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D waves in the nucleon: A test of color magnetism

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The Becchi-Morpurgo selection rule that the decay $\Delta \rightarrow N\gamma$ is pure $M1$ was an early success of the quark model. We show that quark hyperfine interactions produce D waves in the nucleon and Δ which lead to a small violation of this rule. The observation of this effect, especially when combined with earlier tests for mixed-symmetry S waves in the nucleon, would provide good evidence for the presence of the color magnetism expected from one-gluon exchange. Existing measurements give some indications for the predicted effect.

I. INTRODUCTION

The study of low-energy hadron physics has been strongly influenced by the present hegemony of quantum chromodynamics. While the rigorous derivation of the connection between the realms of high-q^2 phenomena and soft hadronic properties is lacking, it is nonetheless possible to make some plausible conjectures about the structure of this latter regime.[1] In the resulting models the unknown long-distance properties of QCD are subsumed into an unknown confining potential or bag, while it is assumed that the remaining interquark forces will essentially be dominated by the calculable and electromagnetic-type one-gluon exchange.[2]

While the fate of such conjectures is for the moment uncertain, at least one aspect of the resulting models has received support from almost every quarter: the color hyperfine interaction. This interaction, which is the color analog of the magnetic-dipole—magnetic-dipole interaction of electromagnetism, is of the form[3] (see Fig. 1)

$$H_{\rm hyp}^{ij}=A^{ij}\left[\frac{8\pi}{3}\vec{S}_i\cdot\vec{S}_j\delta^3(\vec{r}_{ij})+\frac{1}{r_{ij}^{\;3}}\left[\frac{3\vec{S}_i\cdot\vec{r}_{ij}\vec{S}_j\cdot\vec{r}_{ij}}{r_{ij}^{\;2}}-\vec{S}_i\cdot\vec{S}_j\right]\right], \qquad (1)$$

where in QCD in lowest order

$$A^{ij}=\begin{cases}\dfrac{2}{3}\dfrac{\alpha_s}{m_im_j} & \text{between } q_i \text{ and } q_j \text{ in a baryon },\\[2ex] \dfrac{4}{3}\dfrac{\alpha_s}{m_im_j} & \text{between } q_i \text{ and } \bar{q}_j \text{ in a meson }.\end{cases} \qquad (2)$$

This interaction has a number of distinctive parameter-independent features including (A) its sign in baryons and mesons, (B) its relative strength in baryons and mesons, (C) the $(m_im_j)^{-1}$ dependence on the quark masses, (D) the short

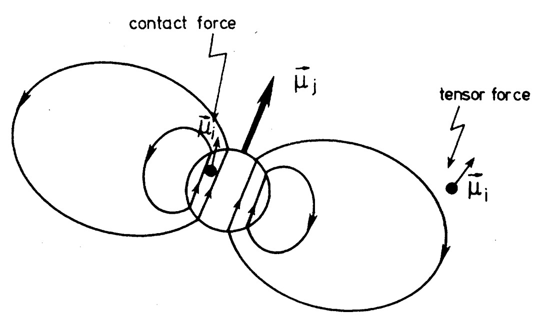

FIG. 1. The hyperfine interaction, showing the contact and tensor forces and the relation between them.

range of the $\vec{S}_i \cdot \vec{S}_j$ (contact) interaction, and (E) the relative strength and sign of the tensor and contact terms. Of these features, (A) and (B) reflect the underlying color symmetry; (C), (D), and (E) are characteristic of elementary-massless-vector exchange.

Among the successes of this interaction are the following.

(1) It automatically predicts[1] $M_\rho > M_\pi$ and $M_\Delta > M_N$ and relates[2,4] the sizes of these two splittings (A,B).

(2) It explains[1,5] the Σ-Λ mass difference (C).

(3) It correctly gives[1] mass differences such as $\Sigma^* - \Sigma$, $\Xi^* - \Xi$, $K^* - K$, and $D^* - D$.

(4) It provides an explanation for the pattern of P-wave [i.e., $(70,1^-)$] baryon masses (A,C,D) (Ref. 2, 5, and 6): The fact that the $S = \frac{1}{2}$ states $\Delta(1650)\frac{1}{2}^-$ and $\Delta(1670)\frac{3}{2}$ lie so near the (dominantly) $S = \frac{3}{2}$ states $N(1700)\frac{1}{2}^-$, $N(1700)\frac{3}{2}^-$, and $N(1670)\frac{5}{2}^-$ is, for example, due to the short range of the contact term; for a long-range $\vec{S}_i \cdot \vec{S}_j$ interaction the Δ's would lie at ~ 1500 MeV near the (dominantly) $S = \frac{1}{2}$ nucleonic resonances.

(5) It explains the mixing angles (i.e., decay patterns) of P-wave baryons (A,E) (Refs. 2, 5, and 7): For example, the mixing angle in the $N^* \frac{1}{2}^-$ sector between the ${}^2P_M$ and ${}^4P_M$ SU(6) states (our notation is ${}^{2S+1}L_\pi$ where $\pi = S,M,A$ is the symmetry of the spatial wave function) constitutes quite a direct measurement of the relative sign and magnitude of the contact and tensor terms and supports the $+8\pi/3$ of Eq. (1).

(6) It provides an explanation for the observed pattern of low-lying positive-parity baryon resonances (A,C,D) and for their decay patterns (A,C,D,E), (Refs. 2, 5, and 7): For example, the tensor force provides just the $(56,2^+)$-$(70,2^+)$ mixing angle suggested by phenomenological analyses to make the $\Delta(1890)\frac{5}{2}^+$ decay dominantly to an F-wave $\Delta\pi$ and to decouple the other predicted $\Delta\frac{5}{2}^+$ resonance from πN scattering.

(7) It predicts the existence of ${}^2S_M$ [i.e., $(70,0^+)$] components in the nucleon which explain the charge radius of the neutron[8,9] and violations of the SU(6) selection rules[9] $\Lambda\, {}^4P_M \not\to \bar{K}N$ and $N\, {}^4P_M \not\to p\gamma$ (A,D).

Of the five characteristics of hyperfine interactions we have listed above, we believe showing that the tensor term is present with the correct relative strength is the most convincing way of proving that the interactions are really of magnetic-dipole type, as illustrated by Fig. 1. Some of the evidence we have mentioned in favor of quark hyperfine interactions, namely (5) and (6) above, do indicate the feature (E). Indeed, we would be willing to accept this evidence as conclusive if it were not for one unsettling fact: one-gluon exchange also predicts the presence of spin-orbit effects from color magnetism, but the observed effects are much smaller than the predicted ones.[5,7,10] There may well be a simple reason for this failure: In a pure r^{-1} potential it is well known that Thomas precession effectively reduces the magnetic spin-orbit effects by a factor of 2; in an (r^{-1} + confinement) potential it can be shown that the reduction will always be greater. While it is very difficult to make these observations quantitative in baryons, there has been some success in showing both in charmonium and old mesons that this may indeed be the mechanism by which spin-orbit effects are suppressed.[10]

Nevertheless, in view of these uncertainties the tests (5) and (6) cannot be considered conclusive. It is the purpose of this paper to point out a test for quark hyperfine interactions that is independent of the uncertainties surrounding spin-orbit effects.

II. E2 ADMIXTURES IN THE DECAY $\Delta \to N\gamma$ AND RELATED EFFECTS

The hyperfine interaction (1) violates SU(6) and so introduces "impurities" into the ${}^2S_S$ [i.e., $(56,0^+)$] baryonic ground states. Since spin-orbit interactions have neither diagonal nor off-diagonal matrix elements in S waves these impurities are produced *solely* by hyperfine interactions. The effects of the ${}^2S_M$ impurity, which is produced by the contact term, have recently been discussed[9] and found to be in good agreement with the experimental data on the charge radius of the neutron[8,9] and on violations of the SU(6) selection rules[9] for $\Lambda\, {}^4P_M \to \bar{K}N$ and $N\, {}^4P_M \to p\gamma$. Here we point out that the tensor term will induce D waves in the Δ and the N which should have an observable effect on the decay $\Delta \to N\gamma$ allowing a test of the relative sign and magnitude of the tensor and contact terms independent of uncertainties arising from spin-orbit terms.[11]

As in Ref. 9 our discussion is framed in terms of the harmonic-oscillator model; we reserve consideration of the resulting model dependence of our conclusions to Sec. III. The interaction (1) will lead to a physical nucleon and delta with the approximate structures

$$|N\rangle \simeq 0.93\,|N\,^2S_S\rangle - 0.29\,|N\,^2S_S'\rangle - 0.23\,|N\,^2S_M\rangle - 0.04\,|N\,^4D_M\rangle + 0.00\,|N\,^2P_A\rangle \quad (3)$$

$$|\Delta\rangle \simeq 0.97\,|\Delta\,^4S_S\rangle + 0.20\,|\Delta\,^4S_S'\rangle - 0.10\,|\Delta\,^4D_S\rangle + 0.07\,|\Delta\,^2D_M\rangle\,, \quad (4)$$

where we have truncated the mixing series by including only the $N=2$ harmonic-oscillator levels.[12] These compositions are based on the matrix elements of (1) listed in Table I and on an extensive analysis[5] of the positive-parity baryons. The relevant wave functions have been given elsewhere.[2,5,7] These compositions differ somewhat from those given earlier[7,9] as a consequence of our having included here the effects of mixings which occur in the $N=2$ band as calculated in Ref. 5.

If we now define[2,7,13]

$$A_{3/2} = 3\sqrt{2}\mu_p \langle N(+\tfrac{1}{2})|\left[\frac{e_3}{e}\right]\left[q\frac{\sigma_3}{2} - i(\tfrac{2}{3})^{1/2}\overleftarrow{\nabla}^{\lambda}\right]_- \exp[i(\tfrac{2}{3})^{1/2}q\lambda_z]\,|\Delta(+\tfrac{3}{2})\rangle\,, \quad (5)$$

$$A_{1/2} = 3\sqrt{2}\mu_p \langle N(-\tfrac{1}{2})|\left[\frac{e_3}{e}\right]\left[q\frac{\sigma_3}{2} - i(\tfrac{2}{3})^{1/2}\overleftarrow{\nabla}^{\lambda}\right]_- \exp[i(\tfrac{2}{3})^{1/2}q\lambda_z]\,|\Delta(+\tfrac{1}{2})\rangle\,, \quad (6)$$

[see Ref. 7 for a full explanation of our notation; $-i(\frac{2}{3})^{1/2}\overleftarrow{\nabla}_\lambda$ is the momentum of the third quark in the final state] then we can calculate the radiative amplitudes between various components of the Δ and the N; the results are listed in Table II (the effects of the transitions that are second order in the mixing coefficients are negligible so they are not shown).

The two amplitudes $A_{3/2}$ and $A_{1/2}$ may be measured in the reaction $\gamma N \to \pi N$. If we define the magnetic dipole (M) and electric quadrupole (E) amplitudes

$$M \equiv \frac{\sqrt{3}}{2}A_{3/2} + \tfrac{1}{2}A_{1/2}\,, \quad (7)$$

$$E \equiv \tfrac{1}{2}A_{3/2} - \frac{\sqrt{3}}{2}A_{1/2}\,, \quad (8)$$

and the quantity

$$\xi \equiv \frac{E}{M}\,, \quad (9)$$

then $\xi = 0$ in the absence of D-wave components in the nucleon and Δ (Ref. 14), but with hyperfine interactions we expect from Eqs. (3) and (4) and Table II that

$$\xi_{\text{hyp}} \simeq +0.007\,. \quad (10)$$

TABLE I. Hyperfine mixing matrix elements.

$\langle N\,^2S_S\,|H_{\text{hyp}}|\,N\,^2S_S\rangle = -\dfrac{2\alpha_s\alpha^3}{3\sqrt{2\pi}m_d{}^2} \equiv -\tfrac{1}{2}\delta$
$\langle N\,^2S_S'\,|H_{\text{hyp}}|\,N\,^2S_S\rangle = +\dfrac{\sqrt{3}}{4}\delta$
$\langle N\,^2S_M\,|H_{\text{hyp}}|\,N\,^2S_S\rangle = +\dfrac{\sqrt{6}}{4}\delta$
$\langle N\,^4D_M\,|H_{\text{hyp}}|\,N\,^2S_S\rangle = +\dfrac{\sqrt{15}}{20}\delta$
$\langle \Delta\,^4S_S\,|H_{\text{hyp}}|\,\Delta\,^4S_S\rangle = +\tfrac{1}{2}\delta$
$\langle \Delta\,^4S_S'\,|H_{\text{hyp}}|\,\Delta\,^4S_S\rangle = -\dfrac{\sqrt{3}}{4}\delta$
$\langle \Delta\,^4D_S\,|H_{\text{hyp}}|\,\Delta\,^4S_S\rangle = +\dfrac{\sqrt{30}}{20}\delta$
$\langle \Delta\,^2D_M\,|H_{\text{hyp}}|\,\Delta\,^4S_S\rangle = -\dfrac{\sqrt{15}}{20}\delta$

TABLE II. Some radiative transition amplitudes in $\Delta \to N\gamma$ [in units of $(-2/\sqrt{3})\mu_p q e^{-q^2/6\alpha^2}$].

	$A_{3/2}$	$A_{1/2}$
$\Delta\,^4S_S \to N\,^2S_S$	1	$\dfrac{1}{\sqrt{3}}$
$\Delta\,^4S_S \to N\,^2S_S'$	$-\dfrac{\sqrt{3}}{18}\dfrac{q^2}{\alpha^2}$	$-\dfrac{1}{18}\dfrac{q^2}{\alpha^2}$
$\Delta\,^4S_S \to N\,^2S_M$	$-\dfrac{\sqrt{6}}{36}\dfrac{q^2}{\alpha^2}$	$-\dfrac{\sqrt{2}}{36}\dfrac{q^2}{\alpha^2}$
$\Delta\,^4S_S \to N\,^4D_M$	$+\dfrac{\sqrt{15}}{30} + \dfrac{\sqrt{15}}{90}\dfrac{q^2}{\alpha^2}$	$-\dfrac{\sqrt{5}}{10} - \dfrac{\sqrt{5}}{45}\dfrac{q^2}{\alpha^2}$
$\Delta\,^4S_S' \to N\,^2S_S$	$-\dfrac{\sqrt{3}}{18}\dfrac{q^2}{\alpha^2}$	$-\dfrac{1}{18}\dfrac{q^2}{\alpha^2}$
$\Delta\,^4D_S \to N\,^2S_S$	$-\dfrac{\sqrt{30}}{90}\dfrac{q^2}{\alpha^2}$	$+\dfrac{\sqrt{10}}{90}\dfrac{q^2}{\alpha^2}$
$\Delta\,^2D_M \to N\,^2S_S$	$+\dfrac{\sqrt{15}}{30}$	$-\dfrac{\sqrt{5}}{10} + \dfrac{\sqrt{5}}{90}\dfrac{q^2}{\alpha^2}$

The D waves in the Δ will also cause it to have a nonzero static quadrupole moment. One may easily calculate that

$$\frac{Q_\Delta}{\langle \sum_i e_i r_i^2 \rangle_\Delta} \equiv \frac{\langle \sum_i e_i (3z_i^2 - r_i^2) \rangle_\Delta}{\langle \sum_i e_i r_i^2 \rangle_\Delta} \simeq -0.07 \left| \frac{e_\Delta}{e} \right| , \tag{11}$$

a quite substantial moment. Finally we mention that there are related consequences of D-wave mixing in the semileptonic decay of the Ω^-: with mixing this otherwise purely axial-vector (Gamow-Teller) decay will have a small vector coupling amplitude.

III. DISCUSSION AND CONCLUSIONS

That the result (10) is so small is already a significant prediction. Its size is the result of a strong cancellation: if we were to reverse the relative sign of the contribution of the D waves in the N and the Δ from the prediction of our QCD-like model, ξ_{hyp} would become ± 0.03, almost five times larger; the minus sign especially would then already be in contradiction to experiment (see below).

There are at least two sources of uncertainty in the predictions (10) and (11). The most serious is probably the accuracy of the D-wave amplitudes in the expansions (3) and (4) for the N and the Δ. Since these expansions are "normalized" to spectroscopic splittings, they are independent of α_s, but they do depend on the use of harmonic-oscillator wave functions, and to some extent on our treatment of the singular hyperfine potentials.[12] However, it should be noted that similar uncertainties are involved in calculating the amplitude of $|N\,{}^2S_M\rangle$ in (3) and this amplitude would seem to be approximately correct.[9] In view of this it is difficult to see why the amplitudes for $|N\,{}^4D_M\rangle$ and $|\Delta\,{}^2D_M\rangle$ should be very wrong, especially considering that all three states belong to the $N=2$ harmonic-oscillator levels. The other cause for concern in these calculations is the radiative amplitudes themselves. The total amplitude for the $\Delta \to N\gamma$ transition is, for example, inaccurate at the level of 30%,[15] and although we may hope that the ratios we are concerned with here will be less inaccurate, without any understanding of the source of this discrepancy we really have no right to this hope. Finally, because the hyperfine-interaction-induced D-wave amplitudes are so small, we must also be concerned that effects of comparable size may arise from other sources. In particular, it is conceivable that mixing through virtual decay channels may be significant at this level.

Using the amplitudes of the Particle Data Group[16] we find that

$$\xi_{\text{exp}} = +0.025 \pm 0.025 \tag{12}$$

consistent with the prediction (10) but unable to test it. On the other hand, the individual measurements of ξ in the Particle Data Group compilation are all positive, and the two most recent analyses of which we are aware give $\xi = +0.017 \pm 0.008$ (Ref. 17) and $+0.019 \pm 0.025$ (Ref. 18). There is to our knowledge, unfortunately, essentially no data relevant to the prediction (11).

In view of the previous evidence in favor of the contact-term-induced $N\,{}^2S_M$ amplitude, and the absence of any complications from spin-orbit effects, we hope it will be possible to check these predictions more stringently and to thereby test the idea of color magnetism embodied in (1).

Note added in proof. While this paper was in the process of publication, S.S. Gershtein and Yu. M. Zinoviev (unpublished) have drawn attention to the quadrupole moment of the Ω^-, which they evaluated in an oscillator model with hyperfine interactions finding $1.8 \times 10^{-28}\,e\,\text{cm}^2$, a value which they claim is susceptible to experimental verification. This estimate has been checked by J. M. Richard (unpublished) using other methods by which he finds typically $0.4 \times 10^{-28}\,e\,\text{cm}^2$. (We believe that this smaller value results from constraining parameters to fit spectroscopic data.) We have repeated the calculations leading to (11) and, constraining our calculation to give the correct proton charge radius (as in Ref. 9), we find the even larger value $3.1 \times 10^{-28}\,e\,\text{cm}^2$. A measurement of the quadrupole moment of the Ω^- would constitute a nice demonstration of the existence of tensor components in the hyperfine interaction. We are indebted to these authors for sending us copies of their work.

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