrections of the same order produces an infrared-finite and small [of order $O(\alpha/\pi)$] but negative correction to $\Gamma(P \to l\nu)$. The IB and structure-dependent (SD) interference term is infrared finite but helicity suppressed and is therefore small. We will not discuss these terms any further, but rather focus on the SD term which is infrared finite and grows with the meson mass (hereafter the subscripts SD will be suppressed).

The vector form factor is closely related to a similar factor in the two-photon decay of flavorless pseudoscalars, $\pi^0, \eta, \eta', \ldots$. Isgur[9] has shown that for such decays the form factor can be quite accurately calculated. We extend his method to calculate the vector as well as the axial-vector form factors in

$P \to \gamma l \nu$ for flavored pseudoscalars. We obtain

$$v_\gamma \approx (f_P/2p)(e_1 L_1 + e_2 L_2) \quad , \tag{2}$$

where f_P is the decay constant; the approximation $L_i = -2Mp/\langle D_i \rangle \approx \ln(E_{i,+}^2/E_{i,-}^2)$ with $E_{i,\pm}^2 = (p \pm M/2)^2 + m_i^2 - p_0^2$ and $p_0 = (m_1^2 - m_2^2)/2M$ results from an angular average of the quark propagator D_i. We follow Ref. 9 and use the average value $p \equiv |\vec{p}| = 0.25 \pm 0.10$ GeV and the decay constant $f_P \approx 0.3$ GeV for heavy mesons.[10] For the axial-vector form factor we obtain an expression which depends quadratically on the relative momentum,[11]

$$a_\gamma \approx -\frac{2f_P}{Mp} \frac{(m_1 - m_2)(e_1 + e_2)(L_1 + L_2)(p_0^2 - p^2)}{[M^2/2 + m_1^2 + m_2^2 - 2(p_0^2 - p^2)]} \quad . \tag{3}$$

The SD radiative decay width[12] is

$$\Gamma(P \to \gamma l \nu) = \Gamma_+ + \Gamma_- \quad ,$$
$$\Gamma_\pm = (\alpha G^2 M^5 / 3840\pi^2)(v_\gamma \pm a_\gamma)^2 \quad . \tag{4}$$

The + and − signs refer, respectively, to final states where the lepton is antiparallel and parallel to the photon.[12] From the formulas given above we obtain[13] $\Gamma(\pi^+ \to \gamma e \nu)/\Gamma(\pi^+ \to e \nu) = (1.1 \pm 0.5) \times 10^{-3}$, $\gamma_\pi \equiv (a_\gamma / v_\gamma)_\pi = -0.030 \pm 0.015$, $\Gamma(K^+ \to \gamma e \nu)_+ / \Gamma(K^+ \to e \nu) = 1.2 \pm 0.7$, and $\gamma_K = -0.12 \pm 0.10$; the corresponding experimental values are[14–16] $\sim 0.5 \times 10^{-3}$, 0.15 ± 0.11 or -2.07 ± 0.11, 1.0 ± 0.2, and < -1.86 or > -0.54. Except for an ambiguity[11] in the sign of γ_π, the agreement of our results with the data is good. The uncertainties given in the calculated results reflect the uncertainty in p.

We now consider gluon radiative decay $P \to Gq\bar{q}$ for heavy mesons. This process is permitted because the effective four-quark operator for weak decay has a color-octet term through Fierz transformation.[5] We can employ (2)–(4) to compute the decay width if we replace α by $4\alpha_s/3$ and let the color charges $e_1 = e_2 = 1$. For heavy mesons we ignore the weak p dependence in (2) and (3) by going to the limit $p \to 0$ (the static quark model). For charged pseudoscalars, one finds that $\Gamma(P \to Gq\bar{q})/\Gamma(P \to \gamma q\bar{q}) \sim 10^3$. However, this large enhancement is still not sufficient to make $P \to Gq\bar{q}$ competitive with the quark-decay process $Q \to 3q$ (i.e., $Q \to qq\bar{q}$), where Q is the heavy quark in P: for the D meson, and ignoring the renormalization of the Wilson coefficients for the moment, we find[17] $\Gamma(D \to Gq\bar{q})/\Gamma(c \to 3q) = (2\pi/45) \times (M_D/m_c)^5 \alpha_s (v_G^2 + a_G^2) \approx 0.2$. A more general gluon-radiative weak decay process is $P \to GX$, where X is a hadronic jet of invariant mass m_f generated from the weak current J^μ; the process is analogous to the decay of η particles and quarkonia to two gluons. If we sum over all quantum numbers of X, including its polarizations, then we have

$$\sum_X \langle 0|J^\mu|X\rangle \langle X|J^\nu|0\rangle = -m_f^2 W(x_f) g^{\mu\nu} \quad , \tag{5}$$

where $x_f = m_f/M$, and to leading order the function $W \approx W_0$ is a constant. If we ignore the x_f dependence of W we can integrate over x_f^2 and obtain $(v^2 \equiv v_G^2 + a_G^2)$

$$\Gamma(P \to GX) = (\alpha_s G^2 M^5/240) W_0 v^2 \times [1 - y^2(10 - 20y + 15y^2 - 4y^3)] \quad , \tag{6}$$

where $\sqrt{y} = x_0$ is the minimum value of x_f. When y is small, we find $\Gamma(P \to GX)/\Gamma(P \to Gq\bar{q}) = 6\pi^2 W_0$ and $\Gamma(P \to GX)/\Gamma(Q \to 3q) = (4\pi^3 \alpha_s/15)(M/m_Q)^5 \times W_0 v^2$. The factor π^2 in the first ratio is indicative of the kinematical advantage of a two-body final state over that of a three-body final state. Another reason to believe the above ratios could be large is that unlike a leptonic $l\nu$ pair, a $q\bar{q}$ pair can form more than one hadronic state. However, in the absence of a theory for hadronization at relatively low energies we must be content to treat W_0 as a parameter. The important point to realize is that the width for gluon emission can be significantly greater than $\Gamma(P \to Gq\bar{q})$.

We derive the effective weak Hamiltonian for charmed- and bottom-meson decays from the standard six-quark model of Kobayashi and Maskawa.[18] Specifically, the coefficient for the $b \to c$ decay is $U_c = U_c = s_3 - s_2 e^{i\delta}$ and that for the $b \to u$ decay is $U_u = s_1 s_3$; s_i and δ are the Kobayashi-Maskawa mixing coefficients.[18] It can be shown[5] that, depending on the flavor structure of the decay, the SD amplitude for $P \to GX$ is proportional to the octet coefficient $f_\pm = (c_+ \pm c_-)/2$, where c_- and $c_+ = 1/\sqrt{c_-}$ are, respectively, the renormalized Wilson coefficients[5] of the antisymmetric and symmetric four-quark operators; we use $c_{-,\text{charm}} = 2.1 \pm 0.1$ and $c_{-,\text{bottom}} = 1.7 \pm 0.1$.

We determine W_0 by attributing the large hadronic D^0 decay rate to $D^0 \to GX$. This yields $W_0 \sim 0.25$, and $\Gamma(D^0 \to GX)/\Gamma(c \to 3q) \sim 3$ and $\Gamma(B^0 \to GX)/\Gamma(b \to 3q) \sim 2$. We consider $Q \to ql\nu$, $Q \to 3q$, $P \to \tau\nu$, $P \to q\bar{q}$, and $P \to GX$, and use $m_q = m_{q,\text{current}} + \Lambda$, $\Lambda = 0.2 \pm 0.1$ GeV for the light (u,d,s) quarks to compute phase-space integrals; Λ is the momentum of minimum subtraction in QCD and represents an estimate of the effects of confinement on the quark masses.[19] For the heavy quarks we use $m_c = 1.5$ and $m_b = 5.0$ GeV.

The results are given in Table I, where the entries are, respectively, the lifetime τ, branching fraction for semileptonic decay $B(e\nu X)$ (same for $\mu\nu X$), kaon multiplicity N_K (i.e., number of s, $\bar{s}$, c, and $\bar{c}$ quarks in the final state), K^+ multiplicity N_{K^+} (one-half the number of c and s quarks in the final state). The uncertainties in τ and $B(e\nu X)$ result from the

TABLE I. Inclusive decay properties of D, F, and B mesons. See text for net strangeness and charm. For B mesons, unbracketed (bracketed) results are obtained with $|U_u/U_c| = 0.22$ (1.0).

	τ (10^{-13} s)	$B(e\nu X)$ (%)	N_K	$100N_{K^+}$
		Calculated		
D^0	1.5 ±0.2	2.6±0.6	1.3	9.5
D^+	10 ±3.4	17 ±3	0.97	2.5
F^+	3.5 ±0.2	5.7±1.2	1.0	24
B^0	0.16±0.04	5.4±1.0	1.5	14
	[1.5 ±0.4]	[5.5±1.0]	[1.0]	[13]
B^-	0.41±0.05	15 ±1.5	1.1	7.8
	[3.0 ±0.4]	[11 ±1.5]	[0.8]	[12]
B_s^0	0.31±0.03	10 ±1.3	2.3	57
	[2.4 ±0.2]	[8.8±1.0]	[1.8]	[51]
		Naive model		
D^0,D^+	10	17	1	1.5
F^+	10	17	2	50
B^0,B^-	0.46 [3.5]	17 [15]	1.2 [0.74]	7.0 [8.2]
B_s^0	0.46 [3.5]	17 [15]	2.2 [1.8]	57 [58]
		Experiment		
D^0	$2.3^{+0.8}_{-0.5}$ [a]	< 4 [b]	0.92±0.16 [c]	8 ±3 [c]
		5.5 ±3.7 [c]		
D^+	$10.3^{+10.3}_{-4.2}$ [a]	$22^{+4.4}_{-2.2}$ [b]	0.77 ±0.19 [c]	6 ±4 [c]
		16.8 ±6.4 [c]		
F^+	$2.0^{+1.8}_{-0.8}$ [a]	. . .	. . .	. . .
B^0,B^-	. . .	13 ±3 [d]	1.7 ±0.7 [d]	. . .
		9 ±3.6 [d]		

[a]Ref. 3; also $\tau(D^0) = 0.58^{+0.8}_{-0.2}$, $\tau(D^+) \sim 4.4$, Ref. 20, and $\tau(F^+) \sim 1.4$, Ref. 21.
[b]Ref. 2.
[c]Ref. 1.
[d]Ref. 4.

assigned uncertainties in the confinement mass Λ and in $c_\pm$, and reflect the sensitivity of the theory to these quantities. The theoretical uncertainties in N_K and N_{K^+} are not given but are relatively smaller. The characteristics of the B-meson decays depend strongly on whether the term proportional to U_c is singly or doubly Cabibbo suppressed; the term proportional to U_u is definitely doubly suppressed[18] and to be specific we used $|U_u| = (0.22)$.[2] The two sets of results for B mesons correspond to (i) unbracketed, where we set the ratio $\eta \equiv |U_u/U_c| = 0.22$ to represent the case when $b \to c$ is dominant, and (ii) bracketed, where $\eta = 1$ for an equal mixture of $b \to c$ and $b \to u$ decays. Also contained in Table I are the predictions of the naive model, where only the quark-decay modes $Q \to ql\nu$ and $Q \to 3q$ are considered, and available experimental data. Other than noting that there is overall qualitative agreement between theory and data we offer the following comments:

(1) Our results on lifetimes and $B(e\nu X)$ are qualitatively similar to those in Ref. 5 where the process $P \to Gq\bar{q}$ is considered. However, our calculation shows that $P \to Gq\bar{q}$ is too weak but $P \to GX$ can be competitive with or may even dominate over $Q \to 3q$. One area where the final states $Gq\bar{q}$ and GX may be distinguishable experimentally is energy distribution: that of GX should be nearly bipolar, whereas those of $Gq\bar{q}$ and $3q$ should be nearly isotropic. This is especially true for the charmed mesons where the final state contains no heavy particles. This difference could explain why more than 80% of the $K\pi\pi$ final states in D^0 decay where (we believe) GX is dominant come from either $K^*\pi$ or $K\rho$, while less than 15% of those in D^+ decay where GX is flavor-suppressed come from $K^*\pi$ or $K\rho$.

(2) Gluon emission has a large effect on N_{K^+} which appears to be borne out by experimental data[1] on D^0 and D^+.

(3) The measured (mean of B^- and B^0) value[4] of $N_K = 1.7 \pm 0.7$ favors $\eta << 1$. If this is the case then we predict $\tau(B^-)/\tau(B^0) \approx B(B^- \rightarrow e\nu X)/B(B^0 \rightarrow e\nu X) \approx N_K + (B^0)/N_K + (B^-) \approx 3$, with $\tau(B) \lesssim 0.5 \times 10^{-13}$ s.

(4) Our prediction for $B(B \rightarrow e\nu X)$ is insensitive to the ratio η: The average for B^- and B^0 of $(9 \pm 2)\%$ is significantly less than the $(16 \pm 1)\%$ of the naive model. At present the error in the experimental mean[4] of $(11 \pm 3)\%$ is too large to allow a firm conclusion.

(5) The net strangeness can be obtained from Table I using the relation $\Sigma_s = -(N_K - 4N_{K^+})$. For D^0, D^+, and F^+ we obtain $\Sigma_s = -0.91$, -0.86, and ~ 0, respectively; the measured values[1] for D^0 and D^+ are -0.60 ± 0.20 and -0.53 ± 0.22. For the B mesons we find Σ_s to be strongly dependent only on η: if $\eta << 1$, then $\Sigma_s(B^-$ and $B^0) \approx -0.9$ and $\Sigma_s(B_s) \approx 0$; if $\eta \approx 1$ then $\Sigma_s \approx -0.3$ and 0.2, respectively.

(6) Another measurement from which a value of η could be inferred is the net charm Σ_c. Our calculation shows that for B^0 and B^-, Σ_c is sensitive only to η: $\Sigma_c \approx 0.9$ and 0.45 when $\eta << 1$ and ≈ 1, respectively. On the other hand, due to $B^0 \rightarrow GX$, $\Sigma_c(B^0) \approx 0.5$ regardless of η.

[1] R. H. Schindler *et al.*, Phys. Rev. D 24, 78 (1981); see also A. J. Lanford, in *High Energy Physics—1980,* proceedings of the XXth International Conference, Madison, Wisconsin, edited by L. Durand an L. G. Pondrom (AIP, New York, 1981), p. 373.

[2] W. Bacino *et al.*, Phys. Rev. Lett. 45, 329 (1980).

[3] N. Ushida *et al.*, Phys. Rev. Lett. 45, 1049 (1980); 45, 1053 (1980); J. M. Trischuk *et al.* (unpublished).

[4] C. Bebek *et al.*, Phys. Rev. Lett. 46, 84 (1981); K. Chadwick *et al.*, *ibid.* 46, 88 (1981); W. A. Loomis, invited talk at Canadian Association of Physicists Congress, Halifax, Canada, 1981 (unpublished).

[5] M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. 44, 7, 962(E) (1980); H. Fritzsch and P. Minkowski, Phys. Lett. 90B, 455 (1980); V. Barger, J. P. Leveille, and P. M. Stevenson, in *High Energy Physics—1980* (Ref. 1), p. 390; Q. Ho-Kim and H. C. Lee, Phys. Lett. 100B, 420 (1981), and references therein.

[6] H. Lipkin, Phys. Rev. Lett. 44, 710 (1980); A. N. Kamal and E. D. Cooper, Z. Phys. C 8, 67 (1981); C. Sorensen, Phys. Rev. D 23, 2618 (1981).

[7] The process $P \rightarrow Gq\bar{q}$ had been advanced by Bander *et al.* (see Ref. 5) as the main hadronic decay mechanism of D^0. Our calculation leads us to conclude that this is not the case. See also Ref. 17.

[8] T. Kinoshita, Phys. Rev. Lett. 2, 477 (1959); W. J. Marciano and A. Sirlin, *ibid.* 36, 1425 (1976); T. Goldman and W. J. Wilson, Phys. Rev. D 15, 709 (1977).

[9] N. Isgur, Phys. Rev. D 13, 129 (1976); M. Moreno and J. Pestieau, *ibid.* 13, 175 (1976).

[10] The value $f_P \approx 0.3$ GeV is well within the upper limits of theoretical estimates: see N. G. Deshpande and D. Iskandar, Phys. Rev. D 19, 3457 (1979) and references therein; V. A. Novikov *et al.*, Phys. Rev. Lett. 38, 577 (1977); 38, 1037(E) (1977); S. S. Gershtein and M. Yu. Khlopov, Pis'ma Zh. Eksp. Teor. Fiz. 23, 374 (1976) [JETP Lett. 23, 338 (1976)]. On the other hand, potential-model estimates yield smaller values for f_P; see H. Krasemann, Phys. Lett. 96B, 397 (1980).

[11] Recall that $f_P \propto \psi(0)$, the relative wave function of the constituent quarks at $r = 0$. Equation (3) implies that a_γ depends on $\nabla^2\psi(\vec{r})|_{r=0}$. Thus a positive value for $\gamma_\pi \equiv (a_\gamma/v_\gamma)_\pi \propto (\nabla^2\psi)/\psi|_{r=0}$ implies that the nonrelativistic $q\bar{q}$ potential is *repulsive* at the origin. In this work we do not pursue this point but replace $-\nabla^2$ by the average $\vec{p}^2$ so that γ_π and γ_K are always negative. For heavy mesons, where $p_0{}^2 >> p^2$, the value of γ is not affected by this approximation.

[12] D. E. Neville, Phys. Rev. 124, 2037 (1961).

[13] For the form factors, we follow Ref. 9 and use the constituent quark masses 340, 343, and 480 MeV for u, d, and s, respectively. For a discussion of $m_u - m_d$, see R. Aaron and H. Goldberg, Phys. Rev. Lett. 45, 1752 (1980).

[14] See analysis by Goldman and Wilson, Ref. 8.

[15] A. Stetz *et al.*, Phys. Rev. Lett. 33, 1455 (1974).

[16] J. Heintze, Nucl. Phys. B149, 365 (1979).

[17] We use $\alpha_s \approx 0.4$ and 0.3 for D and B, respectively. Our $\Gamma(P \rightarrow Gq\bar{q})$ is much smaller (by a factor of ~ 20) than that calculated by Bander *et al.* (Ref. 5) because we obtain a smaller phase-space factor and use smaller and more realistic values for α_s and f_p.

[18] M. Kobayashi and K. Maskawa, Prog. Theor. Phys. 49, 652 (1973). The values or limits of the coefficients are $|\sin\theta_C| \approx |s_1| \approx 0.22$, $|s_3| = 0.28^{+0.21}_{-0.28}$, $|s_2| \lesssim 0.2$, $|\sin\delta| \lesssim 10^{-2}$; see V. Barger *et al.*, Phys. Rev. Lett. 42, 1585 (1979); R. E. Shrock *et al.*, *ibid.* 42, 1589 (1979).

[19] S. Wolfram, Report No. CALT-68-790 (unpublished).

[20] M. I. Adamovich *et al.*, Phys. Lett. 99B, 271 (1981).

[21] R. Ammar *et al.*, Phys. Lett. 94B, 118 (1980).