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Quantum radiation by moving mirrors

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The quantum radiation of scalar particles by a moving mirror is discussed. A perturbation method is developed which allows one to calculate the radiation produced by a boundary of arbitrary shape in the nonrelativistic limit. It is applied to the case of a mirror in two-dimensional space-time and reproduces the results of Fulling and Davies. It is also applied to the case of a plane mirror in four-dimensional space-time, and a formula is obtained for the total energy radiated by an arbitrary nonrelativistic motion.

I. INTRODUCTION

Radiation by a moving mirror is a purely quantum effect which has no analog in classical field theories. It arises as a result of the interaction of the mirror with zero-point fluctuations of a quantized field. By a mirror we mean a surface on which the field operator must satisfy a boundary condition corresponding to perfect reflection. For a scalar field ϕ, the boundary condition we consider is

$$\phi\,|_S = 0 \; . \tag{1.1}$$

This condition provides a coupling between the field ϕ and the mirror. (In fact, the specific form of the condition is unimportant: Any boundary condition on a timelike surface would give rise to vacuum polarization and quantum radiation effects). Even for a static mirror, the modes of the field ϕ which satisfies Eq. (1.1) are different from plane waves. As a result, the vacuum energy density and pressure acquire nonzero expectation values and can lead to observable effects (in particular, to the well-known Casimir effect[1]). If the mirror moves with acceleration, then one cannot, in general, define a stable vacuum state. In and out vacuum states can be defined if the motion of the mirror is bounded in time. Modes of the field ϕ which have positive frequency at $t \to -\infty$ become a mixture of positive- and negative-frequency components at $t \to +\infty$. This means that creation and annihilation operators and the vacuum states in the two asymptotic regions are different, and that the motion of the mirror creates particles.

Quantum radiation from moving mirrors is an interesting effect in its own right, but it can also have important cosmological applications. Phase transitions in the early universe can produce macroscopic vacuum structures, vacuum domain walls, or strings.[2] The evolution of these structures depends on the efficiency of various energy-loss mechanisms, in particular on the rate of particle creation. Vacuum domain walls and strings are not perfect mirrors (with respect to electromagnetic waves they act like dielectrics rather than mirrors). Nonetheless, a study of radiation from moving mirrors can help develop an intuition about quantum radiation and indicate the magnitude of the effects in other situations.

Radiation from a moving mirror in two-dimensional space-time has been studied by Fulling and Davies.[3] They found an exact expression for the vacuum expectation value of the energy-momentum tensor $\langle T_{\mu\nu} \rangle$, which enabled them to calculate the radiation reaction force and the rate of energy loss by the mirror. In four-dimensional space-time the problem is much more complicated and has been solved only in the special case of a uniformly accelerated mirror.[4] In the present paper we develop a perturbation method, which consists of perturbation of the boundary conditions, and which allows one to calculate $\langle T_{\mu\nu} \rangle$ and the radiated power for arbitrary nonrelativistic motion of the mirror. Our perturbation technique is described in Sec. II. It is applied to mirrors in two- and four-dimensional space-times in Secs. III and IV, respectively. Quantum radiation from moving dielectrics and implications for the cosmological evolution of vacuum structures will be discussed elsewhere.

II. PERTURBATION OF THE BOUNDARY CONDITIONS

For simplicity, we consider a real, massless scalar field $\phi(x)$ satisfying the equation[5]

$$\Box\phi(x)=0 \tag{2.1}$$

and the boundary condition

$$\phi(x)\,|_{x\in S}=0 . \tag{2.2}$$

The surface S is a timelike surface describing the world history of the mirror. The initial condition for the field operator $\phi(x)$ is

$$\phi(x)\to\phi_{\rm in}(x) \quad (t\to\infty) , \tag{2.3}$$

where $\phi_{\rm in}(x)$ is the field operator for a stationary mirror. (We assume that the motion of the mirror is bounded in time, and thus the mirror is stationary at $t\to\pm\infty$.) In simple geometries, the field $\phi_{\rm in}(x)$ can be easily found.

Suppose that the surface S differs only slightly from a simple surface S_0, for which the solution of Eqs. (2.1)–(2.3) can be found, and suppose that $\phi_0(x)$ is such a solution;

$$\Box\phi_0(x)=0, \quad \phi_0(x)\,|_{x\in S_0}=0 . \tag{2.4}$$

If the surface S_0 describes a static mirror, then $\phi_0=\phi_{\rm in}$. Below we assume that this is the case. We can rewrite the boundary condition (2.2) as

$$\phi(x^\mu+\xi^\mu(x))\,|_{x\in S_0}=0 , \tag{2.5}$$

where $\xi^\mu(x)$ is the displacement vector which translates a point $x\in S_0$ to a point in S and ξ^μ is orthogonal to S_0 and small in the sense to be specified below. Expanding (2.5) in powers of ξ^μ, we obtain

$$[\phi(x)+\xi^\mu(x)\partial_\mu\phi(x)+\cdots]_{x\in S_0}=0 . \tag{2.6}$$

Let

$$\phi=\phi_0+\phi_1 ; \tag{2.7}$$

then to first order we obtain

$$\Box\phi_1(x)=0 , \tag{2.8a}$$

$$\phi_1(x)\,|_{x\in S_0}=-\xi^\mu(x)\partial_\mu\phi_0(x)\,|_{x\in S_0} , \tag{2.8b}$$

where

$$\phi_1(x)\to 0 \quad (t\to\infty) . \tag{2.8c}$$

Instead of the vanishing boundary condition Eq. (2.2) on a complicated surface S, we now have a more complicated boundary condition (2.8b) on a simple surface S_0. The solution of Eqs. (2.8) is

$$\begin{aligned}\phi_1(x)&=\int_{S_0}\phi_1(x')\partial'_\mu\Delta_0^R(x,x')d\Sigma^{\mu'}\\&=-\int_{S_0}\xi^\mu(x')\partial'_\mu\phi_0(x')\,\partial'_\nu\Delta_0^R(x,x')d\Sigma^{\nu'} ,\end{aligned} \tag{2.9}$$

where the first equality can be easily derived using Green's identity.[6] The retarded Green's function $\Delta_0^R(x,x')$ satisfies the equations

$$\Box\Delta_0^R(x,x')=\Box'\Delta_0^R(x,x')=\delta(x-x') \tag{2.10a}$$

and

$$\Delta_0^R(x,x')\,|_{x\in S_0}=\Delta_0^R(x,x')\,|_{x'\in S_0}=0 , \tag{2.10b}$$

and

$$\Delta_0^R(x,x')=0 \quad \text{if } t<t' . \tag{2.10c}$$

We want to find the vacuum expectation value of the energy-momentum tensor,

$$\langle 0,\text{in}\,|\,T_{\mu\nu}\,|\,0,\text{in}\rangle\equiv\langle T_{\mu\nu}\rangle , \tag{2.11}$$

where, for the minimal energy-momentum tensor,

$$T_{\mu\nu}=\tfrac{1}{2}(\phi_{,\mu}\phi_{,\nu}+\phi_{,\nu}\phi_{,\mu}-\eta_{\mu\nu}\phi_{,\sigma}\phi^{,\sigma}) . \tag{2.12}$$

Substituting Eqs. (2.7) and (2.9) into Eq. (2.12), we can express $\langle T_{\mu\nu}\rangle$ in terms of $\xi^\mu(x)$, $\Delta_0^R(x,x')$, and

$$\Delta_0^1(x,x')\equiv\langle\phi_0(x)\phi_0(x')+\phi_0(x')\phi_0(x)\rangle . \tag{2.13}$$

Of course, the resulting integrals will be divergent, and $\langle T_{\mu\nu}\rangle$ should be regularized and renormalized by subtracting the $\langle T_{\mu\nu}\rangle$ of infinite space without a mirror. We expect $\langle T_{\mu\nu}\rangle$ renormalized in this way to be finite. Here we adopt the point-separation regularization technique[1]: The field operators in all the products of Eq. (2.12) will be taken at different points, x^μ and $x^\mu+\epsilon^\mu$. The limit $\epsilon^\mu\to 0$ will be taken in the final result after renormalization.

From Eqs. (2.9) and (2.12) we find, neglecting second and higher powers of ξ,

$$\langle T_{\mu\nu}\rangle=\langle T_{\mu\nu}\rangle_0+\langle T_{\mu\nu}\rangle_1 , \tag{2.14}$$

where $\langle T_{\mu\nu}\rangle$ is the energy-momentum tensor for a static mirror, and

$$\langle T_{\mu\nu}\rangle_1=D_{\mu\nu}-\tfrac{1}{2}\eta_{\mu\nu}D_\sigma^\sigma , \tag{2.15}$$

with

$$D_{\mu\nu}(x)=-\tfrac{1}{2}\int_S[\partial_\mu\partial'_\alpha\Delta_0^R(x+\epsilon,x')\partial_\nu\partial'_\beta\Delta_0^1(x,x')+\partial_\nu\partial'_\alpha\Delta_0^R(x,x')\partial_\mu\partial'_\beta\Delta_0^1(x+\epsilon,x')]\xi^\alpha(x')d\Sigma^{\beta'}\,|_{\epsilon\to 0}\,. \tag{2.16}$$

To proceed further we need to specify the shape of the surface S_0. In the remainder of this paper we consider the special case in which S_0 is a plane. In this case we can write explicit expressions for Δ_0^R and Δ_0^1:

$$\Delta_0^R(x,x')=\Delta^R(x-x')-\Delta^R(x-\bar{x}')\,, \tag{2.17a}$$

$$\Delta_0^1(x,x')=\Delta^1(x-x')-\Delta^1(x-\bar{x}')\,, \tag{2.17b}$$

where $\Delta^R(x)$ and $\Delta^1(x)$ are the corresponding functions in the free space without a mirror and the point $\bar{x}'$ is the mirror image of the point x' with respect to the plane S_0. If n^μ is a unit normal to S_0, $n^\mu n_\mu=-1$, then it follows from (2.17) that

$$n^\mu\partial'_\mu\Delta_0^R(x,x')\,|_{x'\in S_0}=2n^\mu\partial'_\mu\Delta^R(x-x')\,|_{x'\in S_0} \tag{2.18a}$$

and

$$n^\mu\partial'_\mu\Delta_0^1(x,x')\,|_{x'\in S}=2n^\mu\partial'_\mu\Delta(x-x')\,|_{x'\in S_0}\,. \tag{2.18b}$$

We can also represent $\xi^\alpha(x)$ and $d\Sigma^\beta$ as

$$\xi^\alpha=\xi n^\alpha,\quad d\Sigma^\beta=n^\beta d\Sigma\,, \tag{2.19}$$

where the sign in the last equation is chosen so that n^μ points in the direction of the point of observation, x. Combining Eqs. (2.16), (2.18), and (2.19), we obtain

$$D_{\mu\nu}(x)=-2n^\alpha n^\beta\int_{S_0}[\partial_\mu\partial_\alpha\Delta^R(x-x'+\epsilon)\partial_\nu\partial_\beta\Delta^1(x-x') +\partial_\nu\partial_\alpha\Delta^R(x-x')\partial_\mu\partial_\beta\Delta^1(x-x'+\epsilon)]\xi(x')d\Sigma'\,|_{\epsilon\to 0}\,. \tag{2.20}$$

Although Eq. (2.20) is valid in four dimensions only if S_0 is a plane, it always applies in two dimensions because in this latter case, S_0 is a straight line.

The perturbation method presented here can be extended to higher orders in ξ, to massive fields, or higher spins. Finally let us briefly discuss the range of validity of the method. We consider the case when the mirror moves as a whole, so that the displacement ξ is a function only of t, and not of the position on the mirror. The perturbation expansion is an expansion in powers of such dimensionless quantities as ξ/z, $\dot{\xi}^2$, and $\xi\ddot{\xi}$, where $\dot{\xi}=d\xi/dt$ and z is the distance to the mirror. Thus a necessary condition for the perturbation method to apply is that the motion be nonrelativistic so that $\dot{\xi}^2<<1$. As will be shown below, in those situations where an exact solution is known, the perturbation method obtains the nonrelativistic limit of the exact solution.

III. TWO DIMENSIONS

In two-dimensional space-time, the trajectory of the mirror is a timelike curve, and the "surface" S_0 is a straight line parallel to the x axis. (We assume that the mirror is stationary as $x\to\pm\infty$.) The functions $\Delta^1(x)$ and $\Delta^R(x)$ are given by

$$\Delta^1(x)=(2\pi)^{-1}\ln x^2 \tag{3.1}$$

and

$$\Delta^R(x)=\tfrac{1}{2}\theta(x_0)[\theta(x_0-x_1)+\theta(x_0+x_1)-1]\,, \tag{3.2}$$

where $x^2={x_0}^2-{x_1}^2$. From Eqs. (2.12), (2.13), (2.17b), and (3.1) it is easily verified that the energy-momentum tensor vanishes for a static mirror,

$$\langle T_{\mu\nu}\rangle_0=0\,. \tag{3.3}$$

In this equation we have already made the subtraction of $\langle T_{\mu\nu}\rangle$ for infinite space, which amounts to keeping only the second term on the right-hand side of Eq. (2.17b). The problem thus reduces to the calculation of the integral (2.20) for $\langle T_{\mu\nu}\rangle_1$. In particular, for $\langle T_{01}\rangle_1$ we have

$$\langle T_{01}(x)\rangle = -2\int_{-\infty}^{\infty} dx'_0\,\xi(x')[\partial_0\partial_1\Delta^R(x-x'+\epsilon)\partial_1{}^2\Delta^1(x-x')+\partial_1{}^2\Delta^R(x-x')\partial_0\partial_1\Delta^1(x-x'+\epsilon)]_{\substack{\epsilon\to 0\\ x'_1=0}}\,, \tag{3.4}$$

where the unperturbed position of the mirror is at $x_1=0$. This integral is expected to be finite and independent of the direction of ϵ^μ. Substituting Eqs. (3.1) and (3.2) in (3.4) with $\epsilon^0=\epsilon$, $\epsilon^1=0$, and integrating by parts, we obtain

$$\langle T_{01}(x)\rangle_1 = \frac{1}{2\pi}\{\,\xi'(u+\epsilon)[\epsilon^{-2}+(2x_1-\epsilon)^{-2}]+\xi(u+\epsilon)[-2\epsilon^{-3}+2(2x_1-\epsilon)^{-3}]+\xi'(u)[\epsilon^{-2}-(2x_1+\epsilon)^{-2}]$$
$$+\xi(u)[2\epsilon^{-3}-2(2x_1+\epsilon)^{-3}]\,\}_{\epsilon\to 0}\,, \tag{3.5}$$

where $u=x_0-x_1$. Expanding (3.5) in powers of ϵ, we find that all negative powers of ϵ cancel. The final result is

$$\langle T_{01}(x)\rangle = (12\pi)^{-1}\xi'''(x_0-x_1)\,. \tag{3.6}$$

The remaining components of $\langle T_{\mu\nu}\rangle_1$ can be found most easily from the conservation laws $\partial^\mu\langle T_{\mu\nu}\rangle=0$. We find

$$\langle T_{00}(x_0,x_1)\rangle_1 = \partial_1\int_{-\infty}^{x_0}\langle T_{01}(x'_0,x_1)\rangle dx'_0\,, \tag{3.7a}$$

$$\langle T_{11}(x_0,x_1)\rangle_1 = -\partial_0\int_{x}^{\infty}\langle T_{01}(x_0,x'_1)\rangle dx'_1\,, \tag{3.7b}$$

where $\langle T_{\mu\nu}\rangle_1\to 0$ as $x_0\to-\infty$ and as $x_1\to-\infty$. From Eqs. (3.6) and (3.7) we obtain

$$\langle T_{00}(x)\rangle_1 = \langle T_{11}(x)\rangle_1 = -(12\pi)^{-1}\xi'''(x_0-x_1)\,. \tag{3.8}$$

Equations (3.6) and (3.8) apply when the point x is to the right of the mirror ($x_1>0$). For $x_1<0$,

$$\langle T_{00}\rangle_1 = \langle T_{11}\rangle_1 = \langle T_{01}\rangle_1 = +(12\pi)^{-1}\xi'''(x_0+x_1)\,. \tag{3.9}$$

Fulling and Davies[3] have found an exact expression for $\langle T_{\mu\nu}\rangle$ in the two-dimensional case. In the nonrelativistic approximation our results, Eqs. (3.6), (3.8), and (3.9), are in agreement with their result.

Let us now find the radiation reaction force and the radiative energy loss by the mirror. If P^μ is the energy-momentum vector of the mirror, then

$$dP^\mu/dx_0 = -dP^\mu_F/dx_0\,, \tag{3.10}$$

where

$$P^\mu_F = \int_{-\infty}^{\xi}\langle T^{\mu 0}\rangle dx_1 + \int_{\xi}^{\infty}\langle T^{\mu 0}\rangle dx_1 \tag{3.11}$$

is the energy-momentum vector of the field and $\xi=\xi(x_0)$. From (3.10) and (3.11) and the energy-conservation law, we have

$$dP^\mu/dx_0 = [\xi'\langle T^{\mu 0}\rangle + \langle T^{\mu 1}\rangle]_{x_1=\xi-0} - [\xi'\langle T^{\mu 0}\rangle + \langle T^{\mu 1}\rangle]_{x_1=\xi+0}\,. \tag{3.12}$$

In our approximation we can neglect terms proportional to ξ'. Then, using Eqs. (3.6), (3.8), and (3.9), we obtain

$$dP^0/dx_0 = 0\,, \tag{3.13}$$

$$F = dP^1/dx_0 = (6\pi)^{-1}\xi'''(x_0)\,, \tag{3.14}$$

where F is the radiation reaction force. The total energy loss vanishes to first order in ξ; however, it is possible to find the energy radiated in second order using the result (3.14) for the reaction force. Let W be work done by the force F, and E be the total energy radiated; then

$$E = -W = -\int_{-\infty}^{\infty} F\xi' dx_0 = (6\pi)^{-1}\int_{-\infty}^{\infty}\ddot{V}^2 dx_0\,, \tag{3.15}$$

where $V=\xi'$ is the velocity of the mirror. Finally we note that, using the exact $\langle T_{\mu\nu}\rangle$ found by Fulling and Davies,[3] we can obtain an exact relativistic expression for the radiative force,

$$\frac{dP^\mu}{d\tau} = \frac{1}{6\pi}\left[\frac{d^2u^\mu}{d\tau^2} - u^\mu u_\nu\frac{d^2u^\nu}{d\tau^2}\right]\,, \tag{3.16}$$

where $u^\mu = d\xi^\mu/d\tau$ and τ is the proper time. Note that Eq. (3.16) has the same form as the relativistic radiative force in classical electrodynamics.[7]

IV. FOUR DIMENSIONS

In this section, we shall calculate $\langle T_{\mu\nu}\rangle_1$, the reaction force and the radiated energy for a plane mirror in four-dimensional space-time. The surface S_0 describing the unperturbed world history of the mirror is the plane $z=0$. The functions $\Delta^1(x)$ and $\Delta^R(x)$ are

$$\Delta^1(x) = -(2\pi^2 x^2)^{-1} , \tag{4.1}$$

$$\Delta^R(x) = (4\pi|\vec{x}|)^{-1}\delta(t-|\vec{x}|) , \tag{4.2}$$

where $x^2 = t^2 - \vec{x}^2$. From Eqs. (2.12), (2.13), (2.17b), and (4.1) we find the energy-momentum tensor for a static mirror:

$$\langle T_{\mu\nu}\rangle_0 = (16\pi^2 z^4)^{-1}\mathrm{diag}(-1,1,1,0) . \tag{4.3}$$

Equation (2.20) yields $\langle T_{03}\rangle_1$,

$$\langle T_{03}\rangle_1 = -2\int d^3x'\xi(t')[\partial_t\partial_z\Delta^R(x-x'+\epsilon)\partial_z{}^2\Delta^1(x-x') + \partial_z{}^2\Delta^R(x-x')\partial_t\partial_z\Delta^1(x-x'+\epsilon)]_{\substack{z'=0\\ \epsilon\to 0}} , \tag{4.4}$$

where $d^3x' = dt'dx'dy'$. As in the previous section, let $\epsilon^0 = \epsilon$, $\vec{\epsilon} = 0$.

Equation (4.4) is valid even if ξ is a function of x and y, the coordinates in the mirror, as well as t. Similar expressions may be given for the remaining components of $\langle T_{\mu\nu}\rangle_1$. Thus it is possible, using this formalism, to study the radiation produced by arbitrary small deformations of the initial surface S_0. Because of the presence of the retarded Green's function Δ^R, $\langle T_{\mu\nu}\rangle_1$ depends upon ξ evaluated at retarded times. Thus to first order in ξ the radiation seen by an observer depends only upon ξ and its derivatives along the intersection of the world history of the mirror S, with the observer's past light cone. In this sense it is correct to say that the quantum radiation emanates from the mirror's surface. In the remainder of this paper we restrict our attention to the case of a plane mirror which moves rigidly so $\xi = \xi(t)$.

Substitute (4.1) and (4.2) in (4.4), integrate by parts and expand in powers of ϵ; after a lengthy but straightforward calculation one finds

$$\langle T_{03}\rangle_1 = (16\pi^3)^{-1}\int d^2x'[z^3R^{-9}(7\xi' + 7R\xi'' + 3R^2\xi''' + \tfrac{2}{3}R^3\xi^{(4)} + \tfrac{1}{15}R^4\xi^{(5)}) - zR^{-7}(\tfrac{5}{2}\xi' + \tfrac{5}{2}R\xi'' + R^2\xi''' + \tfrac{1}{6}R^3\xi^{(4)})] , \tag{4.5}$$

where $d^2x' = dx'dy'$, $R^2 = (x-x')^2 + (y-y')^2 + z^2$, and ξ is evaluated at the retarded time $\xi = \xi(t-R)$.

As expected, all negative powers of ϵ have canceled. If we convert Eq. (4.5) to an integral on R and repeatedly use the relation

$$\int_z^\infty dR\, R^{-n}\xi^{(m)}(t-R) = \frac{\xi^{(m)}(t-z)}{(n-1)z^{n-1}} - \frac{1}{n-1}\int_z^\infty \frac{dR}{R^{n-1}}\xi^{(m+1)}(t-R) , \tag{4.6}$$

we obtain

$$\langle T_{03}\rangle = (16\pi^2 z^4)^{-1}(\xi' + z\xi'' + \tfrac{7}{15}z^2\xi''' + \tfrac{2}{15}z^2\xi^{(4)}) , \tag{4.7}$$

where now $\xi = \xi(t-z)$. All off-diagonal components of $\langle T_{\mu\nu}\rangle$ expect $\langle T_{03}\rangle$ are equal to zero by symmetry. As before, the components $\langle T_{00}\rangle_1$ and $\langle T_{33}\rangle_1$ can be found from the conservation law, recalling that $\langle T_{\mu\nu}\rangle$ is a function of t and z only.

The results are

$$\langle T_{00}\rangle_1 = -(16\pi^2 z^5)^{-1}(4\xi + 4z\xi' + \tfrac{29}{15}z^2\xi'' + \tfrac{3}{5}z^3\xi''' + \tfrac{2}{15}z^4\xi^{(4)}) \tag{4.8}$$

and

$$\langle T_{33}\rangle_1 = -(48\pi^2 z^3)^{-1}(\xi'' + z\xi''' + \tfrac{2}{5}z^2\xi^{(4)}) . \tag{4.9}$$

The remaining components, $\langle T_{11}\rangle = \langle T_{22}\rangle$, would require a separate calculation, but are unimportant for the calculation of the reaction force. Equations (4.7), (4.8), and (4.9) apply for all points to the right of the mirror, i.e., for $z>0$. For $z<0$, we have to replace ξ by $-\xi$ and make a coordinate reflection $z\to -z$. This gives

$$\langle T_{00}(t,z)\rangle = -\langle T_{00}(t,-z)\rangle,\ \ \langle T_{33}(t,z)\rangle = -\langle T_{33}(t,-z)\rangle\ ,\ T_{03}(t,z)\rangle = \langle T_{03}(t,-z)\rangle\ . \tag{4.10}$$

As a check of our results for $\langle T_{\mu\nu}\rangle_1$, one can verify that terms proportional to ξ and to ξ' in Eqs. (4.7) andd (4.8) can be obtained from the result Eq. (4.3), for a static mirror by a translation and a Lorentz transformation.

From Eqs. (4.7), (4.8), and (4.9) we see that $\langle T_{\mu\nu}\rangle_1$ becomes infinite as $z\to 0$. This means that not only is the field energy infinite for a static mirror, but it changes by an infinite amount as the mirror begins to move. These infinities can be attributed to the fact that the classical boundary condition Eq. (1.1) is not strictly compatible with the quantum theory. Forcing ϕ to vanish on a surface requires its conjugate momentum to be totally indeterminate. Thus $\langle T_{\mu\nu}\rangle$ is singular on the mirror for the same reason that single-particle quantum mechanics would require a position eigenstate to have infinite energy. This singularity should disappear in a more realistic model for the mirror.[4,8] Nevertheless, it will be shown that the total energy radiated by the mirror is finite if the motion is bounded in time.

The radiation reaction force per unit on the mirror is

$$F=\langle T_{33}\rangle_{z=-a}-\langle T_{33}\rangle_{z=a}=-2\langle T_{33}\rangle_{z=a}\ , \tag{4.11}$$

where we have introduced a cutoff at $z=a$. Using Eq. (4.9) and expanding in powers of a we find

$$F(t)=(24\pi^2)^{-1}[a^{-3}\xi''(t)-\tfrac{1}{10}a^{-1}\xi^{(4)}(t) -\tfrac{1}{15}\xi^{(5)}(t)]\ . \tag{4.12}$$

The total energy radiated by the mirror is

$$E=-\int_{-\infty}^{\infty}F(t)\xi'(t)dt =\frac{1}{360\pi^2}\int_{-\infty}^{\infty}\dddot{V}^2dt\ , \tag{4.13}$$

where we have integrated by parts, assumed that $\xi^{(n)}\to 0$ as $t\to\pm\infty$, and let $a\to 0$. We see that, although the reaction force (4.12) diverges as $a\to 0$, the radiated energy per unit area is finite. This infinite force creates an infinite energy in the space surrounding the mirror, but this energy is bound to the mirror and is reabsorbed when the motion ceases.

We have also calculated $\langle\theta_{\mu\nu}\rangle$, F, and E for the conformal energy-momentum tensor,

$$\theta_{\mu\nu}=\tfrac{1}{2}(\phi_{,\mu}\phi_{,\nu}+\phi_{,\nu}\phi_{,\mu}-\eta_{\mu\nu}\phi_{,\sigma}\phi^{,\sigma}) -\zeta(\partial_0\phi_\nu-\eta_{\mu\nu}\partial_\sigma\partial^\sigma)\phi^2 \tag{4.14}$$

with $\zeta=\frac{1}{6}$. The results are

$$\langle\theta_{03}\rangle_1=(720\pi^2z^2)^{-1}(\xi'''+z\xi^{(4)})\ , \tag{4.15a}$$

$$\langle\theta_{00}\rangle_1=-(720\pi^2z^2z^3)^{-1}(2\xi''+2z\xi'''+z^2\xi^{(4)})\ , \tag{4.15b}$$

$$\langle\theta_{33}\rangle_1=-(720\pi^2z)^{-1}\xi^{(4)}\ , \tag{4.15c}$$

where, as above, $\xi=\xi(t-z)$. The vacuum expectation value of the energy-momentum tensor for arbitrary conformal parameter ζ can be found as a linear combination of $\langle T_{\mu\nu}\rangle_1$ and $\langle\theta_{\mu\nu}\rangle_1$. We can find the remaining nonzero components of $\langle\theta_{\mu\nu}\rangle$ from the relation $\theta^\mu_\mu=0$. They are

$$\langle\theta_{11}\rangle_1=\langle\theta_{22}\rangle_1=-(720\pi^2z^3)^{-1}(\xi''+z\xi''')\ . \tag{4.16}$$

From Eqs. (4.11) and (4.15c), the radiative reaction force is given by

$$F(t)=(360\pi^2)^{-1}[a^{-1}\xi^{(4)}(t)-\xi^{(5)}(t)] \tag{4.17}$$

and the energy radiated by the mirror is the same as for the minimal tensor Eq. (4.13). The fact that E is independent of the conformal parameter ζ is not surprising, because the difference between $\theta_{\mu\nu}$ and $T_{\mu\nu}$ is a total divergence.

For constant acceleration ($\xi''=$ constant), our result for $\langle\theta_{\mu\nu}\rangle$ is in agreement with the nonrelativistic limit of that of Candelas and Deutsch.[4] Note that the reaction force and the radiated energy vanish in this case.

To illustrate the result Eq. (4.13), let us consider a surface which oscillates so that $\xi(t)=l\cos\omega t$. Then the energy radiated per unit area per oscillation is

$$E=\frac{1}{720\pi^2}l^2\omega^5\ . \tag{4.18}$$

Because $l\omega<1$, we must have $l^2E<10^{-4}\omega$. Because one expects the photon produced in this case to have a typical frequency of the order of ω, the mirror radiates fewer than 10^{-4} photons per oscillation per area l^2. Thus quantum radiation by mirrors is a small effect under normal circumstances. However, such structures as domain walls and vacuum strings might be expected to be subjected to large accelerations and hence radiate strongly. Another object which is similar to an accelerating mirror and which can create quantum radiation is the expanding bubble associated with a phase transition in various gauge theories.[9]

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[1]See, e.g., B. S. DeWitt, Phys. Rep. 19C, 295 (1975).

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[5]We use units in which $\hbar = c = 1$. The signature of the metric tensor is $(+,-,-,-)$ in four dimensions and $(+,-)$ in two dimensions; $\Box = \partial_0{}^2 - \vec{\partial}^2$.

[6]The Green's identity is

$$\int_\Omega (\phi \Box \psi - \psi \Box \phi) d\Omega = \oint_S (\phi \partial^\mu \psi - \psi \partial^\mu \phi) d\Sigma_\mu ,$$

where $d\Omega$ is the space-time volume element. With $\psi(x) = \Delta^R(x,x')$ we obtain Eq. (2.9).

[7]See, e.g., L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Pergamon, Oxford, 1971).

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