New tests of quark-parton model

Rudolph C. Hwa
Institute of Theoretical Science and Department of Physics, University of Oregon, Eugene, Oregon 97403

M. Sajjad Zahir
Department of Physics, University of Victoria, Victoria, British Columbia, Canada V8W 2Y2

(Received 1 February 1982)

It is shown that a simple constraint between $F_2^{\mu n}/F_2^{\mu p}$ and $F_2^{\nu p}/F_2^{\mu p}$ derived from the basic properties of QCD that are contained in the quark-parton model is not satisfied by the present experimental data. The measurement of semi-inclusive cross sections that may clarify the issue on the d/u ratio is suggested.

Recent experiments on lepton-nucleon deep-inelastic scattering have been used to check the predictions of quantum chromodynamics, in particular, the Q^2 dependences of the structure functions.[1] However, because of the existence of adjustable parameters in the theory, the comparison with data has essentially led to the determination of those parameters (e.g., Λ) without subjecting QCD to a decisive test. We suggest here a very simple test on the basic tenets of the theory that are contained even in the quark-parton model. Preliminary experimental results seem to show inconsistency.

We consider three basic structure functions $F_2^{\mu p}$, $F_2^{\mu n}$, and $F_2^{\nu p}$ for the purpose of illustrating our procedure; other structure functions can also be considered so long as they have adequate accuracy. They can be expressed as

$$F_2^{\mu p,n}(x,Q^2) = \sum_i e_i^2 x q_{i/p,n}(x,Q^2) + H_{p,n}(x,Q^2) \ , \qquad (1)$$

$$F_2^{\nu p}(x,Q^2) = \sum_{i=d,\bar{u},s,\ldots} 2x q_{i/p}(x,Q^2) + H_p'(x,Q^2) \ , \qquad (2)$$

where $q_{i/p,n}(x,Q^2)$ is the quark or antiquark distribution in the nucleon, while H and H' are higher-twist terms that are of order M^2/Q^2, M being some hadronic mass scale. Except for the H and H' terms and the Q^2 dependences, (1) and (2) are expressions of the quark-parton model. With the factorization property having been established for QCD, (1) and (2) are indeed the basic consequences of QCD even though the theory can go much further and specify definite characteristics in the dependences on Q^2. Our focus is on the validity of (1) and (2) themselves.

In our following discussion we shall consider Q^2 as being high enough to justify the neglect of the higher-twist terms H and H'. Recent experimental results at high Q^2 have provided two ratios

$$R_1(x,Q^2) \equiv F_2^{\nu p}(x,Q^2)/F_2^{\mu p}(x,Q^2) \ , \qquad (3)$$

$$R_2(x,Q^2) \equiv F_2^{\mu n}(x,Q^2)/F_2^{\mu p}(x,Q^2) \ . \qquad (4)$$

In terms of the quark distribution, we have from (1) and (2)

$$R_1 = \frac{18(r+bS)}{4+r+aS}, \quad R_2 = \frac{1+4r+aS}{4+r+aS} \ , \qquad (5)$$

where

$$r(x,Q^2) = d(x,Q^2)/u(x,Q^2) \ , \qquad (6)$$

$$S(x,Q^2) = \bar{q}(x,Q^2)/u(x,Q^2) \ . \qquad (7)$$

The constants a and b are 7, 15, 17, and 2, 3, 4, for the number of flavors f being 3, 4, 5, respectively. In (6) and (7) the u- and d-quark distributions refer to the proton and include both valence and sea quarks. A symmetric sea has been assumed so $\bar{q}$ represents a sea-quark distribution of any flavor. Note that S is bounded by 0 and 1, and is usually neglected for $x > 0.2$.

The data[2,3] on R_1 and R_2 are shown in Figs. 1 and 2. We have included in Fig. 1 not only the theoretical curves[4,5] exhibited originally in Ref. 2, but also two other predictions (Buras-Gaemers[6] and valon model[7]), calculated according to (5)–(7) and the quark distributions parametrized in Refs. 6 and 7. The latter two predictions are, strictly speaking, Q^2 dependent, but being ratios of structure functions their dependences on Q^2 are too weak to be discernable in the plot. The agreement between data and the valon-model prediction is evidently quite striking. In Fig. 2, the theoretical curve which is calculated also in accordance to the predictions of the valon model does not agree very well with the preliminary high-Q^2 data on σ_n/σ_p from the European Muon Collaboration (EMC).[3] In both figures the solid curves are obtained assuming $f=3$.

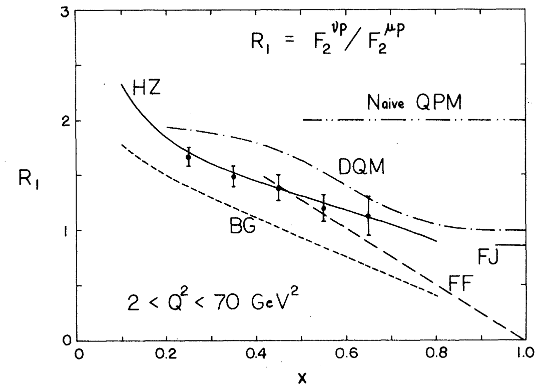

FIG. 1. Ratio $F_2^{\nu p}/F_2^{\mu p}$ vs x. Data are from Ref. 2. Long-dashed line is the prediction according to Field-Feynman (Ref. 4) parametrization of quark distributions, dash-dotted line is in accordance with the diquark model (Ref. 5), short-dashed line is from Buras-Gaemars (Ref. 6), solid line from Hwa-Zahir (Ref. 7), and FJ stands for the $x \to 1$ limit predicted by Farrar-Jackson (Ref. 9).

To check whether the discrepency in Fig. 2 is because of poor parametrization of the quark distributions or possible inaccuracy of the data, we eliminate r between R_1 and R_2 in (5) and obtain

$$R_2 = \frac{1 + (a - 4b)S + (5 + aS)R_1/6}{4 + (a - b)S} \quad . \tag{8}$$

For $x > 0.25$ the sea quarks are negligible. Thus, setting $S = 0$ yields

$$R_2 = \tfrac{1}{4}(1 + \tfrac{5}{6}R_1) \quad , \tag{9}$$

which is independent of any parametrization of the quark distributions. Coincidentally, for $f = 4$, (9) remains valid for any S. For $f = 3$ and 5, R_2 is lower at $S = 1$ than at $S = 0$.

The straight line for (9) is shown in Fig. 3 by the solid line. Deviation from the line is permitted only when S is not negligible (for $f = 3$ and 5); that occurs at small x, i.e., when R_1 and R_2 are both large. The lower bounds (corresponding to $S = 1$) are shown by the long-dashed ($f = 3$) and short-dashed ($f = 5$) lines in the figure. The data in Fig. 1 and 2, fortunately, have points for common values of x and ranges of Q^2. They can therefore be plotted in Fig. 3. All five points are for $x > 0.25$ and should fall on the solid line, but they do not. Consideration of the sea contributions makes the disagreement even worse if $f = 3$ and 5, and no better for $f = 4$.

Evidently, the discrepancy in Fig. 2 is not due to the predicted quark distributions. The data on R_1 and R_2 are not jointly consistent with the basic properties of QCD or the quark model as expressed in (1) and (2), independent of the detailed nature of any of the quark distributions. If the data are confirmed to be accurate, then the only way out is that the higher-twist terms H and H' are important. If that is the case, then the determination of Λ and α_s thus far considered in numerous publications is meaningless. However, that option is not very likely since the result of the diquark model[5] (an example of higher-twist contribution) does not agree well with data, as is indicated in Fig. 1.[2] Thus, we are faced with a serious problem of fundamental importance.

Similar considerations can be applied to other structure functions (F_2 and xF_3) involving ν and $\bar{\nu}$. Experimental groups with accurate neutrino data should test their results against other constraints analogous to the one derived in (8). We propose in the following a useful quantity to examine, which not only reveals the d/u ratio directly but also is readily measurable within the same experiment. In a deep-inelastic semi-inclusive leptoproduction experiment it is the ratio of the integrated cross sections of π^- to

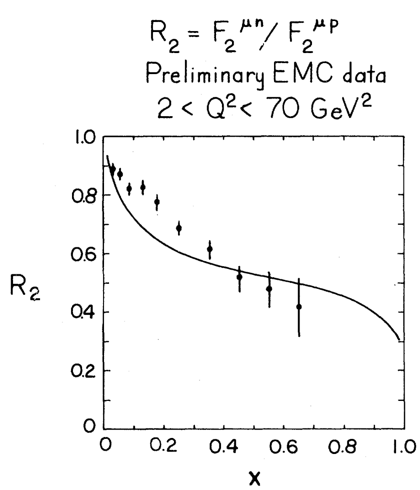

FIG. 2. Ratio $F_2^{\mu n}/F_2^{\mu p}$ vs x. Data are preliminary result from EMC (Ref. 3). Solid curve was originally predicted in Ref. 7.

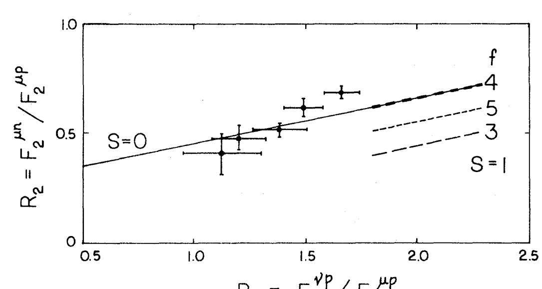

FIG. 3. R_2 vs R_1. Data are from Refs. 2 and 3 for $x > 0.2$ and $2 < Q^2 < 70$ GeV^2. $S = \bar{q}/u$ is assumed zero for the solid line. The broken lines are for $S = 1$ corresponding to the value of f indicated.

π^+ produced in the current fragmentation region. To be specific, consider first the the probability of producing $\pi^\pm$ at fixed x and Q^2

$$J^{\pm}(x,Q^2) = \int_{z_0}^{z_1} dz \frac{d^3\sigma^{\pi^\pm}}{dx\, dQ^2 dz} \Big/ \frac{d^2\sigma^{\pi^\pm}}{dx\, dQ^2} \quad . \tag{10}$$

The integration in z can be over any convenient range with end points at z_0 and z_1; here, z may be the usual energy ratio E_h/ν or the hadron momentum fraction in either the Breit frame, $z_B = 2p_\parallel^B/Q$, or the c.m. frame. Assuming that Q^2 is high enough so that factorization into quark distribution and fragmentation function (D) is justified, then we have for μp semi-inclusive reactions

$$J_{\mu p}^{\pm}(x,Q^2) = \frac{\sum_i e_i^2 q_{i/p}(x,Q^2) A_i^{\pm}(Q^2)}{\sum_i e_i^2 q_{i/p}(x,Q^2)} \quad , \tag{11}$$

where

$$A_i^{\pm}(Q^2) = \int_{z_0}^{z_1} dz\, D_i^{\pm}(z,Q^2) \quad . \tag{12}$$

By charge symmetry we have

$$A_1 \equiv A_u^+ = A_d^- = A_{\bar{d}}^+ = A_{\bar{u}}^- \quad , \tag{13a}$$

$$A_2 \equiv A_d^+ = A_u^- = A_{\bar{u}}^+ = A_{\bar{d}}^- \quad , \tag{13b}$$

$$A_3 \equiv A_s^{\pm} = A_{\bar{s}}^{\pm} \quad . \tag{13c}$$

Since A_2 and A_3 describe unfavored hadronization they are small compared to A_1. If we define

$$t(Q^2) = A_2(Q^2)/A_1(Q^2) \quad , \qquad t'(Q^2) = A_3(Q^2)/A_1(Q^2) \quad , \tag{14}$$

then they are small numbers, especially if the range (z_0, z_1) is narrow and situated on the upper end of the z range. Furthermore, being ratios they are also very mildly dependent on Q^2.

We now consider the quantity

$$R_3(x,Q^2) = 4J_{\mu p}^-(x,Q^2)/J_{\mu p}^+(x,Q^2) \quad . \tag{15}$$

Assuming $f=3$ we obtain

$$R_3 = \frac{r + 4t + S(4 + t + 2t')}{1 + rt/4 + S(1 + 4t + 2t')/4} \quad . \tag{16}$$

If $x > 0.2$ where S is negligible, $R_3 - 4t$ is very nearly r, particularly if t is small. The value of t can be determined either directly from experimental data or from parametrizations of the fragmentation functions. For the purpose of illustration consider the D function determined theoretically in QCD and in the recombination model for hadronization[8]; it has a concrete parametrization in z and Q^2 that agrees with data. Adopting for definiteness $z_0 = 0.2$ and $z_1 = 0.8$, we obtain from Ref. 8

$$t = (5.27 + 0.73s)^{-1} \quad , \tag{17}$$

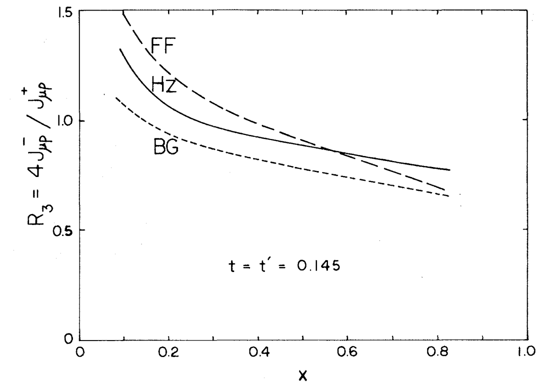

FIG. 4. R_3 vs x for parametrizations of quark distributions: FF (Ref. 4), BG (Ref. 6), and HZ (Ref. 7). The cuts on fragmentation functions are characterized by t and t', defined in Eq. (14).

where $s = \ln[(\ln Q^2/\Lambda^2)/(\ln Q_0/\Lambda^2)]$ and for s in the range $2.1 < s < 2.4$. In practice, we may regard t as a parameter around 0.145 for the limits chosen. If the statistics of data is good, one may prefer a higher value of z_0, resulting in an even smaller value of t. The point is that t can be regarded as a known number so (16) reveals directly the d/u ratio, when S is small.

Instead of (15) one can consider other ratios involving neutrinos. They all have similar expressions in terms of r. They should be combined with other ratios such as the ones in (5) through the elimination of r, thereby yielding direct constraints among the measurable quantities. Although an agreement between data and such constraints does not prove the validity of QCD, any firm disagreement would be detrimental to QCD.

In order to exhibit the differences in various quark distributions, we show in Fig. 4 the predictions of R_3 according to three parametrizations: Field-Feynman,[4] Buras-Gaemers,[6] and Hwa-Zahir,[7] using $t = t' = 0.145$. Dependence on Q^2 is insignificant. Accurate data for $x > 0.2$ can easily select the best parametrization, if any.

In conclusion, we remark that the simple relation, (8) or (9), and others that can easily be derived, are constraints on measurable quantities that are necessary conditions for the validity of QCD. Disagreement at the level indicated in Fig. 3 is sufficiently disturbing that a concerted effort to clarify the discrepancy seems warranted.

A discussion with D. Soper has been helpful. The work was supported in part by U.S. Department of Energy under Contract No. EY-76-S-06-2230, and the Natural Science and Engineering Research Council of Canada.

[1]See, for example, F. Sciulli, in *High Energy Physics—1980,* proceedings of the XXth International Conference, Madison, Wisconsin, edited by L. Durand and L. G. Pondrom (AIP, New York, 1981), p. 1278; also, various review talks in *Proceedings of the 1981 International Symposium on Lepton and Photon Interactions at High Energies, Bonn,* edited by W. Pfeil (Universitat Bonn, Bonn, 1981).
[2]P. Allen *et al.,* Phys. Lett. 103B, 71 (1981).
[3]G. Smadja, in *Proceedings of the 1981 International Symposium on Lepton and Photon Interactions at High Energies, Bonn* (Ref. 1), p. 444.
[4]R. D. Field and R. P. Feynman, Phys. Rev. D 15, 2590 (1977).
[5]A. Donnachie and P. V. Landshoff, Phys. Lett. 95B, 437 (1980); F. E. Close and R. G. Roberts, Z. Phys. C 8, 57 (1981).
[6]A. J. Buras and K. J. F. Gaemers, Nucl. Phys. B132, 249 (1978).
[7]R. C. Hwa and M. S. Zahir, Phys. Rev. D 23, 2539 (1981).
[8]V. Chang and R. C. Hwa, Phys. Rev. D 23, 728 (1981).
[9]G. R. Farrar and D. R. Jackson, Phys. Rev. Lett. 35, 1416 (1975).