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Brief Reports

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Two-fireball model for hadron production in lepton-hadron scattering

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A two-fireball model for hadron production, related to the model developed in Phys. Rev. D 20, 1093 (1979), reproduces the average charged-secondary-hadron multiplicity and the dispersion $D = (\langle n^2\rangle - \langle n\rangle^2)^{1/2}$ observed in deep-inelastic neutrino-proton scattering.

Even if quantum chromodynamics turns out to be the correct theory of strong interactions, phenomenological models of secondary-hadron production may be necessary or useful. This is because hadronization is a soft process involving large values of the coupling constant, and direct calculations are likely to be extremely difficult.

Preceding papers[1] have discussed a simple two-fireball model for secondary-hadron production in e^+e^- annihilation which successfully reproduced the general features of the experimental data. More complex fireball models for secondary-hadron production in nucleon-nucleon scattering have also been developed.[2]

As part of the process of determining whether fireball models of the type discussed in Refs. 1 and 2 are generally useful for describing secondary-hadron production, this paper reports on a two-fireball model used to calculate the average multiplicity and the dispersion $D = (\langle n^2\rangle - \langle n\rangle^2)^{1/2}$ of charged secondary hadrons produced in neutrino-proton scattering.

Let W be the invariant mass of the excited hadronic system generated in an inelastic lepton-nucleon scattering event. Assume for simplicity that all quarks have an effective mass m_Q equal to one third of the nucleon mass m_N and that the final-state hadrons consist of one nucleon and one or more pions.

The excited hadronic system immediately after a deep-inelastic lepton-hadron scattering event comprises a quark and a diquark flying apart in opposite directions in the total hadronic c.m. frame with momentum p determined by

$$p^2 = \left(\frac{W^2 - 3m_Q{}^2}{2W}\right)^2 - m_Q{}^2 .$$

The quark and the diquark move apart until the potential energy of interaction between them grows so large that a new quark-antiquark ($Q\bar{Q}$) pair can be formed. By a mechanism similar to that discussed in Ref. 1, this leads to a color-neutral $Q\bar{Q}$ pair (the mesonic fireball) and a color-neutral excited three-quark system (the baryonic fireball) flying apart with negligible interaction in comparison with the superstrong color forces acting within the fireballs. The initial relative momentum of the two systems is $p_{mB} = 2p$ and the change in relative momentum during the time t_F between the scattering and the formation of the new $Q\bar{Q}$ pair (when the force between the two systems becomes negligible) is

$$\Delta p_{mB} = \int_0^{t_F} F_{mB}\, dt ,$$

where F_{mB} is the superstrong force between the two systems. Since the relative velocity of the two systems is approximately equal to the velocity of light, $dr_{mB} = c\, dt$, where r_{mB} is the distance between the centers of momentum of the two systems. Then, in units where $c = 1$,

$$\Delta p_{mB} = \int_0^{t_F} F_{mB} dt = -\int \frac{dV_{mB}}{dr_{mB}} dr_{mB}$$
$$= -\int_{V_E}^{V_F} dV_{mB} = -(V_F - V_E) ,$$

where V_{mB} is the potential energy of the interaction between the quark and the diquark, V_E is its equilibrium value when the quark and diquark are bound in a nucleon, and V_F is the value of the potential at the moment when the new $Q\overline{Q}$ pair is formed. Since the new $Q\overline{Q}$ pair is formed within a volume of roughly hadronic dimensions, the uncertainty principle requires the pair to have a certain minimum momentum p_Q in the overall hadronic c.m. frame. Therefore, as in Ref. 1, it can be assumed that

$$V_F - V_E = 2(p_Q^{\,2} + m_Q^{\,2})^{1/2} = \Delta .$$

Then, the final momentum of each of the fireballs in the overall hadronic c.m. frame is $p' = p - \Delta/2$ and the invariant mass of the mesonic fireball (m_m^*) and the baryonic fireball (m_B^*) is found from

$$m_m^{*2} = p\Delta - \Delta^2/4 + m_Q^{\,2} ,$$

$$m_B^{*2} = p\Delta - \Delta^2/4 + 4m_Q^{\,2} .$$

The two fireballs decay into secondary hadrons, producing back-to-back jets in the total hadronic c.m. frame. The mesonic fireball must produce at least one pion, and the available energy A_m for producing additional pions from that fireball is $A_m = m_m^* - m_\pi$, where m_π is the pion mass. The baryonic fireball must produce at least one nucleon and the available energy A_B for producing additional pions from the baryonic fireball is $A_B = m_B^* - m_N$.

The fireballs are assumed to decay isotropically in their own rest frame with a (truncated) Poisson distribution in multiplicity. The probability of a fireball producing n additional pions is given by

$$p_x(n) = \alpha_x^{-1}[A_x/(p_H^{\,2} + m_\pi^{\,2})^{1/2}]^n (1/n!) \times \exp[-A_x/(p_H^{\,2} + m_\pi^{\,2})^{1/2}] , \quad 0 \le n \le N_x ,$$

where x denotes m or B, respectively, $(p_H^{\,2} + m_\pi^{\,2})^{1/2}$ is the (constant) average energy needed to produce a secondary pion in the fireball rest frame, $p_H^{\,2}$ is the square of the (constant) average momentum of a secondary pion in the fireball rest frame, N_x is the largest integer less than or equal to A_x/m_{π^0}, and the normalization constant α_x is given by

$$\alpha_x = \sum_{n=0}^{N_x} [A_x/(p_H^{\,2} + m_\pi^{\,2})^{1/2}]^n (1/n!) \times \exp[-A_x/(p_H^{\,2} + m_\pi^{\,2})^{1/2}] .$$

Since the probability of a pion pair being charged is $\frac{2}{3}$, the probability $P_x'(j)$ of producing j additional charged pion pairs from a fireball is

$$P_x'(j) = \sum_{i=j}^{N_x/2} \left(\frac{2}{3}\right)^j \left(\frac{1}{3}\right)^{1-j} \frac{i!}{j!\,(1-j)!} \times [P_x(2i) + P_x(2i+1)] .$$

For comparison with experimental data on $\nu + p \to \mu^- + \text{hadrons}$,[3] where there at least two positively charged hadrons in the final state, the probability of finding $2n+2$ charged hadrons in the final state is found by convoluting the probability distributions for the two fireballs

$$P_c(2n+2) = \sum_{j=0}^{n} P_m'(n-j) P_B'(j) .$$

Using $\langle n^a \rangle = \sum_{n=0}^{\infty} n^a P_c(2n+2)$, the average charged-hadron multiplicity $\langle n \rangle$ and the dispersion $D = (\langle n^2 \rangle - \langle n \rangle^2)^{1/2}$ can be calculated for comparison with experiment.

The average momentum of a secondary pion in one of the fireball rest frames is chosen as $|\vec{p}_H| = 440$ MeV/c.[1,2] This implies that secondary hadrons are produced in back-to-back jets in the total hadronic c.m. frame with a constant average secondary-hadron momentum transverse to the jet direction which is approximately equal to $(\frac{2}{3})^{1/2} |\vec{p}_H| = 360$ MeV/c.

The calculated results for the charged-hadron multiplicity and dispersion in $\nu + p \to \mu^- + \text{hadrons}$, denoted by crosses, are compared with the experimental results from Ref. 3, denoted by dots, in Fig. 1. As in Ref. 1, $|\vec{p}_Q|$ is assumed equal to $|\vec{p}_H|$ for simplicity, so $\Delta = 1.08$ GeV/c. When the charged-hadron probabilities $P_c(m)$ are calculated, they show the same general behavior as the experimental probabilities given in Ref. 3, but a graphical display is not particularly illuminating.

As in Refs. 1 and 2, this calculation assumes that (a) hadron jets are produced by the isotropic decay in their own rest frame of fireballs gener-

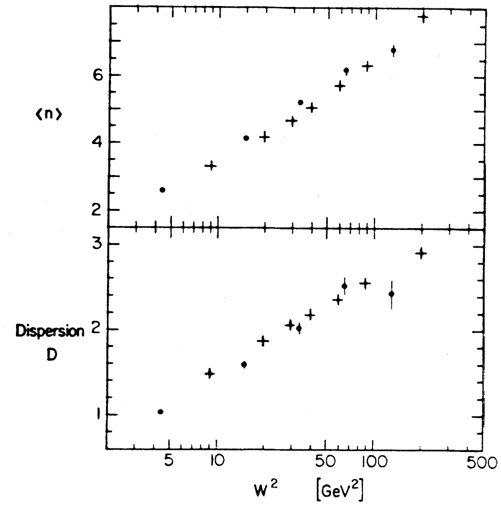

FIG. 1. The average charged-hadron multiplicity and the dispersion $D = (\langle n^2 \rangle - \langle n \rangle^2)^{1/2}$ in $\nu + p \to \mu^- + \text{hadrons}$, plotted against the square of the invariant mass of the final-state hadronic system. The dots are the experimental points from Ref. 4 and the crosses are the calculated points.

ated in quark scattering processes; (b) fireball decay multiplicities follow truncated Poisson distributions; (c) the average decay multiplicity increases linearly with fireball invariant mass; and (d) the average momentum of secondary hadrons transverse to the jet direction (fireball momentum vector) is constant. The calculation demonstrates that a simple two-fireball model reproduces the average charged-secondary-hadron multiplicity and the dispersion $D = (\langle n^2\rangle - \langle n\rangle^2)^{1/2}$ observed in deep-inelastic neutrino-proton scattering. Consequently, charged-hadron multiplicities in lepton-hadron scattering and e^+e^- annihilation at presently accessible energies can be estimated with similar very simple models.

[1]T. R. Mongan, Phys. Rev. D 20, 1093 (1979); 21, 1431 (1980).

[2]T. R. Mongan, Phys. Rev. D 22, 1538 (1980); 23, 2554 (1981).

[3]H. Saarikko, in *Neutrino '79*, proceedings of the International Conference on Neutrinos, Weak Interactions, and Cosmology, Bergen, Norway, 1979, edited by A. Haatuft and C. Jarlskog (Univ. of Bergen, Bergen, 1980), Vol. 2, p. 507.