PHYSICAL REVIEW D VOLUME 25, NUMBER 9 1 MAY 1982

Statistical bootstrap duality. II

Robert K. Logan, Matthew Bates, and Stephan Kogitz
Department of Physics, University of Toronto, Toronto M5S 1A7, Ontario, Canada

(Received 6 April 1981)

The dual statistical bootstrap model describing general two-body inelastic-scattering cross sections proposed by Kogitz, Logan, and Tanaka is tested with the differential-cross-section data for 48 new differential cross sections representing 42 different reactions, thus extending the total number of reactions satisfactorily described by the model to 55.

Hagedorn, using the thermodynamical concept of hadrons, first calculated[1] the density of hadronic states as a function of energy. This was subsequently verified through phenomenological tests.[2] Chiu and Heimann[3] have extended these results by calculating the density of state as a function of both energy and angular momentum. Using these results, Kogitz, Logan, and Tanaka[4] were able to describe two-body hadronic inelastic-scattering amplitudes as a sum of direct-channel resonances whose distributions are given by the Chiu-Hiemann density of states. The model successfully described the two-body inelastic-scattering results for 27 cross sections and 23 reactions.

In this paper we present the results of fitting an additional 48 differential cross sections representing 42 different reactions using the dual statistical bootstrap model of Kogitz, Logan, and Tanaka.[4] These new results (1) extend the range of reactions for which the model is applicable to 75 cross sections representing 55 different reactions, (2) place added weight on the validity of the statistical bootstrap duality description of two-body inelastic-scattering reactions, and (3) provide a further phenomenological test of the bootstrap approach.[1,3,5]

The bootstrap model of Kogitz, Logan, and Tanaka as explained in Ref. 4 builds the high-energy scattering amplitude using the duality concept, by summing the direct-channel resonances whose energy and angular momentum distribution are given by the Chiu and Heimann solution to the bootstrap equations[3]

$$\tilde{\rho}(E,l) = -\frac{d}{dl_z}\rho(E,l_z)\bigg|_{l_z=l}, \tag{1}$$

$$\rho(E,l_z) = \frac{\rho(E)}{2DE}\left[\cosh\left(\frac{\pi l_z}{2DE}\right)\right]^{-1}, \tag{2}$$

$$0 < D < 7 \text{ GeV}.$$

The positive- and negative-parity partial-wave scattering amplitudes as explained in Ref. 4 are parametrized in terms of $\tilde{\rho}(E,l)$

$$f_{l^+} = C\sin\alpha\,\tilde{\rho}(l,m) \quad\text{and}\quad f_{l^-} = C\cos\alpha\,\tilde{\rho}(l,m). \tag{3}$$

A similar form of the amplitudes is also used for NN and $\bar{N}N$ reactions. The parameter D in Eq. (2) was found in Ref. 4 to be 1.6 GeV for all reactions and to be responsible for the average behavior of all scattering cross sections. $C\sin\alpha$ ($C\cos\alpha$) is proportional to the coupling constant of the resonances to the different s-channel amplitudes and differs from reaction to reaction.

Forty-eight additional cross sections have been analyzed using the above model. The literature from late 1974 was searched and new reactions with the following beam and target particles were found: $K^{\pm}p$, $K^{\pm}n$, $K^0_l p$, pp, pn, nn, $\bar{p}p$, $\bar{p}n$, $\pi^{\pm}p$, and $\pi^{\pm}n$.

The free parameters not determined by the model are the angle α and the scale factor C. The scale factor was fixed through normalization. Normalizations to the low-t data points were examined and the best fit chosen. The angle α was varied from 0° to 180° in increments of 10°. The difficulty with χ^2 fitting is that a best χ^2 fit does not necessarily imply a best structural fit. In our case the predicted function varied slowly enough with respect to α that structural analysis was not lost through χ^2 fitting. Fits were judged through χ^2 minimization and found to be comparable to those of Ref. 4. They exhibit dip-bump structure and are valid up to the first dip but are not accurate for large t where the model breaks down. The parameter D was found to have the same value independent of the quantum numbers of the incoming particles and was once again found to be equal to 1.6 GeV for this new grouping of cross sections. D is the most important factor in determining the width of the diffraction peak of the differential cross section, which seems to have a universal value independent of the quantum numbers of the scattered particles.

Both data and predicted scattering cross sections are presented in Fig. 1. We have noticed that there is a tendency to obtain a poor fit for those

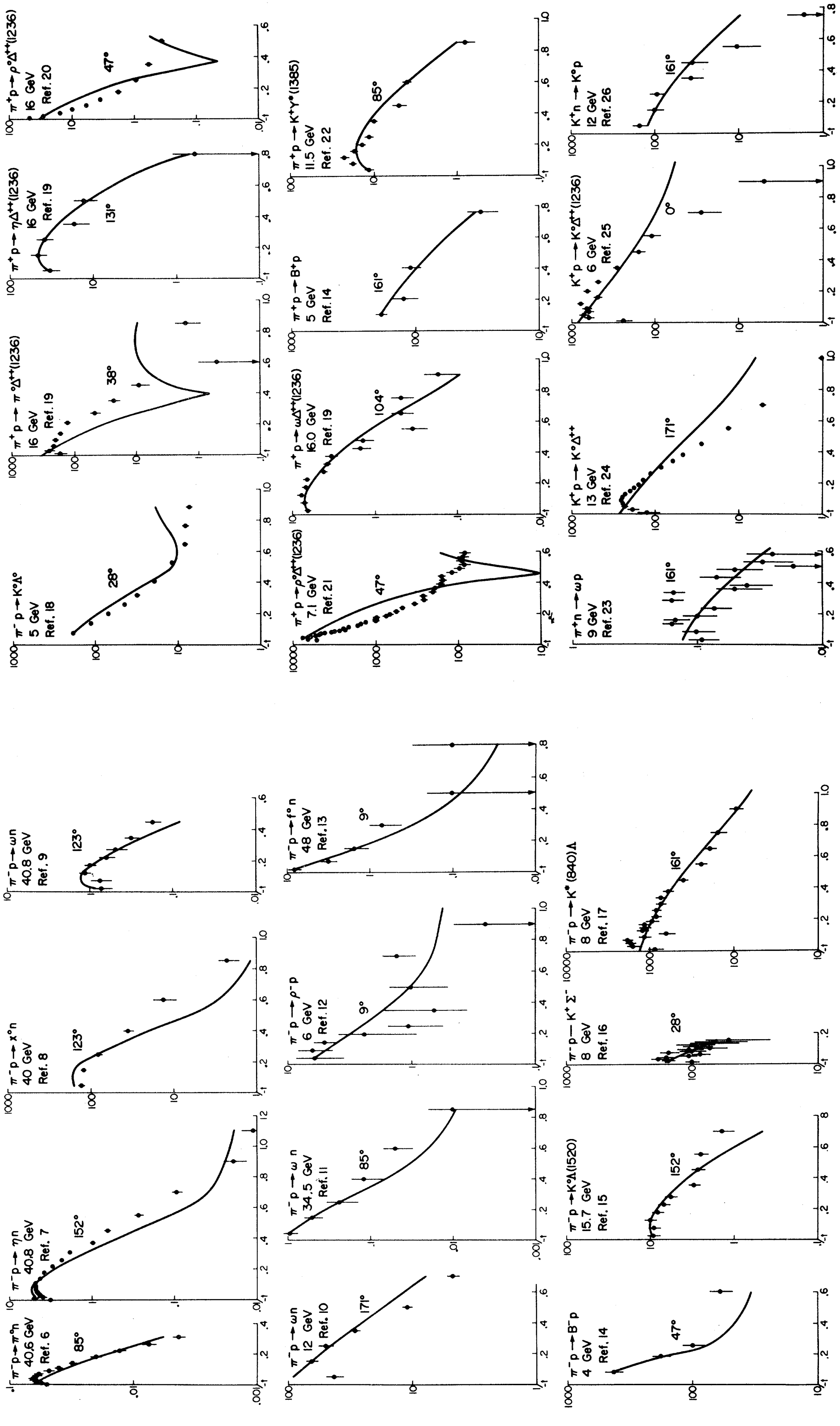

FIG. 1. Fit to inelastic differential cross sections.

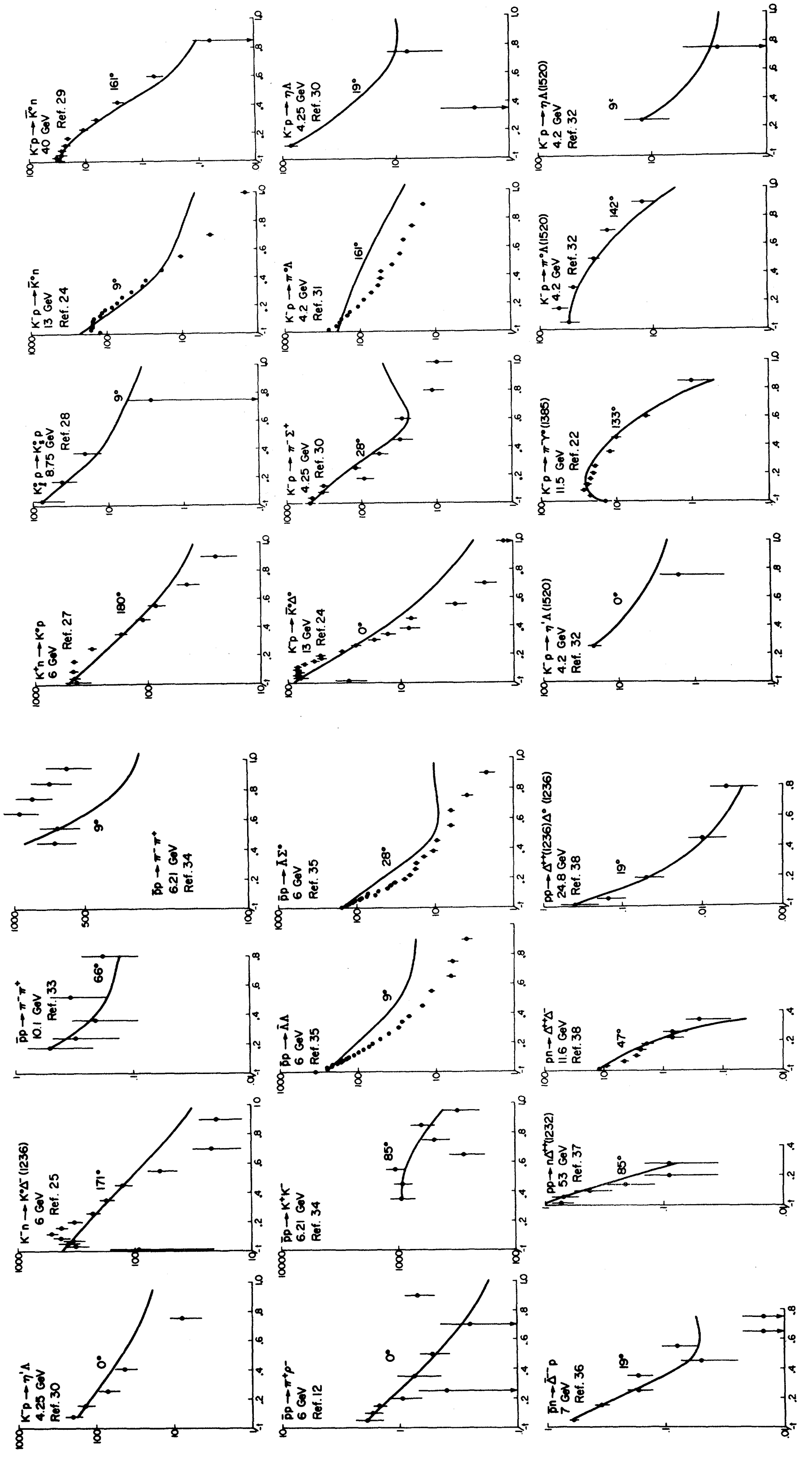

FIG. 1. (*Continued.*)

reactions for which there is the exchange in the t channel of the quantum numbers of the K meson. We have no explanation for this phenomenon. Equally mysterious is the fact that we are able to describe reactions such as $K^+n \to K^0p$ and $K^+n \to K^0\Delta^{++}(1236)$ where the s channel is exotic and hence there are no s-channel resonances.

---

[1] R. Hagedorn, Nuovo Cimento Suppl. 3, 147 (1965).

[2] F. Johns *et al.*, Phys. Rev. Lett. 29, 756 (1972).

[3] C. B. Chiu and R. L. Heimann, Phys. Rev. D 4, 3184 (1971); C. B. Chiu, *ibid.* 13, 2091 (1976).

[4] S. Kogitz, R. K. Logan, and S. Tanaka, Phys. Rev. D 14, 1420 (1976).

[5] S. Frautschi, Phys. Rev. D 3, 2821 (1971).

[6] B. R. Desai *et al.*, Phys. Rev. D 11, 2449 (1975).

[7] O. I. Dahl *et al.*, Phys. Rev. Lett. 39, 1243 (1977).

[8] W. D. Apel *et al.*, Yad. Fiz. 25, 561 (1977) [Sov. J. Nucl. Phys. 25, 300 (1977)].

[9] O. I. Dahl *et al.*, Phys. Rev. Lett. 38, 54 (1977).

[10] J. D. Dowell *et al.*, Nucl. Phys. B108, 30 (1976).

[11] V. N. Bolotov *et al.*, Yad. Fiz. 21, 316 (1975) [Sov. J. Nucl. Phys. 21, 166 (1975)].

[12] D. R. Green *et al.*, Phys. Rev. Lett. 39, 1243 (1977).

[13] V. N. Bolotov *et al.*, Yad. Fiz. 20, 1214 (1974) [Sov. J. Nucl. Phys. 20, 635 (1975)].

[14] A. C. Irving and V. Chaloupka, Nucl. Phys. B89, 345 (1975).

[15] K. J. Foley *et al.*, Phys. Rev. D 15, 609 (1977).

[16] D. Dekkers *et al.*, Phys. Rev. Lett. 40, 681 (1978).

[17] H. Brundiers *et al.*, Nucl. Phys. B119, 349 (1977).

[18] C. E. W. Ward *et al.*, Phys. Rev. D 11, 1802 (1975).

[19] R. Honecker *et al.*, report, 1977 (unpublished).

[20] R. Honecker *et al.*, Nucl. Phys. B106, 365 (1976).

[21] S. U. Chung *et al.*, Phys. Rev. D 12, 693 (1975).

[22] J. Ballam *et al.*, Phys. Rev. Lett. 41, 676 (1978).

[23] P. Bandyopadhyay *et al.*, Acta Phys. Pol. B6, (1975).

[24] G. W. Brandenburg *et al.*, Phys. Rev. D 15, 617 (1977).

[25] J. J. Phelan *et al.*, Phys. Lett. 61B, 483 (1976).

[26] A. Firestone *et al.*, Phys. Rev. Lett. 25, 958 (1970).

[27] R. Diebold *et al.*, Phys. Rev. Lett. 32, 904 (1974).

[28] G. W. Brandenburg *et al.*, Phys. Rev. D 9, 1939 (1974).

[29] W. D. Apel *et al.*, Nucl. Phys. B129, 275 (1977).

[30] R. Blokziji *et al.*, Nucl. Phys. B51, 535 (1973).

[31] F. Marzano *et al.*, Nucl. Phys. B123, 203 (1977).

[32] S. J. M. Barlag *et al.*, Phys. Rev. D 12, 693 (1975).

[33] A. Berglund *et al.*, Nucl. Phys. B137, 276 (1978).

[34] T. Buran *et al.*, Nucl. Phys. B116, 51 (1976).

[35] H. Becker *et al.*, Nucl. Phys. B141, 48 (1978).

[36] R. W. B. Ardill *et al.*, Acta Phys. Austriaca 44, 119 (1976).

[37] H. DeKerret *et al.*, Phys. Lett. 69B, 372 (1977).

[38] R. W. B. Ardill *et al.*, Acta Phys. Austriaca 46, 27 (1976).