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Magnetic moments of the nucleon octet calculated in the cloudy bag model

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We calculate the lowest-order pionic corrections to the magnetic moments of the strange members of the nucleon octet. The overall agreement is remarkably good, but one would like to see an improvement in the data for Σ^- and Ξ^-.

In our search to understand the structure of the hadrons, their magnetic moments provide some very significant information. The large moments of neutral baryons like the n and Ξ^0, as well as the anomalous magnetic moments of charged particles like the proton, are clear evidence for important internal structure. Indeed, a major success of the naive quark model was its prediction of the ratio μ_p/μ_n as $-\frac{3}{2}$.[1] More sophisticated dynamical theories, such as the potential model of Isgur and Karl, have hardly altered this result.[2] Although one might question the validity of an essentially nonrelativistic approach, some justification for the procedures of the constituent quark model has been provided recently by the bag model,[3] or more general relativistic considerations.[4]

With the availability of excellent hyperon beams in the last couple of years there has been a dramatic improvement in the precision with which we know the hyperon magnetic moments.[5] The only exception is the Σ^-, for which the decay asymmetry is very small. Therefore in this case we must rely on the less precise determinations using exotic-atom techniques.[6] Unfortunately, from the theoretical point of view the Σ^- magnetic moment is quite significant, as is that of the Ξ^-, for which only a preliminary number is presently available. In a recent Letter,[7] Franklin discussed some inconsistencies in the theoretical understanding of the magnetic moments in the nucleon octet, and concluded that the "present Σ^- moment determination [was] incompatible with [his] analysis."

Even more interest in the value of the Σ^- magnetic moment was engendered by the study of Brown and co-workers.[8] Within the framework of one particular chiral bag model,[9] they obtained values of $\mu(\Sigma^-)$ in the range $-0.54\mu_N$ to $-0.64\mu_N$ (μ_N = nuclear magneton), in comparison with the experimental determination of $(-1.41 \pm 0.25)\mu_N$ (Ref. 6). In agreement with Lipkin[4] the authors suggested that a remeasurement of $\mu(\Sigma^-)$ would provide "a crucial test of [their] model."

The hybrid bag models[10–13] have been constructed in the past two years as a response to the observation that the MIT bag model (or indeed any model which confines quarks through a scalar potential) badly violates chiral symmetry.[14] By introducing the pion as an approximate Goldstone boson, associated with an as-yet-unknown dynamical symmetry-breaking mechanism, one can restore the $SU(2)\times SU(2)$ symmetry (when $m_\pi = 0$). The cloudy bag model (CBM) developed at TRIUMF and the University of Washington, has already been applied to the problem of the nucleon magnetic moments,[15,16] which were a long-standing puzzle for the MIT bag model.

One of the beauties of the bag model[17] is that there are no extra parameters (such as constituent quark masses) which can be used to fit (say) the p, n, and Λ magnetic moments. The massless quarks in the bag model have a magnetic moment as a result of confinement, which is proportional to the radius of the confining volume. It was one of the major puzzles of the original MIT work[17] that with a radius of order 1 fm, as determined by spectroscopy, the proton magnetic moment was only $1.9\mu_N$. (If one scaled all predictions by μ_p, however, the predictions for all other members of the octet were invariably an improvement on naive quark-model predictions.) The recently calculated recoil correction of Donoghue and Johnson[18] improved the situation [μ_p (corrected) $=2.24\mu_N$], but there was still a significant discrepancy. It was therefore satisfying that the inclusion of lowest-order pion-loop corrections improved the situation even more, giving $\mu_p=2.60\mu_N$ (Ref. 15). In fact, the remaining corrections from configuration mixing[2] and sea quarks[19] are sufficiently large that one could not really expect better agreement from the model.

In view of this success it seems natural to extend the CBM calculation to the rest of the octet. As in Refs. 12, 15, and 16, we use the linearized Lagrangian density (for a static spherical bag of radius R)

$$\begin{aligned}\mathcal{L}(x) = {} & (i\bar{q}\partial\!\!\!/ q - B)\theta(R-r) - \tfrac{1}{2}\bar{q}q\,\delta(r-R) \\ & - \frac{i}{2f}\bar{q}\gamma_5\vec{\tau}q\cdot\vec{\phi}\,\delta(r-R) \\ & + \tfrac{1}{2}(\partial_\mu\vec{\phi})^2 - \tfrac{1}{2}m_\pi^2\phi^2 \ . \end{aligned} \tag{1}$$

Here q and ϕ are the quark and pion fields, and f the pion decay constant (93 MeV). Equation (1) clearly leads to a Hamiltonian of the form

$$H = H_{\mathrm{MIT}} + H_{\mathrm{int}} + H_\pi \ , \tag{2}$$

where H_{int} describes the surface coupling of the (for the present) elementary pion field to the quarks. By defining a P space of three-quark baryons, and ignoring the corrections due to Q space (essentially the effects of sea quarks), we obtain a Hamiltonian describing the emission and absorption of pions by extended (bag model) hadrons. The resultant theory is completely renormalizable, and the convergence properties have been rigorously established for the nucleon.[20]

In order to generalize Eqs. (1) and (2) to the other members of the nucleon octet we simple redefine q as a three-component field (u,d,s) where the s has a mass of 279 MeV. While one could generalize the CBM to $SU(3)_L\times SU(3)_R$ and calculate corrections from a virtual-kaon "cloud," we have chosen not to do so. The mass of the kaon is so much larger than that of the pion that there is no longer such a clean separation between the phenomenology of the bag surface and the mesonic corrections. For the present we calculate only the longest-range (that is, pionic) corrections.

The coupling of the quark and pion fields to the photon occurs through the usual minimal coupling, and the pionic current was presented in detail in Ref. 15. For simplicity we take SU(6) wave functions for all octet members in order to calculate both the ratio of the coupling constants (e.g., $\Sigma\Lambda\pi$, $\Xi\Xi\pi$, etc.) to the $NN\pi$ coupling constant, and the magnetic coupling to the bag ($\gamma\Lambda\Lambda$, $\gamma\Lambda\Sigma^0$, etc.). The $NN\pi$ coupling constant itself was fixed at the usual value of $f^2/4\pi=0.081$, and the radius of the bag for all hyperons was taken to be 1 fm, in agreement with the MIT analysis. [Of course the radii may change when pionic corrections are included. However, as the pion self-energy is a factor of 2 smaller for the Σ (6 for the Ξ), we expect the changes for the hyperons to be much smaller than for the nucleon. In view of the insensitivity to bag radius noted below, the neglect of such corrections seems quite reasonable.]

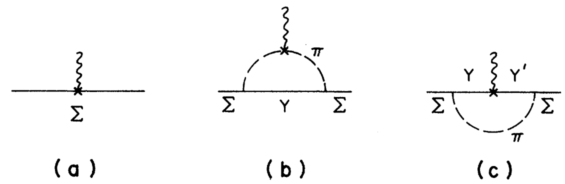

FIG. 1. Contributions to the magnetic moment of the Σ hyperon, including pionic corrections to $O(f^2)$ ["Y" denotes either Σ, Λ, or Σ^*, and the combinations (Y,Y') include (Λ,Λ), (Σ,Σ), (Σ^*,Σ^*), (Λ,Σ), (Λ,Σ^*), and (Σ,Σ^*)].

At this stage the model has no free parameters! For the Σ, for example, we calculate all the graphs shown in Fig. 1 in exactly the way described in Ref. 15. We observe that although there are a large number of graphs in which the photon couples to the bag with the pion "in the air," these are usually small, and in any case there is considerable cancellation. (In calculating the Donoghue-Johnson[18] recoil correction, the appropriate value of $\langle p^2\rangle$ and mass is used for each baryon.) The results of the calculation are summarized in Table I, together with the most recent experimental values.[5,6]

TABLE I. Comparison of the magnetic moments of the members of the nucleon octet calculated in the CBM, in comparison with the most recent data (Refs. 5 and 6) (all numbers in nuclear magnetons).

	CBM	Experiment
p	2.60[a]	2.793
n	−2.01[a]	−1.913
Λ	−0.58	−0.614 ± 0.005
Σ^-	−1.08	−1.41 ± 0.27
Σ^+	2.34	2.33 ± 0.13
Ξ^-	−0.51	−0.75 ± 0.07[b]
Ξ^0	−1.27	−1.25 ± 0.014

[a]From Ref. 15, using $R=0.82$ fm as determined from pion-nucleon scattering.
[b]Preliminary result.

Clearly the overall agreement with experiment is excellent. One remarkable feature of the calculation not shown in Table I is that once pionic corrections are included there is little sensitivity to a small change in the bag radius. For example, arbitrarily reducing R from 1.0 to 0.9 fm changes $\mu(\Sigma^+)$ and $\mu(\Sigma^-)$ to $2.21\mu_N$ and $-1.07\mu_N$ (i.e., by 5% and 1%), respectively. To some extent, therefore, the extra pion contribution for a small bag compensates for the decrease in the contribution from the core.

Unlike Brown *et al.*,[8] we do not find any significant disagreement with the Σ^- magnetic moment. This would appear to rule out the phenomenological isoscalar contribution assumed in Ref. 8. With regard to the analysis of Franklin we note that the combination

$$\mu_s'(\Sigma) = -\Sigma^+ - 2\Sigma^- \ , \tag{3}$$

[Eq. (2′) of Ref. 7] includes a pionic-loop correction or order $0.6\mu_N$. Such corrections would be expected to violate SU(6) constraints, and thus reduce the usefulness of such sum rules in extracting quark moments.

It must be emphasized that there has been a great deal of work on other corrections to baryon magnetic moments, arising from effects such as configuration mixing[2] and sea quarks.[19] In view of the theoretical uncertainties associated with both these effects and our pionic corrections,[21] it appears unlikely that theory will match experiment in precision for some time. Nevertheless, it does seem reasonable to conclude from Table I that the inclusion of the lowest-order pionic corrections associated with chiral $SU(2)\times SU(2)$ results in good overall agreement with the presently available data. More accurate measurements for the Σ^- and the Ξ^- would certainly be welcome.

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[1]H. J. Lipkin, Phys. Rep. 8C, 174 (1973).

[2]N. Isgur and G. Karl, Phys. Rev. D 21, 3175 (1980).

[3]T. A. De Grand, in *Baryon 1980,* proceedings of the IVth International Conference on Baryon Resonances, Toronto, edited by N. Isgur (Univ. of Toronto, Toronto, 1981), p. 209.

[4]H. J. Lipkin, in *Baryon 1980* (Ref. 3), p. 461.

[5]O. E. Overseth, in *Baryon 1980* (Ref. 3), p. 259.

[6]B. L. Roberts *et al.*, Phys. Rev. D 20, 2154 (1979); G. Dugan *et al.*, A254, 396 (1975); T. Hansl *et al.*, *ibid.* B132, 45 (1978).

[7]J. Franklin, Phys. Rev. Lett. 45, 1607 (1980).

[8]G. E. Brown, M. Rho, and V. Vento, Phys. Lett. 97B, 423 (1980).

[9]G. E. Brown and M. Rho, Phys. Lett. 82B, 177 (1979); I. Hulthage, F. Myhrer, and Z. Xu, Nucl. Phys. A364, 322 (1981).

[10]R. L. Jaffe, lectures at the 1979 Erice Summer School "Ettore Majorana," 1979 (unpublished).

[11]M. V. Barnhill and A. Halprin, Phys. Rev. D 21, 1916 (1980).

[12]S. Théberge, A. W. Thomas, and G. A. Miller, Phys. Rev. D 22, 2838 (1980); 23, 2106E (1981).

[13]C. DeTar, Phys. Rev. D 24, 752 (1981); 24, 762 (1981).

[14]A. Chodos and C. B. Thorn, Phys. Rev. D 12, 2733 (1975).

[15]A. W. Thomas, S. Théberge, and G. A. Miller, Phys. Rev. D 24, 216 (1981).

[16]S. Théberge, G. A. Miller, and A. W. Thomas, Can. J. Phys. (to be published).

[17]A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn, Phys. Rev. D 10, 2599 (1974); T. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, *ibid.* 12, 2060 (1975).

[18]J. F. Donoghue and K. Johnson, Phys. Rev. D 21, 1975 (1980).

[19]J. F. Donoghue and E. Golowich, Phys. Rev. D 15, 3421 (1977).

[20]L. R. Dodd, A. W. Thomas and R. F. Alvarez-Estrada, Phys. Rev. D 24, 1961 (1981).

[21]Although the formal convergence has been established in Ref. 20, we can not rule out corrections of order 5% from terms of $O(f_{NN\pi}{}^4)$.