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# Neutron electric form factor

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The electric form factor of the neutron is calculated within the framework of a simple vector-meson-dominance model where certain additional constraints are also applied.

The determination of the electromagnetic structure of the nucleon has been an important experimental and theoretical endeavor for almost four decades.[1] For a given nucleon, i.e., proton or neutron, this structure is given in terms of two functions, the electric and magnetic form factors, which are related to the respective electric and magnetic distributions. However, the electric form factor of the neutron, $G_{NE}(t)$, has been particularly difficult to measure; this is because the total charge of the neutron is zero and the corresponding electric form factor is small. In fact, the most accurate experimental measurements on $G_{NE}(t)$ only give the value of its slope at $t=0$, i.e., $G'_{EN}(0)$.[2] It should be pointed out that a knowledge of the neutron electric form factor is important both in nuclear physics, where it is needed for accurate calculations of the charge structure of nuclei,[3-9] and in elementary-particle physics, where it provides a very sensitive test of the breaking of symmetry in the nucleon spatial wave function.[10]

The purpose of this paper is to show, within the framework of a simple vector-meson-dominance model[11,12] where additional constraints are applied, that the electric form factor of the neutron can be determined for spacelike values of $t$. This calculation gives a simple representation for $G_{NE}(t)$ which is very suitable for use in, for example, nuclear-structure calculations.[3-9]

As our starting point, we make use of the fact that both quantum chromodynamics[13] (QCD) and quark-counting rules[14] lead to the following asymptotic behavior for the electric form factor of the neutron (up to logarithmic corrections):

$$G_{NE}(t) \underset{t\to-\infty}{\sim} ct^{-2} . \quad (1)$$

This means that $G_{NE}(t)$ satisfies an unsubtracted dispersion relation[11,15]

$$G_{NE}(t)=\int_{4\mu^2}^{\infty}\frac{w(x)dx}{x-t} , \quad (2)$$

where $\mu$ is the mass of the pion and $w(x)$ is the spectral function. Since the neutron has zero charge, we have

$$G_{NE}(o)=0 . \quad (3)$$

The conditions given by Eqs. (1) and (3) lead to the following two sum rules for the spectral function:

$$\int_{4\mu^2}^{\infty} w(x)dx=0 , \quad (4a)$$

$$\int_{4\mu^2}^{\infty}\frac{w(x)dx}{x}=0 . \quad (4b)$$

If, in addition, we use the fact that the slope of $G_{NE}(t)$ is known at $t=0$, then a third sum rule is obtained:

$$G'_{NE}(o)=\int_{4\mu^2}^{\infty}\frac{w(x)dx}{x^2} . \quad (4c)$$

It should be noted that the sum rules given by Eqs. (4) are completely general and do not depend on any particular model for the structure of the neutron.

Our model consists of taking the above three sum rules and assuming that the spectral function is saturated by the three lowest-mass vector mesons, namely, the $\rho$, $\omega$, and $\phi$. Therefore, in the zero-width approximation, the spectral function is given by the expression

$$w(x)=\sum_i m_i^2 g_i \delta(x-m_i^2) \quad (i=\rho,\omega,\phi) , \quad (5)$$

where the $g$'s are, for the present, unknown constants. The substitution of Eq. (5) in Eqs. (4) gives

$$\sum_i m_i^2 g_i=0 , \quad (6a)$$

 

$$\sum_i g_i = 0 \; , \tag{6b}$$

$$\sum_i \frac{g_i}{m_i^{\,2}} = G'_{NE}(o) \; . \tag{6c}$$

The Eqs. (6) are three linear equations in the three $g$'s and, therefore, can be easily solved. Doing this and substituting the values for the $g$'s into Eqs. (5) and (2) gives the following result for the neutron electric form factor:

$$G_{NE}(t) = \frac{m_\rho^{\,2} m_\omega^{\,2} m_\phi^{\,2} G'_{NE}(o) t}{A - Bt + Ct^2 - t^3} \; , \tag{7}$$

where

$$A = (m_\rho m_\omega m_\phi)^2 \; ,$$

$$B = (m_\rho m_\omega)^2 + (m_\rho m_\phi)^2 + (m_\omega m_\phi)^2 \; ,$$

$$C = m_\rho^{\,2} + m_\omega^{\,2} + m_\phi^{\,2} \; .$$

Thus, in this model $G_{NE}(t)$ is determined uniquely by the masses of the three vector mesons and the value of the slope of the form factor at $t=0$.

Let us now discuss the consequences which follow from the representation of the neutron electric form factor as given by Eq. (7):

(i) $G_{NE}(t)$ has the correct value of the slope at $t=0$. (This was one of the constraints which was built into the model.)

(ii) $G_{NE}(t)$ has the proper asymptotic behavior as determined by QCD. (Again, this feature was built into the model.)

(iii) If we use for $G'_{NE}(o)$ the value[2]

$$G'_{NE}(o) = -2\times 10^{-2}\ \text{fm}^2 \; , \tag{8}$$

then we find that $G_{NE}(t)$ is positive for spacelike values of $t$ and takes its maximum value of 0.0542 at $t=-0.35$ (GeV/$c$).$^2$

(iv) The predicted values for $G_{NE}(t)$, using Eqs. (7) and (8), are consistent with all available experimental measurements[16,17] ; see Fig. 1.

(v) The corresponding charge density,[1,11] which is essentially the Fourier transform of $G_{NE}(t)$, is given by the following expression:

$$\rho(r) = \left[\frac{1}{4\pi r}\right] \sum_i m_i^{\,2} g_i \exp(-m_i r) \; . \tag{9}$$

Note that the charge density is finite[18] at $r=0$ and has the value $\rho(o)=0.0089$. In addition, $\rho(r)$ has a simple zero at $r_0=0.67$ fm. This means that the neutron has a charge distribution which is positive for $0 \le r < r_0$ and negative for $r > r_0$. The maximum of the positive charge density occurs at the origin, while the maximum of the negative charge density is at $r_m=0.90$ fm where $\rho(r_m)=-6\times 10^5$. These results are shown in Fig. 2.

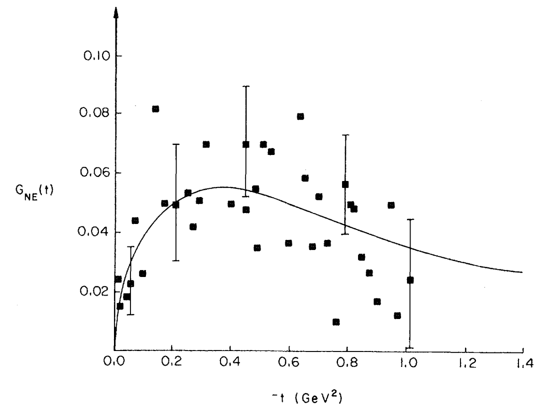


FIG. 1. Comparison of the predicted values of $G_{NE}(t)$ with selected data taken from Ref. 17. Typical error flags are shown.

In summary, we have constructed a model which yields a simple functional form for the neutron electric form factor. Our model has built into it both the correct asymptotic behavior, as given by QCD, and the correct threshold behavior, as determined from experiment. In addition, it gives values for $G_{NE}(t)$ which are consistent with all available experimental results. For these reasons, we suggest that $G_{NE}(t)$, as given by Eq. (7), be used in calculations where a knowledge of the neutron electric form factor is needed.

Finally, it should be pointed out that the model of this paper gives no information on the electric form factor of the proton. It was constructed

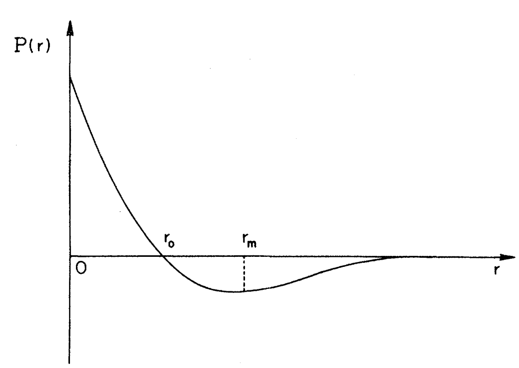


FIG. 2. A (not to scale) representation of the charge density of the neutron: $\rho(o)=0.0089$, $r_0=0.67$ fm, $r_m=0.90$ fm, $\rho(r_m) = -6\times 10^{-5}$.

specifically to provide a *good phenomenological representation* of only the neutron's electric form factor. All of the input for the model was information on $G_{NE}(t)$, except for the asymptotic behavior which followed from QCD and quark-counting rules. The representation of $G_{NE}(t)$ by Eq. (7) is superior to other formulas which have appeared in the literature in that it satisfies all of the theoretical constraints which one can incorporate into this kind of model. Again, it is to be noted that the distinctive feature of the model, which sets it apart from other works which have appeared in the literature, is the imposition of the asymptotic behavior $t^{-2}$. The comparison of $G_{NE}(t)$, as given by Eq. (7), with the data shows it to be a good phenomenological representation of the actual neutron electric form factor.

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[18]If, in Eq. (9), the limit as $r \to 0$ is taken, then Eqs. (6) may be used to show that the charge density is finite at $r = 0$.