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Abelian dominance and quark confinement in Yang-Mills theories

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We define the Abelian component of the gauge field as a gauge-invariant object in the Yang-Mills theory. Then, by assuming that the Abelian component dominates in the theory at a long-distance scale, we demonstrate that quarks as well as gluons are confined by electric vortices. The vacuum structure is shown to depend on resolution R. That is, the vacuum has two phases in R and monopole condensation occurs for $R \geq R_c$. The string tension of mesons is obtained as $\sigma_q = g_c{}^2/3\pi R_c{}^2$, where g_c is the effective coupling constant at the critical resolution R_c. We estimate $R_c{}^{-1}$ to be 0.6 GeV in the presence of static quarks, and the bag constant $B^{1/4}$ to be 0.2–0.4 GeV. We also derive a relation $\alpha_g' = \frac{1}{3}\alpha'$ between Regge slopes of mesons (α') and of gluonia (α_g'). This relation agrees with experimental data remarkably well provided that gluonia are identified with states lying on the Pomeranchuk trajectory.

I. INTRODUCTION

The SU(N) Yang-Mills theory contains magnetic monopoles as topological excitations in space-time, which are labeled by the magnetic root lattice of SU(N).[1] They are Wu-Yang monopoles in SU(2). It has been believed that the confinement of quarks will be realized in a condensed phase of these monopoles.[2] If this is indeed the case, the Abelian component of the theory must be important in analyzing the problem of confinement. This is so because classical configurations of these monopoles are constructed within the Cartan subalgebra of SU(N),[1,3,4] which is the maximal Abelian subalgebra of SU(N). Then, it would be reasonable to speculate that the Abelian component dominates in the Yang-Mills theory at a long-distance scale. We call this the hypothesis of Abelian dominance. In this paper, it is demonstrated that gluons as well as quarks are confined by electric vortices on the basis of this hypothesis. Our major concern is to obtain quantitative results with respect to the structure of hadrons, though yet very rough, within the Yang-Mills theory.

In previous papers,[5] we have analyzed a similar problem in the SU(N) Higgs model, where the Abelian component is extracted by way of a spontaneous symmetry breakdown. However, this method is not applicable to the Yang-Mills theory. It is necessary to define the Abelian component in a gauge-invariant manner. We are able to do this as follows. Suppose that field $X(x)$ takes values in the Lie algebra of SU(N) and transforms according to the adjoint representation:

$$X(x) \to U(x)X(x)U(x)^{-1} . \tag{1.1}$$

We now diagonalize $X(x)$ by

$$X(x) = T(x)\hat{X}(x)T(x)^{-1}, \quad T \in \mathrm{SU}(N) , \tag{1.2}$$

which is always possible. Here, the elements of the diagonal field $\hat{X}(x)$ are gauge invariant since they are eigenvalues of $X(x)$. We note that the gauge transformation (1.1) is generated by the left shift operation of $T(x)$ such that

$$T(x) \to U(x)T(x) , \tag{1.3}$$

and that the field $\hat{X}(x)$ is determined up to an element of the Weyl group of SU(N); under the action of the Weyl group the diagonal elements of $\hat{X}(x)$ permute among themselves. We call $\hat{X}(x)$ and $T(x)$ the Abelian component and the non-Abelian component of $X(x)$, respectively, since $\hat{X}(x)$ takes values in the Cartan subalgebra of SU(N). The hypothesis of Abelian dominance implies that the non-Abelian field $T(x)$ does not propagate at a long-distance scale and hence that only the Abelian component is relevant at a long-distance scale. We emphasize that this hypothesis is a gauge-invariant concept.

Once Abelian dominance is postulated, the mechanism of quark confinement is essentially the same as in the SU(N) Higgs model.[5] By applying the technique developed therein,[5] we integrate over all configurations of magnetic monopoles and derive an effective Lagrangian of the Yang-Mills theory. It should be noted that various parameters in the effective Lagrangian depend on resolution R; R could be the distance between two quarks. The effective Lagrangian describes the Yang-Mills vacuum for $R \geq R_c$, R_c being a critical resolution beyond which monopole condensation takes place.

Based on the effective Lagrangian, we show that the Yang-Mills vacuum is a magnetic superconductor in which electric flux is squeezed into vortices. Then, a relation

$$\alpha_g'/\alpha' = (N-1)/2N \tag{1.4}$$

is derived between Regge slopes of mesons (α') and of gluonia (α_g'). In the case of SU(3), this relation gives rise to

$$\alpha_g' = \tfrac{1}{3}\alpha' \approx 0.3 \text{ GeV}^{-2} , \tag{1.5}$$

where $\alpha' = 0.9$ GeV^{-2} has been used. Although there are other predictions on this relation,[6-8] we emphasize that our prediction gives the best fit for the observed data,[9] provided that gluonia may be identified with states lying on the Pomeranchuk trajectory.

In our scheme, the resolution R is a crucial length parameter. If R is small, the perturbative picture is valid. As R increases, the perturbative picture gradually becomes dubious. Then, we speculate that the Abelian component would dominate. When R reaches at a certain critical distance R_c, the condensation of monopoles occurs. Finally, for $R > R_c$, electric vortices emerge as stable topological excitations and confine quarks. We may rephrase such a situation in terms of the bag picture with R being the separation between two quarks. For $R << R_c$, quarks can move freely in a bag. However, at $R \approx R_c$, the bag itself begins to deform. When quarks are separated sufficiently ($R > R_c$), the bag is deformed into a string, which is an electric vortex in a magnetic superconductor.

In order to determine various mass parameters at the critical point numerically, it is enough just to use one experimental datum as an input, that is, the Regge slope of mesons, $\alpha' = 0.9$ GeV^{-2}. Then, we calculate the critical resolution $R_c \approx$ (0.6 GeV)$^{-1}$, which would measure the size of a bag in the presence of static quarks. We also estimate the width of the boundary of the bag to be 0.1 – 0.2 fm, and the bag constant $B^{1/4}$ to be 0.2 – 0.4 GeV.

This paper is composed as follows. In Sec. II, we review magnetic monopoles in the Yang-Mills theory and then we discuss the hypothesis of Abelian dominance. In Sec. III, an effective Lagrangian of the Yang-Mills theory is derived by integrating over all configurations of monopole excitations on the basis of the hypothesis of Abelian dominance. In Sec. IV, we present some numerical values which account for the structure of hadrons.

II. HYPOTHESIS OF ABELIAN DOMINANCE

We start with a brief review on magnetic monopoles in the SU(N) Yang-Mills theory, where we are able to construct monopole configurations without violating the Bianchi identity:

$$D_\mu F^*_{\mu\nu} = 0 . \tag{2.1}$$

In so doing, it is most convenient to consider Abelian gauge potentials A^H, $H = 1, \ldots, N-1$, which describe Dirac monopoles.[10] Then, as is well known,[11] fields $F^H_{\mu\nu}$ defined by

$$F^H_{\mu\nu} = \partial_\mu A^H_\nu - \partial_\nu A^H_\mu + \rho^H_{\mu\nu} \tag{2.2}$$

satisfy

$$\partial_\mu F^{H*}_{\mu\nu} = k^H_\nu , \tag{2.3}$$

and represent Abelian electromagnetic fields around Dirac monopoles. Here, k^H_μ and $\rho^H_{\mu\nu}$ denote Dirac monopoles and Dirac strings, respectively,

$$\begin{aligned} k^H_\mu(x) &= 4\pi \sum_q \eta^H_q \int d\tau\, \delta^{(4)}(x - z^q)\dot{z}^q_\mu , \\ \rho^{H*}_{\mu\nu}(x) &= 4\pi \sum_q \eta^H_q \int d^2\tau\, \delta^{(4)}(x - z^q)\, \frac{\partial(z^q_\mu, z^q_\nu)}{\partial(\tau_1, \tau_2)} , \end{aligned} \tag{2.4}$$

with η^H_q being the magnetic charges of the qth monopole. It is to be remarked that in the Dirac theory the gauge potential A^H_μ contains string singularities which must be explicitly subtracted out to obtain the electromagnetic field $F^H_{\mu\nu}$ as in (2.2).[10,11] We now embed these Dirac monopoles in the Yang-Mills theory. For this purpose, we take the diagonal Gell-Mann matrices λ^H,

$$\lambda^H = \left[\frac{2}{H(H+1)} \right]^{1/2} \times \mathrm{diag}(1, \ldots, 1, -H, 0, \ldots, 0) , \tag{2.5}$$

and set

$$\begin{aligned} \hat{A}_\mu &= \sum_{H=1}^{N-1} A^H_\mu \lambda^H/2, \quad \hat{F}_{\mu\nu} = \sum_{H=1}^{N-1} F^H_{\mu\nu}\lambda^H/2 , \\ \hat{k}_\mu &= \sum_{H=1}^{N-1} k^H_\mu \lambda^H/2, \quad \hat{\rho}_{\mu\nu} = \sum_{H=1}^{N-1} \rho^H_{\mu\nu}\lambda^H/2 . \end{aligned} \tag{2.6}$$

Then, it follows that

$$\partial_\mu \hat{F}^*_{\mu\nu} = \hat{k}_\nu \ . \tag{2.7}$$

It has been proved[1,3,5] that, provided that the magnetic charge vectors $\vec{\eta} = (\eta^1, \ldots, \eta^{N-1})$ are on the root lattice of SU(N), there is a singular gauge transformation S such that

$$\hat{\rho}_{\mu\nu} = iS^{-1}[\partial_\mu, \partial_\nu]S \ . \tag{2.8}$$

Therefore, when we define

$$A_\mu = S\hat{A}_\mu S^{-1} + i(\partial_\mu S)S^{-1} \ ,$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu] \ , \tag{2.9}$$

we obtain

$$F_{\mu\nu} = S\hat{F}_{\mu\nu}S^{-1} \ . \tag{2.10}$$

It can be seen that A_μ and $F_{\mu\nu}$ are free from string singularities. Moreover, it is straightforward to show that field configurations (2.9) do satisfy the Bianchi identity (2.1). We are able to construct all possible monopole solutions in this way: These configurations have point singularities due to the absence of the Higgs fields.

We note that the singular gauge transformation S is determined up to a regular gauge transformation and that the ambiguity in choosing a matrix S in formula (2.8) is entirely attributed to the gauge degrees of freedom. That is, under the left shift operation of S such that

$$S \to US \ , \tag{2.11}$$

formula (2.8) is invariant, but A_μ and $F_{\mu\nu}$ defined by formula (2.9) undergo a regular gauge transformation:

$$A_\mu \to UA_\mu U^{-1} + i(\partial_\mu U)U^{-1} \ ,$$
$$F_{\mu\nu} \to UF_{\mu\nu}U^{-1} \ . \tag{2.12}$$

It should be emphasized that the element $F^H_{\mu\nu}$ of the Abelian field $\hat{F}_{\mu\nu}$ is an SU(N)-invariant quantity. The reason reads as follows. Suppose that a monopole configuration $F_{\mu\nu}$ is given by solving the Yang-Mills field equations. Then, the Abelian component $\hat{F}_{\mu\nu}$ is obtained by *diagonalizing* $F_{\mu\nu}$. Thus, $\hat{F}_{\mu\nu}$ consists of eigenvalues of $F_{\mu\nu}$, which are SU(N)-invariant quantities.

As we have reviewed, magnetic monopoles are constructed within the Cartan subalgebra of SU(N). It is our common belief that these magnetic monopoles are the essential agents that lead to quark confinement. Hence, we are led to a speculation that the Abelian component would be the dominant part of the theory at a long-distance scale. We call this the hypothesis of Abelian dominance.

Let us describe the hypothesis of Abelian dominance a bit more in detail. We first discuss the problem of extracting the Abelian component from the field strength $F_{\mu\nu}$. In general, $F_{\mu\nu}$ is a very complicated object and it is impossible to choose the Abelian component as in formula (2.10). However, we can define the Abelian component by diagonalizing $F_{\mu\nu}$ as follows:

$$F_{\mu\nu} = T_{\mu\nu}\hat{F}_{\mu\nu}T_{\mu\nu}{}^{-1}, \quad T_{\mu\nu} \in \mathrm{SU}(N) \ . \tag{2.13}$$

Here, $T_{\mu\nu}$ is a gauge-variant quantity, while the elements of $\hat{F}_{\mu\nu}$ are gauge invariant since they are eigenvalues of $F_{\mu\nu}$. Note that $\hat{F}_{\mu\nu}$ takes values in the Cartan subalgebra of SU(N). When we quantize the system in the functional formalism it is possible to obtain the effective theory of the Abelian component $\hat{F}_{\mu\nu}$ by integrating out the non-Abelian component $T_{\mu\nu}$. Obviously, the effective Lagrangian must be invariant under the action of the Weyl group of SU(N). A comment is in order. The decomposition (2.13) may not be useful in a formal argument because $T_{\mu\nu}$ and $\hat{F}_{\mu\nu}$ are not Lorentz covariant in general. There would be other choices of fields by use of which the Abelian component becomes Lorentz covariant. We do not attempt to analyze the problem in this paper. We only emphasize that it is possible to extract the Abelian component as a gauge-invariant quantity in the Yang-Mills theory.

We continue to discuss the hypothesis of Abelian dominance. We are mainly interested in the effective Abelian theory at a long-distance scale. Our basic assumption is that the non-Abelian component does not contribute to the effective Lagrangian at a long-distance scale; the only effect of the non-Abelian component is to smear out the short-distance behaviors of the theory inclusive of the point singularities of magnetic monopoles. This is the hypothesis of Abelian dominance. The Abelian dominance would be achieved if the dynamics make the non-Abelian component "heavier" than the Abelian component so that the non-Abelian component does not propagate at a long-distance scale. Moreover, in such a situation the Abelian component $\hat{F}_{\mu\nu}$ would be effectively a covariant tensor. The reason is as follows. Since quantum fluctuations of low-momentum components in the non-Abelian field $T_{\mu\nu}$ are very small by assumption, it would be possible to fix the direction of $T_{\mu\nu}$ arbitrarily in the group space of SU(N) such that $T_{\mu\nu}=S$ as far as long-range correlations of gauge-invariant quantities are concerned. We shall not try to prove the Abelian dominance in this paper. Let us simply adopt this as a working hypothesis and examine whether the results may account for the structure of hadrons.

In the following arguments, weight vectors and root vectors of SU(N) play important roles. For the reader's convenience, we list the minimal formulas that we need in the rest of the paper. The easiest way of constructing these vectors is to use the diagonal Gell-Mann matrices (2.5).[5] Then, elementary weight vectors $\vec{\epsilon}_j=(\epsilon_j^1, \ldots, \epsilon_j^{N-1})$ are defined by setting $\epsilon_j^H=(\lambda^H/2)_{jj}$. There are N such vectors, among which $N-1$ vectors are independent. They characterize a quantity which transforms according to the fundamental representation of SU(N). Now, weight vectors are constructed by

$$\vec{\epsilon}=\sum_{j=1}^{N-1} n_j\vec{\epsilon}_j \tag{2.14}$$

with n_j being integers. The set of all these vectors constitutes the weight lattice of SU(N). On the other hand, the elementary root vectors $\vec{\eta}_{ij}$ are defined by

$$\vec{\eta}_{ij}=\vec{\epsilon}_i-\vec{\epsilon}_j \; . \tag{2.15}$$

There are $N(N-1)$ nontrivial vectors, among which $N-1$ vectors are independent. They characterize a quantity which transform according to the adjoint representation of SU(N). Then, root vectors are constructed by

$$\vec{\eta}=\sum_{j=1}^{N-1} m_j\vec{\eta}_{jN} \tag{2.16}$$

with m_j being integers, the set of which constitutes the root lattice of SU(N). Obviously, the root lattice is a sublattice of the weight lattice. Note that

$$\begin{aligned} &\vec{\epsilon}_j^{\,2}=(N-1)/2N \quad (\text{ for each } j)\;, \\ &\vec{\eta}_{ij}^{\,2}=1 \quad (\text{ for each } i,j;\; i\neq j)\;, \end{aligned} \tag{2.17}$$

which we shall use when we discuss string tensions of mesons and gluonia.

In this section, we have argued that excitations of magnetic monopoles in the Yang-Mills theory are taken in the Abelian component of the gauge field $F_{\mu\nu}$. We have emphasized that the Abelian component consists of eigenvalues of $F_{\mu\nu}$ which are SU(N)-invariant quantities.

III. EFFECTIVE LAGRANGIAN

In the previous section, by defining the Abelian component of the gauge field, we have proposed a hypothesis that the Abelian component will be the dominant part of the theory at a long-distance scale. In this section, we shall explicitly use this hypothesis to analyze the Wilson loop in order to show quark confinement. Thus, we consider

$$\langle W(C)\rangle = Z^{-1}\int\prod_p [dF_{\mu\nu}(p)]\delta(D_\mu F^*_{\mu\nu})W(C,F_{\mu\nu})\exp\left[-\frac{1}{2g_0{}^2}\int \mathrm{Tr}F_{\mu\nu}{}^2\right] , \tag{3.1}$$

where we have adopted the field strength formulation.[12] Here g_0 and Z stand for the bare coupling constant and the normalization constant. We wish to derive the effective Lagrangian of the Yang-Mills theory at resolution R, R being the distance between a quark and an antiquark. For this purpose we integrate out all the field variables with the momentum components $p\geq R^{-1}$. In the process of integrations, we need to take into account of monopole configurations as well.

It is quite complicated to carry out explicitly the above integrations, which we wish to study in a future paper. In this paper, for the sake of simplicity, we make a working hypothesis of Abelian dominance. That is, at a resolution R which is beyond a certain scale, we assume that the Abelian component $\hat{F}_{\mu\nu}$ becomes dominant. Then, in the presence of monopole excitations, the Abelian field $\hat{F}_{\mu\nu}$ would have a monopole source effectively as in (2.7). Hence, we assume that (3.1) is reduced to

$$\langle W(C)\rangle = Z'^{-1}\sum\!\!\!\!\!\!\int_k \int\prod_p^{R^{-1}} [d\hat{F}_{\mu\nu}(p)]\delta(\partial_\mu\hat{F}^*_{\mu\nu}-\hat{k}_\nu)\delta(\partial_\mu\hat{k}_\mu)W(C,\hat{F}_{\mu\nu})\exp\left[-\frac{1}{2g^2(\Lambda R)}\int \mathrm{Tr}\hat{F}_{\mu\nu}{}^2\right] , \tag{3.2}$$

with Λ being a renormalization mass parameter. Here $\sum\!\!\!\!\!\!\int_k$ indicates the integration measure for collective coordinates of magnetic monopoles parametrized by (2.4), and all fields take values in the Cartan subalgebra of SU(N). The essential point of the hypothesis is that all the non-Abelian effects are summarized by magnetic monopole excitations and effective coupling constant $g(\Lambda R)$ at resolution R.

We proceed to evaluate the Wilson loop (3.2). First, we note that, when the gauge field is taken in the Cartan subalgebra, the fundamental representation (quark) Wilson loop operator reads

$$W(C,\hat{F}_{\mu\nu})=\mathrm{Tr}\exp\left[\frac{i}{2}\sum_{H=1}^{N-1}\int J_{\mu\nu}F_{\mu\nu}^{H}\lambda^{H}/2\right]=\sum_{i=1}^{N}\prod_{H=1}^{N-1}\exp\left[\frac{i}{2}\int \epsilon_i^H F_{\mu\nu}^H J_{\mu\nu}\right], \tag{3.3}$$

where $\epsilon_i=(\epsilon_i^1,\ldots,\epsilon_i^{N-1})$ are the elementary weight vectors of SU(N), and $J_{\mu\nu}$ parametrizes a surface whose boundary is given by the loop C;

$$J_{\mu\nu}(x)=\int d^2\tau\,\delta^{(4)}(x-z)\frac{\partial(z_\mu,z_\nu)}{\partial(\tau_1,\tau_2)} . \tag{3.4}$$

Similarly, the adjoint representation (gluon) Wilson loop operator reads

$$W(C,\hat{F}_{\mu\nu})=\sum_{i\neq j}\prod_{H=1}^{N-1}\exp\left[\frac{i}{2}\int \eta_{ij}^H F_{\mu\nu}^H J_{\mu\nu}\right], \tag{3.5}$$

where $\eta_{ij}=(\eta_{ij}^1,\ldots,\eta_{ij}^{N-1})$ are the nontrivial elementary root vectors of SU(N): Note that the trivial elementary root vector does not contribute to the Wilson loop. Next, we introduce dual potentials B_μ^H by

$$\delta(\partial_\mu F_{\mu\nu}^{H*}-k_\nu^H)=\int[dB_\nu^H]\exp\left[i\int B_\nu^H(\partial_\mu F_{\mu\nu}^{H*}-k_\nu^H)\right] \tag{3.6}$$

and dual Goldstone fields χ^H by

$$\delta(\partial_\mu k_\mu^H)=\int[d\chi^H]\exp\left[i\int \chi^H\partial_\mu k_\mu^H\right] . \tag{3.7}$$

Inserting (3.3), (3.6), and (3.7) into (3.2), and integrating over $\hat{F}_{\mu\nu}^H$, we obtain

$$\langle W(C)\rangle=Z'^{-1}\sum_{i=1}^{N}\sum\!\!\!\!\!\!\int_k\prod_{H=1}^{N-1}\int[dB_\mu^H][d\chi^H]\times\exp\left[-\frac{g^2}{4}\int(G_{\mu\nu}^H-\epsilon_i^H J_{\mu\nu}^*)^2+i\int k_\mu^H(B_\mu^H-\partial_\mu\chi^H)-\int\mathcal{M}\right] \tag{3.8}$$

with $G_{\mu\nu}^H=\partial_\mu B_\nu^H-\partial_\nu B_\mu^H$, where we have extracted the self-energies of magnetic monopoles explicitly as

$$\mathcal{M}(x)=\frac{\pi^2R^{-1}}{g(\Lambda R)^2}\sum_q\vec{\eta}_q^{\,2}\int d\tau\,\delta^{(4)}(x-z_q)\times\left[\dot{z}_\mu^q\dot{z}_\mu^q\right]^{1/2}, \tag{3.9}$$

since they act as the chemical potentials for the excitations of these monopoles.[5] We shall derive formula (3.9) in the next paragraph. When we use (3.5) instead of (3.3), we obtain (3.8), but with ϵ_i^H replaced by η_{ij}^H.

In order to derive the self-energies of magnetic monopoles, we integrate over B_μ^H in formula (3.8). Then, we obtain an interacting system of magnetic monopoles whose total energy is given by

$$\mathcal{E}=\frac{1}{2g^2}\sum_{H=1}^{N-1}\int d^4x\,d^4y\,k_\mu^H(x)\times\Delta_{\mu\nu}(x-y)k_\nu^H(y) , \tag{3.10}$$

where $\Delta_{\mu\nu}(x-y)$ is the massless propagator. Let us consider a static monopole sitting at the origin, i.e.,

$$k_\mu^H(x)=4\pi\eta^H\delta^{(3)}(x)\delta_{\mu 0} . \tag{3.11}$$

In this case, $\mathcal{E}$ is the self-energy of the monopole and hence $\mathcal{E}=\int\mathcal{M}(x)$ with

$$\mathcal{M}(x)=\frac{\pi^2R^{-1}}{g^2}\vec{\eta}^2\delta^{(3)}(x)\ , \tag{3.12}$$

where we have considered only the contribution from the momentum range $R^{-1}\geq p\geq 0$, since it is understood that the momentum range $p>R^{-1}$ has already been integrated out in the formula (3.2). In general, it is straightforward to derive the self-energy term (3.9) from the total energy (3.10). Note that the mass scale of monopoles depends only on resolution R, which implies that monopoles have no intrinsic scales in the Yang-Mills theory. In this sense, they are essentially different from 't Hooft—Polyakov monopoles in the Georgi-Glashow model.

We proceed to integrate over all possible configurations of magnetic monopoles in formula (3.8). We have described a technique for this calculation in detail in previous papers,[5] where we have already anlayzed (3.8). Therefore, we only cite the result. By making use of Poisson resummation formulas, we may change the summation over magnetic monopoles ($\sum\!\!\!\!\!\!\int_k$) labeled by the root lattice of SU(N) into the summation ($\sum\!\!\!\!\!\!\int_\sigma$) over electric vortices labeled by the weight lattice of SU(N). Thus (3.8) is rewritten as

$$\langle W(C)\rangle=Z'^{-1}\sum_{i=1}^{N}\sum\!\!\!\!\!\!\int_{\sigma}\prod_{H=1}^{N-1}\int[dB_\mu^H][d\chi^H]$$
$$\times\exp\left\{-\frac{g^2}{4}\int\left[(G_{\mu\nu}^H-\sigma_{\mu\nu}^{H*}+\epsilon_i^HJ_{\mu\nu}^*)^2+2m^2(B_\mu^H-\partial_\mu\chi^H)^2\right]\right\}, \tag{3.13}$$

where $m^2=8R^{-2}$ (note that we have taken the resolution R as the lattice spacing which defines the integration measure of magnetic monopole excitations[5]), while

$$\sigma_{\mu\nu}^H(x)=\sum_q\epsilon_q^H\int d^2\tau\,\delta^{(4)}(x-z^q)\frac{\partial(z_\mu^q,z_\nu^q)}{\partial(\tau_1,\tau_2)} \tag{3.14}$$

with $\partial_\mu\sigma_{\mu\nu}^H=0$. We may interpret $\sigma_{\mu\nu}^H$ to be electric vortex strings which are closed upon themselves. These strings describe excitations of electric vortex loops labeled by the weight lattice of SU(N), which are stable topological excitations at a long-distance scale $R>R_c$, R_c being a critical length beyond which the monopole condensation occurs.

We wish to remark upon the following observations. We have evaluated the Wilson loop by postulating the Abelian dominance. We could interpret that formula (3.13) has been obtained as a result of the integration over the non-Abelian component. Thus, the formula describes an effective Abelian theory which is gauge invariant. The Abelian theory must be invariant under the action of the Weyl group of SU(N), W(SU(N)), as we have noticed in the previous section. It is trivial to see that the formula (3.13) possesses this symmetry group; the Weyl group denotes the invariance under the permutations of N elementary weight vectors $\vec{\epsilon}_i$ therein. It is notable that besides this symmetry group the formula contains the maximal torus of SU(N), T(SU(N)), as a local gauge symmetry. Thus, the total symmetry group of the effective Abelian theory is given by W(SU(N)) $\otimes\, T$(SU(N)). Bearing these remarks in mind, we extract the effective Lagrangians from formula (3.13) as

$$L=\frac{g^2}{4}[\,(\vec{\mathrm{G}}_{\mu\nu}+\vec{\epsilon}J_{\mu\nu}^*)^2+2m^2(\vec{\mathrm{B}}_\mu-\partial_\mu\vec{\chi})^2] \tag{3.15}$$

in the presence of external quarks, where $\vec{\epsilon}$ denotes generically the elementary weight vector of SU(N), while

$$L=\frac{g^2}{4}[(\vec{\mathrm{G}}_{\mu\nu}+\vec{\eta}J_{\mu\nu}^*)^2+2m^2(\vec{\mathrm{B}}_\mu-\partial_\mu\vec{\chi})^2] \tag{3.15$'$}$$

in the presence of external gluons, where $\vec{\eta}$ denotes generically the nontrivial elementary root vector of SU(N). It should be emphasized that we may use these effective Lagrangians only to calculate gauge-invariant quantities.

We have analyzed the Wilson loops in this section. It is concluded from the effective Lagrangians that gluons as well as quarks are confined by an electric vortex.

IV. HADRONIC STRUCTURE

In the previous section, we have derived the effective Lagrangian of the Abelian component of the Yang-Mills theory by integrating over all configurations of magnetic monopoles. We have shown that the effective Lagrangian describes a magnetic superconductor, where $\epsilon_{ijk}\vec{G}_{ij}$ stands for the electric field.

In our scheme there are several mass scales, such as monopole mass M and B_μ-field mass m:

$$M=\frac{\pi^2}{Rg^2(R)}, \quad m^2=\frac{8}{R^2} . \tag{4.1}$$

These masses are not physical quantities, since they are dependent on resolution R. We may interpret M^{-1} as the coherent length and m^{-1} as the penetration depth of electric field in the magnetic superconductor.

First, let us argue how monopole condensation occurs as a function of resolution R. We assume that the effective coupling constant $g(R)$ is a monotonically increasing function as we expect from perturbative calculations. Then, when $R\to 0$, the monopole mass $M(R)$ is very large, as is obvious in (4.1), and hence there are scarcely any monopole excitations. On the other hand, when $R\to\infty$, the monopole mass is zero, and hence the vacuum will be dominated by monopole excitations. There must be a critical resolution R_c at which the phase transition takes place. This critical point is determined by[13]

$$R_c M_c=\ln(2d) , \tag{4.2}$$

where $M_c=M(R_c)$ and d is the dimension of the space-time; here $d=4$. Thus, monopole excitations are incoherent at short distance ($R<R_c$), while they become coherent at large distance ($R>R_c$).

It is obvious from the effective Lagrangian (3.15) that electric vortices emerge between external charges whose string tensions are given by[14]

$$\sigma_q=\frac{\vec{\epsilon}^{\,2}}{8\pi}g^2(R)m^2(R) \text{ for quarks }, \quad \sigma_g=\frac{\vec{\eta}^{\,2}}{8\pi}g^2(R)m^2(R) \text{ for gluons }, \tag{4.3}$$

where $\vec{\epsilon}$ and $\vec{\eta}$ are the elementary weight and root vectors of SU(N). Recall that $\vec{\epsilon}^{\,2}$ and $\vec{\eta}^{\,2}$ are explicitly given by (2.17). It is remarkable that the ratio σ_q/σ_g is determined without knowing details of the theory. Thus, by using the string-model connection between string tensions and Regge slopes, we obtain

$$\alpha_g'=\frac{N-1}{2N}\alpha' , \tag{4.4}$$

where α' and α_g' stand for the Regge slopes of mesons and gluonia, respectively. Here and hereafter, we only consider the case of SU(3) explicitly. Then, this relation gives rise to

$$\alpha_g'=\tfrac{1}{3}\alpha'=0.3 \text{ GeV}^{-2} , \tag{4.5}$$

where $\alpha'=0.9$ GeV^{-2} has been used.

We emphasize that this ratio has been derived on the basis of Abelian dominance. Let us cite some other predictions on this relation proposed in literature. First, a naive consideration of the Casimir operators in the adjoint and the fundamental representations gives rise to[6]

$$\alpha_g'=\tfrac{4}{9}\alpha'\approx 0.4 \text{ GeV}^{-2} . \tag{4.6}$$

Second, a bag model gives[7]

$$\alpha_g'=\tfrac{2}{3}\alpha'\approx 0.6 \text{ GeV}^{-2} . \tag{4.7}$$

Third, the dual resonance model gives[8]

$$\alpha_g'=\tfrac{1}{2}\alpha'\approx 0.45 \text{ GeV}^{-2} , \tag{4.8}$$

provided that gluonia are identified with states lying on the Pomeranchuk trajectory. On the other hand, the slope of the Pomeron has been observed experimentally as[9]

$$\alpha_P=0.30-0.33 \text{ GeV}^{-2} . \tag{4.9}$$

Therefore, when gluonia and the Pomeron are identified, our prediction (4.5) agrees remarkably well with experimental data.

Let us recapitulate the underlying physics in the Yang-Mills theory. The physics depends crucially upon resolution R with which we are concerned; R could be the distance between two quarks. If R is small, the perturbative picture is valid. As R increases, the perturbative picture becomes gradually dubious. Then, we have speculated that the Abelian component would dominate. When R reaches a certain critical distance R_c, the condensation of monopoles occurs. Finally for $R>R_c$, electric vortices emerge as stable topological excitations and confine quarks. We may rephrase such a situation in terms of the bag picture. For $R<<R_c$, quarks can move freely in a bag. However, at $R\approx R_c$ the bag itself begins to deform. When quarks are separated sufficiently ($R>R_c$), the bag is deformed into a string, which is an electric vortex in a magnetic superconductor.

We go on to examine whether some quantitative results account for the structure of hadrons. Making use of (4.1) and (4.3), we are able to represent various mass scales in terms of the effective coupling constant,

$$\sigma_q = \frac{g^2(\Lambda R)}{3\pi R^2}\ ,$$

$$M(R) = \frac{\pi^2}{R g^2(\Lambda R)}\ , \qquad (4.10)$$

$$m^2(R) = \frac{24\pi\sigma_q}{g^2(\Lambda R)}\ .$$

From (4.2) and (4.10), we first derive that

$$g^2(\Lambda R_c)/4\pi = \pi/4\ln 8 \approx 0.37\ , \qquad (4.11)$$

which fixes the critical resolution R_c in terms of the renormalization point Λ if the effective coupling constant $g(\Lambda R)$ is known. In this case, we are able to calculate numerically σ_q, $M_c = M(R_c)$, and $m_c = m(R_c)$ from (4.10).

Actually, we do not know the functional form of $g(\Lambda R)$, which must be determined in the process of proving the hypothesis of Abelian dominance [see (3.2)]. Thus, it is more convenient to fix the scale of the theory by giving the Regge slope of mesons, that is, $\alpha' = (2\pi\sigma_q)^{-1} = 0.9\ \mathrm{GeV}^{-2}$ as an input. Then, we may easily obtain from (4.10) that

$$R_c^{-1} \approx 0.6\ \mathrm{GeV},\quad M_c \approx 1.2\ \mathrm{GeV},$$
$$m_c \approx 1.6\ \mathrm{GeV}\ . \qquad (4.12)$$

These critical values are physical quantities. We may consider that R_c^{-1} gives the size of a bag in the presence of static quarks. It is quite plausible that the actual size of a hadronic bag will be enlarged to, e.g., $R_c^{-1} \sim 0.2$ GeV in the presence of dynamical quarks. On the other hand, the skin widths of the bag are given by the coherent length ($M_c^{-1} \approx 0.17$ fm) and the penetration depth ($m_c^{-1} \approx 0.13$ fm).

It is remarkable that the effective coupling constant (4.11) is quite small at the critical point. It would be tempting to assume the well-known one-loop formula

$$g^2(\Lambda R)/4\pi = 12\pi/[33\ln(\Lambda R)^{-2}] \qquad (4.13)$$

as an approximation to $g(\Lambda R)$. Then, we may calculate from (4.11)–(4.13) that $\Lambda \approx 0.5$ GeV, which is consistent with the experimental data of Λ obtained in the analysis of perturbative QCD.

Finally we estimate the bag constant $B^{1/4}$. We have shown that the Yang-Mills vacuum is a magnetic superconductor for $R \geq R_c$. Let us write down an effective Landau-Ginzburg Lagrangian:

$$L = \sum_{H=1}^{N-1} \left[\tfrac{1}{4}(G'^{H}_{\mu\nu})^2 + \left| \left[\partial_\mu + i\frac{4\pi}{g} B'^{H}_{\mu} \right] \phi^H \right|^2 + \lambda(|\phi^H|^2 - v^2)^2 \right]\ , \qquad (4.14)$$

which is equivalent to the effective Lagrangian (3.15) if we set $B'^{H}_{\mu} = gB^{H}_{\mu}$ and $\phi^H = v\exp(-i4\pi\chi^H)$, where $v^2 = g^2 m^2/32\pi^2$. Now, bag constant $B^{1/4}$ is given by

$$B = (N-1)\lambda v^4 = N\sigma_q m_S{}^2/8\pi \qquad (4.15)$$

with $m_S = 2v\sqrt{\lambda}$ being the mass of the ϕ field. It would be justified to set either $m_S = R_c^{-1}$ or $m_S = M_c$. Then, in the case of SU(3), we obtain $B^{1/4} = 0.2-0.4$ GeV corresponding to $m_s = 0.5-1$ GeV at the critical point.

We wish to emphasize that all these numerical values have been derived on the basis of the hypothesis of Abelian dominance. We believe that they agree reasonably well with our expectation for the structure of hadrons.

V. DISCUSSION

In this paper we have analyzed the Wilson loop in the Yang-Mills theory on the basis of the hypothesis of Abelian dominance. We have emphasized that the hypothesis is a gauge-invariant concept. By integrating over all configurations of magnetic monopole excitations, we have derived an effective Lagrangian. It is to be remarked that various parameters in our scheme are dependent on resolution R; R could be the distance between two quarks. We have argued that monopole excitations are incoherent at short distance ($R < R_c$), while they are coherent at long distance ($R > R_c$) with R_c being the critical resolution. The effective Lagrangian describes a magnetic superconductor for $R \geq R_c$.

Then, by making use of the effective Lagrangian, we have obtained some numerical results such as the size of a hadron and the bag constant. We have also derived a relation between the Regge slopes of mesons and gluonia. Thus, our scheme of quark confinement has phenomenological applications on the structure of hadrons. It is also possible to calculate various long-range correlation functions, once the hypothesis of Abelian dominance is accepted. We are currently investigating this problem.

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