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Renormalization of quantum chromodynamics

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Renormalization of quantum chromodynamics is carried out by means of a momentum-space subtraction scheme, which is computationally simple, provides a well-defined procedure for the determination of renormalization constants, and is equally applicable to light and heavy quarks. We also obtain the renormalization-group parameters, verify the decoupling theorem for heavy quarks, and compute the effective coupling constant and effective quark masses.

I. INTRODUCTION

Several renormalization prescriptions are currently being used for the choice of the finite parts of renormalization constants in quantum chromodynamics. This leads to prescription dependence in physical results, and there has been controversy regarding the reliability of various results derived from perturbative quantum chromodynamics. Three different renormalization prescriptions are commonly used at present, which are referred to as the minimal subtraction[1] (MS) scheme, the modified minimal subtraction[2] ($\overline{\mathrm{MS}}$) scheme, and the momentum-space subtraction[3] (MOM) scheme. The MS scheme is simple but unphysical, the $\overline{\mathrm{MS}}$ scheme is a distinct improvement over the MS scheme, while the MOM scheme is both physical as well as found to yield reasonably small higher-order corrections in perturbative calculations.[4] The success of the MOM scheme is not surprising because a good deal of the radiative corrections at some typical momentum for a process under consideration are absorbed in the coupling constant.

The MOM scheme, as formulated by Celmaster and Gonsalves,[3] has the following disadvantages: (1) It is computationally difficult. (2) Since renormalization constants resulting from the MOM scheme do not automatically satisfy the Ward identities, several different choices of renormalization constants are possible, but there is no indication as to which choice is to be preferred. (3) Quarks are treated as massless, which is clearly not justified for the heavier quarks. The aim of our paper is to formulate a momentum-space subtraction scheme which is free from these disadvantages.

The computational difficulty in the Celmaster-Gonsalves scheme arises from the fact that the renormalization of three-point vertices is carried out at the symmetric point. But, it is obvious from the well-known analogy between interaction diagrams and electrical circuits that all internal lines in an interaction diagram do not carry the same mean momentum, and the symmetric point does not necessarily represent an ideal choice. We shall, therefore, use a momentum-space subtraction scheme in which the momentum carried by one gluon line at any three-point vertex will be made to vanish, which simplifies the computation of the renormalization constants.[5] Although the Ward identities are not automatically satisfied even in our scheme, we shall show how we can arrive at a preferred choice of the renormalization constants. Moreover, our treatment will be applicable to both light as well as heavy quarks.

Our scheme will be based on a mass-dependent off-mass-shell renormalization procedure which differs from those of earlier authors.[6,7] We shall first show how off-mass-shell renormalization in quantum electrodynamics can be carried out in such a way that Ward's identity is automatically satisfied, and subsequently follow the same treatment in quantum chromodynamics to obtain the renormalization constant for the quark-gluon vertex in a particularly simple form. We shall then perform renormalization by using the off-mass-shell renormalization constants for all the two-point vertices and the three-point quark-gluon vertex, while the remaining renormalization constants will be determined by means of the Ward identities. Such a renormalization procedure seems especially appropriate because the quark-gluon vertex plays a more significant role than the other three-point vertices in applications of quantum chromodynamics to physically interesting processes such as the positron-electron annihilation[8] or the para-

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quarkonium decay.[9]

Besides deriving the renormalization constants to the one-loop level in our scheme, we shall obtain the renormalization-group parameters, verify the Appelquist-Carazzone decoupling theorem,[10] and compute the effective coupling constant and effective quark masses.

It should be mentioned that we shall use dimensional regularization[11] for the evaluation of renormalization constants. However, since we shall introduce the renormalization-scale parameter μ through a momentum-space subtraction scheme, we shall not carry out the usual transformation of the coupling constant, and thus avoid the introduction of an additional parameter into the results for the renormalization constants.

II. OFF-MASS-SHELL RENORMALIZATION OF QUANTUM ELECTRODYNAMICS

The renormalized Lagrangian density in quantum electrodynamics is expressible as

$$\begin{aligned} L = & -\tfrac{1}{4}Z_A(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \tfrac{1}{2}(\partial_\mu A_\mu)^2 \\ & - Z_\psi(\bar{\psi}\gamma_\mu\partial_\mu\psi + m\bar{\psi}\psi) \\ & + iZ'_\psi e A_\mu \bar{\psi}\gamma_\mu\psi - Z_\psi(Z_m - 1)m\bar{\psi}\psi \end{aligned} \tag{2.1}$$

with

$$Z_\psi = Z'_\psi, \quad e = Z_A{}^{1/2}e_0, \quad m = Z_m{}^{-1}m_0 , \tag{2.2}$$

where e_0 and m_0 are the bare charge and bare mass of the electron. We shall describe a procedure for off-mass-shell renormalization in which Ward's identity is automatically satisfied. Our procedure will be valid for massive as well as massless quantum electrodynamics.

Let $\Sigma(p)$ be the contribution of proper electron self-energy parts, and $\Lambda(p,p)$ be the contribution of proper electron-photon vertex parts in which the photon momentum has been made to vanish. We then set

$$\Sigma(p) = A_1(p^2) + A_2(p^2)(ip\cdot\gamma + m) , \tag{2.3}$$

$$\begin{aligned} \Lambda_\mu(p,p) = & B_1(p^2)\gamma_\mu + iB_2(p^2)p_\mu \\ & + iB_3(p^2)p_\mu(ip\cdot\gamma + m) , \end{aligned} \tag{2.4}$$

so that substitution of (2.3) and (2.4) into Ward's identity

$$-i\frac{\partial\Sigma(p)}{\partial p_\mu} = \Lambda_\mu(p,p) \tag{2.5}$$

gives

$$A_2(p^2) = B_1(p^2) , \tag{2.6}$$

and we carry out off-mass-shell renormalization by choosing the counterterms in (2.1) such that $A_1(p^2)$, $A_2(p^2)$, and $B_1(p^2)$ in (2.3) and (2.4) are canceled at

$$p^2 + m^2 = \mu^2 . \tag{2.7}$$

For the contribution $\Pi_{\mu\nu}(p)$ of proper photon self-energy parts, we follow the usual off-mass-shell renormalization procedure by setting

$$\Pi_{\mu\nu}(p) = C(p^2)(p^2\delta_{\mu\nu} - p_\mu p_\nu) , \tag{2.8}$$

and choosing the counterterm in (2.1) such that $C(p^2)$ is canceled at

$$p^2 = \mu^2 . \tag{2.9}$$

The above renormalization procedure yields to the one-loop level,

$$Z_m = 1 - \frac{e^2}{16\pi^2}\left[3\left[\frac{2}{4-n} - \gamma_E + \ln 4\pi - \ln\mu^2\right] + 5 - \frac{m^2}{m^2-\mu^2} - \frac{m^2(3m^2-4\mu^2)}{(m^2-\mu^2)^2}\ln\frac{m^2}{\mu^2}\right] , \tag{2.10}$$

$$Z_\psi = Z'_\psi = 1 - \frac{e^2}{16\pi^2}\left[\frac{2}{4-n} - \gamma_E + \ln 4\pi - \ln\mu^2 + 1 + \frac{m^2}{m^2-\mu^2} - \frac{m^4}{(m^2-\mu^2)^2}\ln\frac{m^2}{\mu^2}\right] , \tag{2.11}$$

$$\begin{aligned} Z_A = 1 - \frac{e^2}{12\pi^2}\Bigg[& \frac{2}{4-n} - \gamma_E + \ln 4\pi - \ln\mu^2 + \tfrac{5}{3} - \frac{4m^2}{\mu^2} - \ln\frac{m^2}{\mu^2} \\ & + \frac{2m^2-\mu^2}{\mu^2}(1+4m^2/\mu^2)^{1/2}\ln\frac{(1+4m^2/\mu^2)^{1/2}+1}{(1+4m^2/\mu^2)^{1/2}-1}\Bigg] . \end{aligned} \tag{2.12}$$

It should be observed that for $\mu^2=0$, our treatment becomes a mass-shell renormalization procedure, and (2.10), (2.11), and (2.12) become

$$Z_m = 1 - \frac{3e^2}{16\pi^2}\left[\frac{2}{4-n} - \gamma_E + \ln 4\pi - \ln m^2 + \tfrac{4}{3}\right] , \tag{2.13}$$

$$Z_\psi = Z'_\psi = 1 - \frac{e^2}{16\pi^2}\left[\frac{2}{4-n} - \gamma_E + \ln 4\pi - \ln m^2 + 2\right], \tag{2.14}$$

$$Z_A = 1 - \frac{e^2}{12\pi^2}\left[\frac{2}{4-n} - \gamma_E + \ln 4\pi - \ln m^2\right], \tag{2.15}$$

where only (2.13) and (2.15) agree with the usual mass-shell results. However, since the equality of Z_ψ and Z'_ψ is maintained, the usual physical results remain unchanged.

We also note that for massless quantum electrodynamics, Z_m drops out from the Lagrangian density (2.1), while (2.11) and (2.12) reduce to

$$Z_\psi = Z'_\psi = 1 - \frac{e^2}{16\pi^2}\left[\frac{2}{4-n} - \gamma_E + \ln 4\pi - \ln \mu^2 + 1\right], \tag{2.16}$$

$$Z_A = 1 - \frac{e^2}{12\pi^2}\left[\frac{2}{4-n} - \gamma_E + \ln 4\pi - \ln \mu^2 + \tfrac{5}{3}\right]. \tag{2.17}$$

III. OFF-MASS-SHELL RENORMALIZATION OF QUANTUM CHROMODYNAMICS

The renormalized Lagrangian density in quantum chromodynamics is given by

$$\begin{aligned} L = &-\tfrac{1}{4}Z_G(\partial_\mu G^i_\nu - \partial_\nu G^i_\mu)^2 - \tfrac{1}{2}(\partial_\mu G^i_\mu)^2 - \tfrac{1}{2}Z'_G g f^{ijk}(\partial_\mu G^i_\nu - \partial_\nu G^i_\mu)G^j_\mu G^k_\nu - \tfrac{1}{4}Z''_G g^2 (f^{ijk}G^j_\mu G^k_\nu)^2 \\ &- \sum_{n_f}[Z_q(\bar{q}\gamma_\mu\partial_\mu q + m\bar{q}q) - iZ'_q g G^i_\mu \bar{q}\gamma_\mu T^i q + Z_q(Z_m - 1)m\bar{q}q] - Z_C\partial_\mu C^{i*}\partial_\mu C^i + Z'_C g f^{ijk} G^i_\mu \partial_\mu C^{j*} C^k \end{aligned} \tag{3.1}$$

with

$$Z_G/Z'_G = Z'_G/Z''_G = Z_q/Z'_q = Z_C/Z'_C\,, \tag{3.2}$$

$$g = (Z_G{}^{1/2} Z_q/Z'_q)g_0, \quad m = Z_m{}^{-1} m_0\,, \tag{3.3}$$

where q, G^i_μ, and c^i are the quark, the gluon, and the ghost fields, n_f is the number of quark flavors, the T^i are $N\times N$ color matrices, and the upper indices take the values $1,2,\ldots,N^2-1$.

We shall determine the off-mass-shell renormalization constants to the one-loop level for the two- and three-point vertices in quantum chromodynamics. These vertices are shown in Fig. 1, and the contributions of the scattering operator for them are given in the Appendix. For the two-point vertices, we require $\Sigma(p)$, $\Pi^{ij}_{\mu\nu}(p)$, and $\Pi^{ij}(p)$, given by (A4), (A10), and (A21). For the three-point vertices, the momentum of one external gluon line is made to vanish in our scheme, and therefore we require $\Lambda^i_\mu(p,p,0)$, $\Lambda^{ijk}_{\mu\nu\lambda}(p,-p,0)$, and $\Lambda^{ijk}_\mu(p,p,0)$, which can be obtained from (A6), (A15), and (A23).

In analogy with quantum electrodynamics, we express $\Sigma(p)$ and $\Lambda^i_\mu(p,p,0)$ in the form

$$\Sigma(p) = A_1(p^2) + A_2(p^2)(ip\cdot\gamma + m)\,, \tag{3.4}$$

$$\Lambda^i_\mu(p,p,0) = B_1(p^2)T^i\gamma_\mu + iB_2(p^2)T^i p_\mu + iB_3(p^2)T^i p_\mu(ip\cdot\gamma + m)\,, \tag{3.5}$$

and choose renormalization constants so as to cancel $A_1(p^2)$, $A_2(p^2)$, and $B_1(p^2)$ in (3.4) and (3.5) at

$$p^2 + m^2 = \mu^2\,. \tag{3.6}$$

We thus obtain

$$Z_m = 1 - C_F\frac{g^2}{16\pi^2}\left[3\left[\frac{2}{4-n} - \gamma_E + \ln 4\pi - \ln\mu^2\right] + 5 - \frac{m^2}{m^2-\mu^2} - \frac{m^2(3m^2-4\mu^2)}{(m^2-\mu^2)^2}\ln\frac{m^2}{\mu^2}\right], \tag{3.7}$$

$$Z_q=1-C_F\frac{g^2}{16\pi^2}\left[\frac{2}{4-n}-\gamma_E+\ln4\pi-\ln\mu^2+1+\frac{m^2}{m^2-\mu^2}-\frac{m^4}{(m^2-\mu^2)^2}\ln\frac{m^2}{\mu^2}\right], \tag{3.8}$$

$$Z_q'=Z_q-C_A\frac{g^2}{16\pi^2}\left[\frac{2}{4-n}-\gamma_E+\ln4\pi-\ln\mu^2+1\right]. \tag{3.9}$$

The evaluation of $\Pi^{ij}_{\mu\nu}(p)$ and $\Lambda^{ijk}_{\mu\nu\lambda}(p,-p,0)$ yields results of the form[12]

$$\Pi^{ij}_{\mu\nu}(p)=C(p^2)\delta^{ij}(p^2\delta_{\mu\nu}-p_\mu p_\nu), \tag{3.10}$$

$$\Lambda^{ijk}_{\mu\nu\lambda}(p,-p,0)=f^{ijk}[D_1(p^2)(p_\mu\delta_{\nu\lambda}+p_\nu\delta_{\mu\lambda}-2p_\lambda\delta_{\mu\nu})+D_2(p^2)(p_\mu\delta_{\nu\lambda}+p_\nu\delta_{\mu\lambda})+D_3(p^2)p_\mu p_\nu p_\lambda/p^2], \tag{3.11}$$

and we choose renormalization constants such that $C(p^2)$ and $D_1(p^2)$ are canceled at

$$p^2=\mu^2, \tag{3.12}$$

which gives

$$\begin{aligned}Z_G=1+\frac{g^2}{16\pi^2}\Bigg[&\tfrac{5}{3}C_A\left[\frac{2}{4-n}-\gamma_E+\ln4\pi-\ln\mu^2+\tfrac{31}{15}\right]\\ &-\tfrac{2}{3}\sum_{n_f}\Bigg[\frac{2}{4-n}-\gamma_E+\ln4\pi-\ln\mu^2+\tfrac{5}{3}-\frac{4m^2}{\mu^2}-\ln\frac{m^2}{\mu^2}\\ &+\frac{2m^2-\mu^2}{\mu^2}(1+4m^2/\mu^2)^{1/2}\ln\frac{(1+4m^2/\mu^2)^{1/2}+1}{(1+4m^2/\mu^2)^{1/2}-1}\Bigg]\Bigg],\end{aligned} \tag{3.13}$$

$$\begin{aligned}Z_G'=1+\frac{g^2}{16\pi^2}\Bigg[&\tfrac{2}{3}C_A\left[\frac{2}{4-n}-\gamma_E+\ln4\pi-\ln\mu^2+\tfrac{23}{12}\right]\\ &-\tfrac{2}{3}\sum_{n_f}\Bigg[\frac{2}{4-n}-\gamma_E+\ln4\pi-\ln\mu^2+\tfrac{2}{3}+\frac{2m^2}{\mu^2}-\ln\frac{m^2}{\mu^2}\\ &-\frac{1+2m^2/\mu^2+4m^4/\mu^4}{(1+4m^2/\mu^2)^{1/2}}\ln\frac{(1+4m^2/\mu^2)^{1/2}+1}{(1+4m^2/\mu^2)^{1/2}-1}\Bigg]\Bigg].\end{aligned} \tag{3.14}$$

For n_l light and n_h heavy quarks, (3.13) and (3.14) reduce to

$$\begin{aligned}Z_G=1+\frac{g^2}{16\pi^2}\Bigg[&\tfrac{5}{3}C_A\left[\frac{2}{4-n}-\gamma_E+\ln4\pi-\ln\mu^2+\tfrac{31}{15}\right]-\tfrac{2}{3}n_l\left[\frac{2}{4-n}-\gamma_E+\ln4\pi-\ln\mu^2+\tfrac{5}{3}\right]\\ &-\tfrac{2}{3}\sum_{n_h}\left[\frac{2}{4-n}-\gamma_E+\ln4\pi-\ln m^2\right]\Bigg],\end{aligned} \tag{3.15}$$

$$\begin{aligned}Z_G'=1+\frac{g^2}{16\pi^2}\Bigg[&\tfrac{2}{3}C_A\left[\frac{2}{4-n}-\gamma_E+\ln4\pi-\ln\mu^2+\tfrac{23}{12}\right]-\tfrac{2}{3}n_l\left[\frac{2}{4-n}-\gamma_E+\ln4\pi-\ln\mu^2+\tfrac{2}{3}\right]\\ &-\tfrac{2}{3}\sum_{n_h}\left[\frac{2}{4-n}-\gamma_E+\ln4\pi-\ln m^2\right]\Bigg].\end{aligned} \tag{3.16}$$

The vertices $\Pi^{ij}(p)$ and $\Lambda^{ijk}_\mu(p,p,0)$ are the simplest ones to deal with. We obtain results of the form

$$\Pi^{ij}(p)=E(p^2)\delta^{ij}p^2, \tag{3.17}$$

$$\Lambda^{ijk}_\mu(p,p,0)=F(p^2)f^{ijk}p_\mu, \tag{3.18}$$

choose renormalization constants such that $E(p^2)$ and $F(p^2)$ are canceled at (3.12), and thus

$$Z_C=1+C_A\frac{g^2}{32\pi^2}\left[\frac{2}{4-n}-\gamma_E+\ln 4\pi-\ln\mu^2+2\right], \tag{3.19}$$

$$Z_C'=1-C_A\frac{g^2}{32\pi^2}\left[\frac{2}{4-n}-\gamma_E+\ln 4\pi-\ln\mu^2+2\right]. \tag{3.20}$$

Let us now examine the compatibility of the above results with the Ward identities (3.2). According to (3.9), (3.15), (3.16), (3.19), and (3.20),

$$Z_q/Z_q'=1+C_A\frac{g^2}{16\pi^2}\left[\frac{2}{4-n}-\gamma_E+\ln 4\pi-\ln\mu^2+1\right]+O(g^4), \tag{3.21}$$

$$Z_G/Z_G'=1+C_A\frac{g^2}{16\pi^2}\left[\frac{2}{4-n}-\gamma_E+\ln 4\pi-\ln\mu^2+\tfrac{13}{6}-\frac{2n_l}{3C_A}\right]+O(g^4), \tag{3.22}$$

$$Z_C/Z_C'=1+C_A\frac{g^2}{16\pi^2}\left[\frac{2}{4-n}-\gamma_E+\ln 4\pi-\ln\mu^2+2\right]+O(g^4), \tag{3.23}$$

and thus, for $C_A=3$ and $n_l=3$ or 4, there is some violation of the Ward identities, which necessitates a special role for one of the three-point vertices in the renormalization procedure.

As explained in Sec. I, it is especially appropriate to use the quark-gluon vertex rather than any other three-point vertex for a preferred choice of the renormalization constants. We shall, therefore, take Z_q, Z_q', Z_G, and Z_C as given by (3.8), (3.9), (3.13), and (3.19), and obtain Z_G' and Z_C' through the relations

$$Z_G'=Z_GZ_q'/Z_q,\quad Z_C'=Z_CZ_q'/Z_q\,. \tag{3.24}$$

It is remarkable that our scheme yields a simple result for Z_q/Z_q' for quarks of arbitrary mass, while earlier authors[13] found renormalization via the quark-gluon vertex too complex to be used in practical applications.

For practical purposes, it is useful to note that the coupling constant in our scheme is related to that in the $\overline{\mathrm{MS}}$ scheme as

$$gZ_q'/Z_qZ_G^{1/2}=g_0=\bar g\bar Z_q'/\bar Z_q\bar Z_G^{1/2}, \tag{3.25}$$

where the barred quantities refer to the $\overline{\mathrm{MS}}$ scheme. It follows that[14]

$$\alpha_s=\bar\alpha_s\left[1+\frac{\bar\alpha_s}{4\pi}\left[2C_A+\tfrac{31}{9}C_A-\tfrac{10}{9}n_l+\tfrac{2}{3}\sum_{n_h}\ln\frac{m^2}{\mu^2}\right]\right], \tag{3.26}$$

where $\alpha_s=g^2/4\pi$.

IV. EFFECTIVE COUPLING CONSTANT AND QUARK MASSES

Let $\Gamma^{(n_q,n_G)}(g,m,\mu,p_1,p_2,\ldots)$ denote the renormalized contribution of proper vertex parts with n_q and n_G external quark and gluon lines carrying the momenta $p_1,p_2,\ldots$. The renormalization-group equation is then given by

$$\left[\mu\frac{\partial}{\partial\mu}+\beta\frac{\partial}{\partial g}+\sum_{n_f}\gamma_m m\frac{\partial}{\partial m}-\tfrac{1}{2}\sum_{n_q}\gamma_q-\tfrac{1}{2}n_G\gamma_G\right]\Gamma^{(n_q,n_G)}(g,m,\mu,p_1,p_2,\ldots)=0 \tag{4.1}$$

with

$$\beta=\mu\frac{dg}{d\mu},\quad \gamma_m=\frac{\mu}{m}\frac{dm}{d\mu},\quad \gamma_q=\frac{\mu}{Z_q}\frac{dZ_q}{d\mu},\quad \gamma_G=\frac{\mu}{Z_G}\frac{dZ_G}{d\mu}, \tag{4.2}$$

where differentiations in (4.2) are to be carried out by treating g_0 and m_0 as constants.

With the use of the results of the preceding section, we obtain from (4.2) to the lowest order

$$\beta=-\frac{g^3}{16\pi^2}\left[\tfrac{11}{3}C_A-\tfrac{2}{3}\sum_{n_f}\left\{1-\frac{6m^2}{\mu^2}+\frac{12m^4/\mu^4}{(1+4m^2/\mu^2)^{1/2}}\ln\frac{(1+4m^2/\mu^2)^{1/2}+1}{(1+4m^2/\mu^2)^{1/2}-1}\right\}\right], \tag{4.3}$$

$$\gamma_m=-C_F\frac{g^2}{8\pi^2}\left[\frac{3-m^2/\mu^2}{(1-m^2/\mu^2)^2}+\frac{(2m^2/\mu^2)(2-m^2/\mu^2)}{(1-m^2/\mu^2)^3}\ln\frac{m^2}{\mu^2}\right], \tag{4.4}$$

$$\gamma_q=C_F\frac{g^2}{8\pi^2}\left[\frac{1-3m^2/\mu^2}{(1-m^2/\mu^2)^2}-\frac{2m^4/\mu^4}{(1-m^2/\mu^2)^3}\ln\frac{m^2}{\mu^2}\right], \tag{4.5}$$

$$\gamma_G=-\frac{g^2}{8\pi^2}\left[\tfrac{5}{3}C_A-\tfrac{2}{3}\sum_{n_F}\left\{1-\frac{6m^2}{\mu^2}+\frac{12m^4/\mu^4}{(1+4m^2/\mu^2)^{1/2}}\ln\frac{(1+4m^2/\mu^2)^{1/2}+1}{(1+4m^2/\mu^2)^{1/2}-1}\right\}\right], \tag{4.6}$$

and the contributions of heavy quarks to the above renormalization-group parameters vanish in the limit $m^2/\mu^2\to\infty$ in accordance with the Appelquist-Carazzone decoupling theorem.[10]

The effective coupling constant and effective quark masses can be determined for various values of μ by solving the coupled equations

$$\mu\frac{dg}{d\mu}=\beta\,, \tag{4.7}$$

$$\frac{\mu}{m}\frac{dm}{d\mu}=\gamma_m\,, \tag{4.8}$$

where β and γ_m, given by (4.3) and (4.4), are functions of g and m/μ. We have computed the solutions for $N=3$ with the use of the following input parameters: at $\mu=3$ GeV, $m_u=m_d=0.02$ GeV, $m_s=0.4$ GeV, $m_c=1.5$ GeV, $m_b=5$ GeV, and $\alpha_s=0.35$. The results, given in Table I, show that as μ decreases, both the effective coupling constant and effective quark masses increase, and the rapid increase in the effective coupling constant for very low values of μ is especially noteworthy.

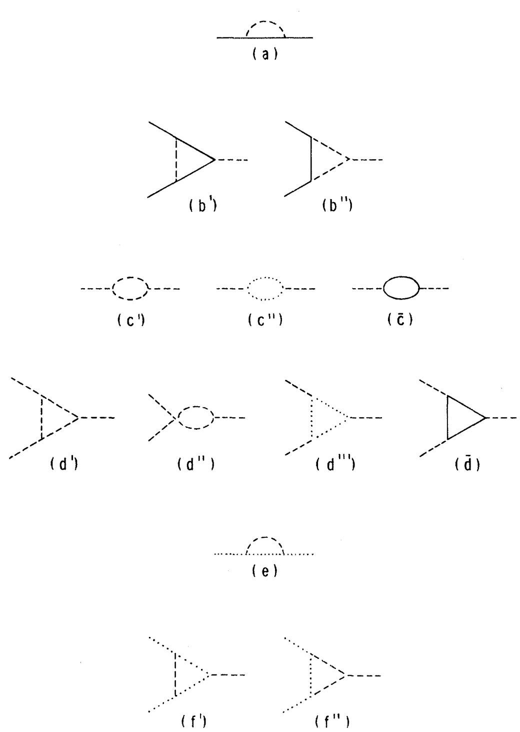

FIG. 1. Two- and three-point vertices in quantum chromodynamics at the one-loop level. Solid lines represent quarks, while broken and dotted lines represent gluons and ghost particles, respectively.

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APPENDIX: TWO- AND THREE-POINT VERTICES IN QUANTUM CHROMODYNAMICS

We give contributions of the scattering operator for the two- and three-point vertices shown in Fig. 1, where tadpole and leaf diagrams with vanishing contributions have been ignored. We have reduced the color factors with the use of the relations

$$[T^i,T^j]=if^{ijk}T^k,\quad \mathrm{Tr}(T^iT^j)=\tfrac{1}{2}\delta^{ij},\quad T^iT^i=C_F\underline{1}\,,$$
$$f^{abi}f^{abj}=C_A\delta^{ij},\quad f^{abi}f^{bcj}f^{cak}=\tfrac{1}{2}C_Af^{ijk}\,, \tag{A1}$$

where

TABLE I. Effective coupling constant and effective quark masses. The renormalization-scale parameter μ and the quark masses are given in GeV.

μ	α_s	m_u,m_d	m_s	m_c	m_b
0.5	4.44	0.06	0.74	1.97	5.41
0.6	1.98	0.04	0.65	1.89	5.36
0.7	1.35	0.04	0.61	1.84	5.33
0.8	1.06	0.03	0.57	1.81	5.31
0.9	0.89	0.03	0.55	1.78	5.29
1.0	0.78	0.03	0.53	1.75	5.27
2.0	0.44	0.02	0.44	1.59	5.12
3.0	0.35	0.02	0.40	1.50	5.00
4.0	0.31	0.02	0.38	1.44	4.90
5.0	0.28	0.02	0.36	1.39	4.82
6.0	0.26	0.02	0.35	1.35	4.74
7.0	0.25	0.02	0.34	1.32	4.68
8.0	0.24	0.02	0.34	1.30	4.62
9.0	0.23	0.02	0.33	1.28	4.57
10.0	0.22	0.02	0.33	1.26	4.52
20.0	0.19	0.01	0.30	1.16	4.21
30.0	0.17	0.01	0.28	1.10	4.04
40.0	0.16	0.01	0.27	1.07	3.92
50.0	0.15	0.01	0.27	1.05	3.84
60.0	0.15	0.01	0.26	1.03	3.77
70.0	0.15	0.01	0.26	1.01	3.72
80.0	0.14	0.01	0.26	1.00	3.68
90.0	0.14	0.01	0.26	0.99	3.64
100.0	0.14	0.01	0.25	0.98	3.60

$$C_F=(N^2-1)/2N,\quad C_A=N\ . \tag{A2}$$

We have also carried out some manipulations to express the results in a compact form.

For the diagram (a), the contribution of the scattering operator is expressible as

$$S_a=-i(2\pi)^n\delta(p-p')\bar{\psi}(p')\Sigma(p)\psi(p) \tag{A3}$$

with

$$\Sigma(p)=C_F\frac{ig^2}{(2\pi)^n}\int dl\frac{\gamma_\mu[i(p-l)\cdot\gamma-m]\gamma_\mu}{l^2[(p-l)^2+m^2]}; \tag{A4}$$

while for the diagrams (b') and (b''),

$$S_b=-g(2\pi)^n\delta(k+p-p')G_\mu^i(k)\bar{\psi}(p')\Lambda_\mu^i(p,p',k)\psi(p) \tag{A5}$$

with

$$\Lambda_\mu^i(p,p',k)=\Lambda_\mu'^{\,i}+\Lambda_\mu''^{\,i}\ , \tag{A6}$$

where

$$\Lambda_\mu'^{\,i}=(C_F-\tfrac{1}{2}C_A)T^i\frac{ig^2}{(2\pi)^n}\int dl\frac{\gamma_\nu[i(p'-l)\cdot\gamma-m]\gamma_\mu[i(p-l)\cdot\gamma-m]\gamma_\nu}{l^2[(p'-l)^2+m^2][(p-l)^2+m^2]}\ , \tag{A7}$$

$$\Lambda_\mu''^{\,i}=-\tfrac{1}{2}C_AT^i\frac{g^2}{(2\pi)^n}\int dl\frac{\gamma_\nu[i(p'-l)\cdot\gamma-m]\gamma_\lambda}{l^2(k-l)^2[(p'-l)^2+m^2]}[\delta_{\nu\lambda}(2l-k)_\mu+\delta_{\lambda\mu}(2k-l)_\nu-\delta_{\mu\nu}(l+k)_\lambda]\ . \tag{A8}$$

Similarly, for the diagrams (c′), (c″), and ($\bar{c}$),

$$S_c=-i(2\pi)^n\delta(p+p')G_\mu^i(p')\Pi_{\mu\nu}^{ij}(p)G_\nu^j(p) \tag{A9}$$

with

$$\Pi_{\mu\nu}^{ij}(p)=\Pi_{\mu\nu}'^{ij}+\Pi_{\mu\nu}''^{ij}+\sum_{n_f}\bar{\Pi}_{\mu\nu}^{ij}\ , \tag{A10}$$

where

$$\Pi'^{ij}_{\mu\nu}=\tfrac{1}{2}C_A\delta^{ij}\frac{ig^2}{(2\pi)^n}\int\frac{dl}{l^2(p-l)^2}[\delta_{\alpha\beta}(2l-p)_\mu+\delta_{\beta\mu}(2p-l)_\alpha-\delta_{\mu\alpha}(l+p)_\beta]$$
$$\times[\delta_{\alpha\beta}(2l-p)_\nu+\delta_{\beta\nu}(2p-l)_\alpha-\delta_{\nu\alpha}(l+p)_\beta]\ , \tag{A11}$$

$$\Pi''^{ij}_{\mu\nu}=C_A\delta^{ij}\frac{ig^2}{(2\pi)^n}\int dl\frac{l_\mu(p-l)_\nu}{l^2(p-l)^2}\ , \tag{A12}$$

$$\overline{\Pi}^{ij}_{\mu\nu}=\tfrac{1}{2}\delta^{ij}\frac{ig^2}{(2\pi)^n}\int dl\frac{\mathrm{Tr}\{(il\cdot\gamma-m)\gamma_\mu[i(l+p)\cdot\gamma-m]\gamma_\nu\}}{(l^2+m^2)[(l+p)^2+m^2]}\ ; \tag{A13}$$

and for the diagrams (d′), (d″), (d‴), and ($\overline{\text{d}}$),

$$S_d=-g(2\pi)^n\delta(p+p'+p'')G^i_\mu(p)G^j_\nu(p')G^k_\lambda(p'')\Lambda^{ijk}_{\mu\nu\lambda}(p,p',p'') \tag{A14}$$

with

$$\Lambda^{ijk}_{\mu\nu\lambda}(p,p',p'')=\Lambda'^{ijk}_{\mu\nu\lambda}+\Lambda''^{ijk}_{\mu\nu\lambda}+\Lambda'''^{ijk}_{\mu\nu\lambda}+\sum_{n_f}\overline{\Lambda}^{ijk}_{\mu\nu\lambda}\ , \tag{A15}$$

where

$$\Lambda'^{ijk}_{\mu\nu\lambda}=\tfrac{1}{2}C_Af^{ijk}\frac{ig^2}{(2\pi)^n}\int\frac{dl}{l^2(l-p)^2(l+p'')^2}[\delta_{\mu\alpha}(l+p)_\beta-\delta_{\alpha\beta}(2l-p)_\mu+\delta_{\beta\mu}(l-2p)_\alpha]$$
$$\times[\delta_{\nu\beta}(l-p+p')_\gamma-\delta_{\beta\gamma}(2l-p+p'')_\nu+\delta_{\gamma\nu}(l-p'+p'')_\beta]$$
$$\times[\delta_{\lambda\gamma}(l+2p'')_\alpha-\delta_{\gamma\alpha}(2l+p'')_\lambda+\delta_{\alpha\lambda}(l-p'')_\gamma]\ , \tag{A16}$$

$$\Lambda''^{ijk}_{\mu\nu\lambda}=\tfrac{9}{4}C_Af^{ijk}\frac{ig^2}{(2\pi)^n}\int dl\left[\frac{\delta_{\mu\nu}p_\lambda-\delta_{\lambda\mu}p_\nu}{l^2(l-p)^2}+\frac{\delta_{\nu\lambda}p'_\mu-\delta_{\mu\nu}p'_\lambda}{l^2(l-p')^2}+\frac{\delta_{\lambda\mu}p''_\nu-\delta_{\nu\lambda}p''_\mu}{l^2(l-p'')^2}\right]\ , \tag{A17}$$

$$\Lambda'''^{ijk}_{\mu\nu\lambda}=\tfrac{1}{2}C_Af^{ijk}\frac{ig^2}{(2\pi)^n}\int dl\ \frac{l_\mu(l-p)_\nu(l+p'')_\lambda+(l-p)_\mu(l+p'')_\nu l_\lambda}{l^2(l-p)^2(l+p'')^2}\ , \tag{A18}$$

$$\overline{\Lambda}^{ijk}_{\mu\nu\lambda}=-\tfrac{1}{2}f^{ijk}\frac{g^2}{(2\pi)^n}\int dl\frac{\mathrm{Tr}\{(il\cdot\gamma-m)\gamma_\mu[i(l-p)\cdot\gamma-m]\gamma_\nu[i(l+p'')\cdot\gamma-m]\gamma_\lambda\}}{(l^2+m^2)[(l-p)^2+m^2][(l+p'')^2+m^2]}\ . \tag{A19}$$

Finally, for the diagram (e),

$$S_e=-i(2\pi)^n\delta(p-p')C^{i*}(p')\Pi^{ij}(p)C^j(p) \tag{A20}$$

with

$$\Pi^{ij}(p)=C_A\delta^{ij}\frac{ig^2}{(2\pi)^n}\int dl\ \frac{p\cdot(p-l)}{l^2(p-l)^2}\ ; \tag{A21}$$

and for the diagrams (f′) and (f″),

$$S_f=g(2\pi)^n\delta(k+p-p')G^i_\mu(k)C^{j*}(p')\Lambda^{ijk}_\mu(p,p',k)C^k(p) \tag{A22}$$

with

$$\Lambda^{ijk}_\mu(p,p',k)=\Lambda'^{ijk}_\mu+\Lambda''^{ijk}_\mu\ , \tag{A23}$$

where

$$\Lambda'^{ijk}_\mu=-\tfrac{1}{2}C_Af^{ijk}\frac{ig^2}{(2\pi)^n}\int dl\ \frac{p'\cdot(p-l)(p'-l)_\mu}{l^2(p-l)^2(p'-l)^2}\ , \tag{A24}$$

$$\Lambda_\mu''^{ijk} = -\tfrac{1}{2}C_A f^{ijk}\frac{ig^2}{(2\pi)^n}\int dl \frac{p'_\nu(p'-l)_\lambda[\delta_{\nu\lambda}(2l-k)_\mu+\delta_{\lambda\mu}(2k-l)_\nu-\delta_{\mu\nu}(l+k)_\lambda]}{l^2(k-l)^2(p'-l)^2} . \tag{A25}$$

These results were used in Sec. III for the evaluation of renormalization constants.

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[12] We express the result for the three-gluon vertex in the form (3.11) so that when $\Lambda^{ijk}_{\mu\nu\lambda}(p,-p,0)$ is multiplied with $G^i_\mu(p)G^j_\nu(-p)\ G^k_\lambda(0)$, the terms excluded from renormalization appear with factors $p_\mu G^i_\mu(p)$ and $-p_\nu G^j_\nu(-p)$.

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[14] For massless quarks, the relation between α_s and $\bar{\alpha}_s$ can be obtained from (3.26) by setting $n_h=0$. We note that this relation for massless quarks differs somewhat from the corresponding relation obtained by Braaten and Leveille in Ref. 5. The difference arises from the fact that we have decomposed the quark-gluon vertex by following an analogy with quantum electrodynamics, while in Ref. 5 another procedure has been followed for the decomposition of this vertex. Further, our result can be compared with that of Celmaster and Gonsalves in Ref. 3 by taking, for instance, $n_l=4$ and $n_h=0$. Then, (3.26) gives, for $N=3$,

$$\alpha_s=\bar{\alpha}_s[1+3.0(\bar{\alpha}_s/\pi)] ,$$

which shows that α_s is practically identical with $\alpha_{\rm MOM}$ for massless quarks.