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Decays of vectors into $\eta'\gamma$ and the structure of η'

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Using the known coupling of η' to two gluons, we predict a sizable branching ratio for $\Upsilon\rightarrow\eta'\gamma$ of approximately 0.1%. We point out the importance of measuring the decays of vectors into $\eta'\gamma$ and present further predictions for $\Upsilon\rightarrow\eta\gamma$, $\psi'\rightarrow\eta'(\eta)\gamma$, and $\phi\rightarrow\eta'\gamma$.

In quantum chromodynamics (QCD) with massless quarks there are eight massless Goldstone bosons π, K, η, as a result of the conservation of the octet of axial-vector currents $\sum_q \bar q\gamma^\mu\gamma_5 T^a q$ (T^a are color matrices with $a=1,\ldots,8$). A ninth singlet axial-vector current $\sum_q \bar q\gamma^\mu\gamma_5 q$, if it were conserved too, would lead to a ninth pseudoscalar Goldstone boson. The η' is, however, more massive than π, K, η and it is unreasonable to expect that its mass will result solely from chiral-symmetry breaking. This is the well known U(1) problem[1] which is resolved[2] through the QCD version of the Adler-Bell-Jackiw anomaly.[3]

The divergence of the singlet axial-vector current is

$$\sum_q 2m_q\bar q\gamma_5 q+\frac{3\alpha_s}{4\pi}G^a_{\mu\nu}\tilde G^{\mu\nu}_a\simeq\frac{3\alpha_s}{4\pi}G^a_{\mu\nu}\tilde G^{\mu\nu}_a \tag{1}$$

in the chiral-symmetry limit ($m_q=0$, $q=u,d,s$). In Eq. (1) α_s is the QCD coupling constant, $G^a_{\mu\nu}$ is the gluon field tensor, and $\tilde G^a_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^{a\alpha\beta}$ is its dual tensor. The solution to the U(1) problem implies[4-7] the importance of the matix element

$$A_{\eta'}=\left\langle 0\left|\frac{3\alpha_s}{4\pi}G^a_{\mu\nu}\tilde G^{\mu\nu}_a\right|\eta'\right\rangle. \tag{2}$$

If perturbative QCD would be meaningful at 1 GeV then one could claim that the annihilation channel in which $q\bar q$ annihilate into gluons is important for the η' (Refs. 8–10), since it is a singlet, thus separating it from the octet. It appears that a more rigorous foundation for the importance of the annihilation channel can be only found in the framework of the $1/N_c$ (N_c is the number of colors) expansion.[11]

As pointed out[4,5,7,12] the importance of $A_{\eta'}$ is not without experimental implications. In particular, the unexpectedly large ratio of $J/\psi\rightarrow\eta'\gamma$ to $J/\psi\rightarrow\eta\gamma$ results, once these decays are assumed to proceed through[13] $J/\psi\rightarrow gg+\gamma\rightarrow\eta'(\eta)\gamma$, from the following relation:

$$\frac{\Gamma(J/\psi\rightarrow\eta'\gamma)}{\Gamma(J/\psi\rightarrow\eta\gamma)}=\left(\frac{A_{\eta'}}{A_\eta}\right)^2\left(\frac{m_\psi{}^2-m_{\eta'}{}^2}{m_\psi{}^2-m_\eta{}^2}\right)^3 \tag{3}$$

and is experimentally[14] 5.9 ± 1.5. In Eq. (3) we define

$$A_\eta=\left\langle 0\left|\frac{3\alpha_s}{4\pi}G^a_{\mu\nu}\tilde G^{\mu\nu}_a\right|\eta\right\rangle, \tag{4}$$

which is fixed by the strong anomaly[4]

$$A_\eta\simeq(\tfrac{3}{2})^{1/2}f_\pi m_\eta{}^2, \tag{5}$$

where $f_\pi\simeq 133$ MeV. However, the evaluation of $A_{\eta'}$ requires more knowledge about gluon dynamics. Using QCD sum rules[4] it was found that

$$A_{\eta'}\simeq(0.5\text{–}1)\sqrt{3}f_\pi m_{\eta'}{}^2, \tag{6}$$

in agreement with other estimates[5,11] and with the value deduced from the experimental result for the ratio in Eq. (3), i.e.,

$$A_{\eta'}=(0.62\pm0.08)f_\pi m_{\eta'}{}^2. \tag{7}$$

In this paper we further exploit the value of $A_{\eta'}$ as given in Eq. (7) and of A_η to predict decays of the type $V\rightarrow\eta'\gamma$ and $\eta\gamma$, where $V=\Upsilon,\psi',\phi$. It has been suggested[15] that the large width for $J/\psi\rightarrow\eta'\gamma$ would imply a large width for $\Upsilon\rightarrow\eta'\gamma$, and an estimate for the branching ratio $B(\Upsilon\rightarrow\eta'\gamma)=3\times10^{-4}$ was obtained[16] under the rather speculative assumption that a 2-GeV glueball saturates both $J/\psi\rightarrow gg+\gamma$ and $\Upsilon\rightarrow gg+\gamma$. We show here that $B(\Upsilon\rightarrow\eta'\gamma)\simeq10^{-3}$, well within experimental feasibility.

Let us first consider $\Gamma(\Upsilon\rightarrow\eta'\gamma)/\Gamma(J/\psi\rightarrow\eta'\gamma)$. For both these decays we assume that $V\rightarrow gg+\gamma\rightarrow\eta'\gamma$. For $V\rightarrow gg+\gamma$ the lowest-order QCD result reads[17,18]

$$\frac{d\Gamma}{dx}=\Gamma_{gg\gamma}\frac{2}{\pi^2-9}F(x), \tag{8}$$

where $x=2E_\gamma/m_V$,

$$F(x)=\frac{x(1-x)}{(2-x)^2}+\frac{2-x}{x}+\frac{2(1-x)^2}{(2-x)^3}\ln\frac{1}{1-x}-2\frac{1-x}{x^2}\ln\frac{1}{1-x}, \tag{9}$$

and

$$\Gamma_{gg\gamma}=\frac{128(\pi^2-9)}{9}\alpha_V{}^2\alpha\left|\frac{e_q\Psi(0)}{m_V}\right|^2. \tag{10}$$

α_V is the QCD coupling at m_V, e_q is the charge of the quark which makes the $q\bar{q}$ bound state, and $\Psi(0)$ is the $q\bar{q}$ wave function at the origin. Using duality arguments the decay width, into a state such as η' which couples to two gluons, is proportional to $\int_{a_V}^{b} F(x)dx$. In Ref. 17 it was suggested that $a_V = 1 - m_R^2/m_V^2$, $b=1$, where m_R is the mass of the two-gluon "resonance" (in the present case it is $m_{\eta'}$). If one integrates instead over some two-gluon mass slice of width Δm,[19] then our results as discussed hereafter will change by a few percent only—except for $\phi \to \eta'\gamma$ (see below), and we can confidently proceed with the first prescription.

We then find

$$\frac{\Gamma(\Upsilon - \eta'\gamma)}{\Gamma(J/\psi \to \eta'\gamma)}$$

$$= \left(\frac{\alpha_\Upsilon}{\alpha_\psi}\right)^2 \frac{\int_{a_\Upsilon}^{1} F(x)dx}{\int_{a_\psi}^{1} F(x)dx} \, \frac{\Gamma(\Upsilon \to \mu^+\mu^-)}{\Gamma(J/\psi \to \mu^+\mu^-)} \left(\frac{P_\Upsilon/m_\Upsilon}{P_\psi/m_\psi}\right)^2 , \tag{11}$$

where the $(P_V/m_V)^3$ factors are due to the p-wave nature of the decays (P_V is the c.m. momentum in $V \to \eta'\gamma$) and we use

$$\Gamma(V \to \mu^+\mu^-) \sim \left| e_q \frac{\Psi(0)}{m_V} \right|^2 . \tag{12}$$

Taking the "experimental" values[20] $\alpha_\Upsilon = 0.17 \pm 0.02$, $\alpha_\psi = 0.19 \pm 0.02$, and recent data[21] for $J/\psi \to \eta'\gamma$ and $V \to \mu^+\mu^-$, we find $B(\Upsilon \to \eta'\gamma) \simeq 3.5 \times 10^{-4}$. However, there is an obvious flaw in this estimate. While the total production rate for $J/\psi \to \gamma +$hadrons is consistent with the lowest-order QCD calculation for $x > 0.6$,[22] the experimental x distribution does not peak at high x as predicted by the lowest-order QCD calculation. However, the values of $F(x > a_V)$ for $V = J/\psi$ as employed in Eq. (11) are approximately three times higher, once folded with the detection efficiency,[22] than the experimental results for $J/\psi \to \gamma +$hadrons, for all $x > a_\psi$ (for lower x values both the shape and the size of the QCD calculation differ from the data). We assume that for $\Upsilon \to gg + \gamma$ lowest-order QCD would explain the yet nonexistent data, an assumption which seems reasonable in view of the agreement between data for $\Upsilon \to$ hadrons[20] and lowest-order QCD calculation for $\Upsilon \to ggg$.[23] We therefore multiply the result of Eq. (11) by a "fudge factor" of 3 and obtain

$$B(\Upsilon \to \eta'\gamma) \simeq 10^{-3} \tag{13}$$

which is three times larger than the estimate of Ref. 16. The reason for the difference is clearly the drastic assumption of dominance of a 2-GeV glueball for both $J/\psi \to gg + \gamma$ and $\Upsilon \to gg + \gamma$, as compared to a mild duality assumption made here.

Following Eq. (3) we find

$$B(\Upsilon \to \eta\gamma) \simeq 1.5 \times 10^{-4} . \tag{14}$$

Turning now to $\psi' \to \eta'\gamma$ we predict, after an analysis similar to the previous one, but without any "fudge factor" since lowest-order QCD is expected to be equally violated in ψ and in ψ' decays, that

$$B(\psi' \to \eta'\gamma) \simeq 6.3 \times 10^{-4} . \tag{15}$$

Experimentally there is only an upper limit[24] $B(\psi' \to \eta'\gamma) < 8 \times 10^{-4}$, thus measurements in the near future will test our prediction. Note that in the present picture

$$\frac{B(\psi' \to \eta'\gamma)}{B(J/\psi \to \eta'\gamma)} \neq \frac{B(\psi' \to \mu^+\mu^-)}{B(J/\psi \to \mu^+\mu^-)} \tag{16}$$

since—apart from the insignificant change in α_V and in the p-wave factors—a ratio of integrals over $F(x)$ is missing. Ignoring this ratio leads to $B(\psi' \to \eta'\gamma) \simeq 9.7 \times 10^{-4}$ which is, considering the errors, barely consistent with the experimental upper limit. Again from Eq. (3) it follows that

$$B(\psi' \to \eta\gamma) \simeq 10^{-4} \tag{17}$$

to be compared with the experimental upper limit[24] $B(\psi' \to \eta\gamma) < 10^{-4}$.

Turning now to ϕ decays, the uncertainties are much larger than before, both experimentally and theoretically. Experimentally $\phi \to \eta'\gamma$ has not been measured, mainly because η' production is barely possible. Theoretically, lowest-order QCD seems even less acceptable than for J/ψ, and the applications of the two duality criteria discussed above lead to different results. Nevertheless, even setting a limit for $\phi \to \eta'\gamma$ will help resolve the following problem: How much of a glue component is there in the η'? Note that all our predictions until now depend on the large value of $A_{\eta'}$ given in Eq. (7) but since the mechanism assumed is[13] $V \to gg + \gamma \to \eta'\gamma$, no assumption is needed for the relative strength of the glue component in η'. Estimates range from the η' as an almost pure pseudoscalar gluonium,[4,5] to about 70%–80% gluonium[10,25,26] down to no glue component at all.[27] The common prediction for $\Gamma(\phi \to \eta'\gamma)/\Gamma(\phi \to \eta\gamma)$ is based on $s\bar{s} \to s\bar{s} + \gamma$. Since for a mixing angle of 10.3° between η and η' the $s\bar{s}$ component of both of them is approximately equal, up to a sign, one finds that the ratio is given by phase-space only and

$$\Gamma(\phi \to \eta'\gamma) \simeq 0.28 \text{ keV} \quad (\text{no glue in } \eta') . \tag{18}$$

This value is consistent with other estimates.[28] However, if η' is a pseudoscalar gluonium one can predict from $J/\psi \to \eta'\gamma$, following the same steps as described above, that $\Gamma(\phi\to\eta'\gamma)\simeq 0.015$ keV. Again a "fudge factor" of 3 has been included in $J/\psi\to\eta'\gamma$ only, since $a_\phi = 0.12$ and the integral $\int_{a_\phi}^{1} F(x)dx$ includes a large part of the spectrum. Although $F(x)$ may differ in shape from the experimental value, the integrals—at least for $J/\psi\to gg+\gamma$ (Ref. 22)—are almost equal. Applying the other prescription for duality[19] will this time decrease the result by a factor of 5. Therefore, we consider the result as an upper limit, i.e.,

$$\Gamma(\phi\to\eta'\gamma)< 0.015\ \text{keV}\quad (\eta' \text{ is a pure gluonium}). \tag{19}$$

Thus, even with all the reservations regarding the estimate for $\phi\to gg+\gamma\to\eta'\gamma$ there is more than an order of magnitude difference between the results in Eqs. (18) and (19). Therefore, if experiment will show that $\Gamma(\phi\to\eta'\gamma)< 0.1$ keV, then η' has a substantial glue component.

Finally, let us comment that predictions for $\Upsilon\to E\gamma$ and $\psi'\to E\gamma$ based on the experimental result[29] for $J/\psi\to E\gamma$ and the (possibly premature) assumption that E is a glueball plus the earlier considerations should await a higher-order QCD calculation for $d\Gamma/dx$. This is so since a_ψ for E production lies below the value for which one can assume that $F(x)$ (lowest-order QCD) $\simeq$ const $\times F(x)$ (experimental).

Let us conclude by reiterating the importance of measuring decays of the type $V\to\eta'\gamma$. These will help in our understanding of gluon dynamics and its relation to the resolution of the U(1) problem. Among the predictions presented in Eqs. (13)–(15) and (17)–(19), the most viable ones are $B(\psi'\to\eta'\gamma)$, where a value significantly smaller than 6×10^{-4} will mean that the whole approach is in trouble, and $B(\Upsilon\to\eta'\gamma)\simeq 0.1\%$. Furthermore, setting a limit for $\Gamma(\phi\to\eta'\gamma)$ will be very useful in settling the problem of the amount of glue in η'.

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