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Do narrow heavy multiquark states exist?

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We discuss the existence of states made of four heavy quarks in the context of potential models already used in the study of heavy mesons and baryons. We first consider the situation where the quarks have the same mass and interact through a two-body potential due to color-octet exchange. In this case, we show that for any reasonable confining potential there is no state below the threshold corresponding to the spontaneous dissociation into two mesons. We investigate in detail different possibilities of modifying this negative result. This concerns the effect of hyperfine corrections, the case of orbitally excited states, the case of unequal quark masses, and the use of the static potential derived from the bag model treated in the adiabatic approximation.

I. INTRODUCTION

In nuclear physics, stable or metastable nuclei exist with a great variety of values for the baryon number. In particular, it is quite remarkable that the α particle lies below the threshold for the decay into two deuterons. In quark physics, one of the most important problems today, experimentally and theoretically, is whether or not narrow multiquark states do exist. In this paper we do not insist (except at the end for P states) on the narrowness due to the dynamical suppression of kinematically allowed decays, since the literature on baryonium, etc.,[1–4] is already very rich. Instead we concentrate more on the mass spectrum than on the disintegration mechanisms and we investigate in what way bunching quarks together affects their binding energy. More explicitly, we compare the masses of four-quark states $(QQ\bar{Q}\bar{Q})$ with the threshold for the superallowed decay into two quarkonia, i.e., $(Q\bar{Q})+(Q\bar{Q})$. Our methods and even some of our results can be generalized to the case of more complicated quark composites.

Here and throughout this article, Q means a heavy quark, i.e., c, b, t,... and possibly the strange quark s. We exclude from the discussion the case of light quarks since we use nonrelativistic potentials, and, in one section, the adiabatic approximation of the bag model, which holds only if the quarks move slower than the surface of the bag. Moreover, the pion is so light that there is little hope of satisfying the inequality $M(qq\bar{q}\bar{q})\leq 2M(q\bar{q})$ with ordinary quarks. From the theoretical point of view, there is an important difference between light- and heavy-quark spectroscopies. In the former case, the chromomagnetic force, or, more generally, the hyperfine interaction, produces mass splittings as large as the quark effective mass itself. For heavy enough quarks, on the other hand, the spin-independent interaction is presumably dominant, so the existence of multiquark states should rely less on a particular spin configuration. As a consequence, flavor enters into the game in a different way. In light-multiquark spectroscopy one prefers antisymmetric flavor combinations which allow attractive coherences in the chromomagnetic interaction. For heavy quarks, flavor acts mostly through the mass of the quarks. As we shall see, mixing quarks with very different masses helps to bind them together and increases the chances of getting a narrow multiquark state.

The main difficulty of our investigation consists of choosing a model for the interaction between the quarks. We first considered additive models made of two-body potentials governed by the simplest color dependence. Within such a frame, in the case of equal masses, the ground state lies above the dissociation threshold as we shall show in quite a general and rigorous way. Examples are given using popular potentials such as "Coulomb-plus-linear" or power laws. An objection could be that, although these models are successful phenomenologically in the mesonic sector, their generalization to the case of multiquarks is rather arbitrary. There are fortunately some quark-antiquark potentials which, although less easily tractable, have a deeper theoretical basis and can in principle be extended to multiquark states without any new parameters or new assumptions. Some attempts have been made in the framework of instantons.[5] Another approach is the bag model, suitably adapted to the case of heavy quarks.[6] It has already been applied for mesons[7] and baryons.[8] A first attempt is made here to use it for four-quark states, and the results are rather encouraging since our rough estimate gives us some masses at the edge of the threshold.

This paper is organized as follows. In Sec. II, we discuss the general properties of the ground state with a pairwise, color-octet exchange potential in the case of equal masses. The effect of mixing quarks with different masses is discussed in Sec. III. We study in Sec. IV the role of the multibody components contained in the potential which is derived from the bag model. In Sec. V, we discuss the effect of hyperfine corrections. We consider in Sec. VI the case of the P states. The last section is devoted to our conclusions.

II. PAIRWISE OCTET-EXCHANGE POTENTIALS

We first consider two-body potentials. Within this drastic restriction, there is a crucial problem concerning the color dependence of the force. From several arguments,[9,10] the only simple and reasonable choice consists of assuming that the potential is due to the exchange of color octets, i.e.,

$$V_{Q_1Q_2}=\tilde{\lambda}_1\tilde{\lambda}_2 V_8(r_{12}) , \tag{2.1}$$

where Q_i denotes a quark or an antiquark. In the latter case, $\tilde{\lambda}_i$ means $-\tilde{\lambda}_i^*$. Of course, a small amount of nonconfining color-singlet admixture cannot be excluded. It would, however, make the model more complicated.

To handle the interaction (2.1), we make an approximation for the treatment of the color degree of freedom inside the color-singlet $QQ\bar{Q}\bar{Q}$ state. In a proper account of color mixing, the spatial wave function would have two components and the potential would be a 2×2 matrix in color space. For instance, if one uses the basis of so-called[2] "true" states $(QQ\text{-}\bar{Q}\bar{Q})=(\bar{3}\text{-}3)$ and "mock" states $(6\text{-}\bar{6})$ where the diquarks have a well-defined color, one should set

$$\psi=\psi_T(\vec{r}_i)\,|\,\bar{3}\text{-}3\rangle+\psi_M(\vec{r}_i)\,|\,6\text{-}\bar{6}\rangle \tag{2.2}$$

and

$$V=\begin{bmatrix} V_{TT} & V_{TM} \\ V_{TM} & V_{MM}\end{bmatrix} , \tag{2.3}$$

with

$$\begin{aligned} V_{TT}&=-\tfrac{8}{3}[V_8(r_{12})+V_8(r_{34})] \\ &\quad -\tfrac{4}{3}\sum_{\substack{i=1,2\\ j=3,4}} V_8(r_{ij}) , \\ V_{MM}&=\tfrac{4}{3}[V_8(r_{12})+V_8(r_{34})] \\ &\quad -\tfrac{10}{3}\sum_{\substack{i=1,2\\ j=3,4}} V_8(r_{ij}) , \\ V_{TM}&=-\frac{4}{\sqrt{2}}[V_8(r_{13})+V_8(r_{24}) \\ &\quad -V_8(r_{14})-V_8(r_{23})] . \end{aligned} \tag{2.4}$$

In our approximation, we assume that the color wave function is factorized, say

$$\psi=\psi(\vec{r}_i)\,|\,\psi_c\rangle \tag{2.5}$$

so each color operator in (2.1) is replaced by its expectation value $\langle\psi_c\,|\,\tilde{\lambda}_i\tilde{\lambda}_j\,|\,\psi_c\rangle$. Note that the color mixing is forbidden or strongly suppressed by the Pauli principle in some cases involving identical quarks. For instance, the ground state with spin 2 has almost pure $\bar{3}$-3 color structure if the two quarks or the two antiquarks are identical since the 6-$\bar{6}$ component is penalized by two degrees of orbital excitation.

For color-singlet multiquark states $\{Q_1,Q_2,\cdots,Q_n\}$, we thus treat the octet-exchange interaction (2.1) as a special case of one-channel potentials of the type

$$V(\vec{r}_1,\vec{r}_2,\cdots,\vec{r}_n)=\sum_{i<j} a_{ij}V_8(r_{ij}) \tag{2.6}$$

with

$$\sum_{i<j} a_{ij}=-\tfrac{8}{3}n ,$$

where $V_8(r)$ is a universal function, the same for any pair inside any hadron. The interaction (2.6) has interesting properties.

(i) Consider first the symmetric case ($a_{ij}=\bar{a}=-[16/3(n-1)]$ $\forall i<j$) for various values of n. Then, if $M_n^{(S)}$ denotes the mass of the ground state

$$\frac{M_2}{2}\le\frac{M_3^{(S)}}{3}\le\cdots\le\frac{M_n^{(S)}}{n}\ . \qquad (2.7)$$

This means, that for instance, with an additive potential of the type (2.1), one has $M(\Omega^-)\ge\frac{3}{2}M(\phi)$ (including the spin-spin corrections if one considers V_8 as being the spin-triplet potential). The proof is easy. Let us denote by E_n and ψ_n the energy and eigenfunction of the ground state of the symmetric Hamiltonian

$$H_n=\sum_{i=1}^{n}\frac{P_i^2}{2m}-\frac{16}{3(n-1)}\sum_{i<j}V_8(r_{ij})\ . \qquad (2.8)$$

H_n may be considered formally as a Hamiltonian for $(n+1)$ particles where the last one plays no rôle. Hence H_n and H_{n+1} act on the same Hilbert space $\mathscr{H}$ and one may use the well-known principle

$$E_n=\min_{\psi\in\mathscr{H}}\langle\psi\,|\,H_n\,|\,\psi\rangle\ . \qquad (2.9)$$

In particular,

$$E_n\le\langle\psi_{n+1}\,|\,H_n\,|\,\psi_{n+1}\rangle$$
$$=\frac{n}{n+1}E_{n+1}\,,\ \text{Q.E.D.}\ ,$$

where the symmetry properties of ψ_{n+1} have been extensively used.

(ii) We now consider the number of quarks n, as well as the total strength $\sum_{i<j}a_{ij}$, as being fixed. The question is what distribution of the a_{ij} gives the lowest mass for the ground state. It is easy to show that the symmetric case always gives the worse result, i.e., for any distribution (a_{ij})

$$M_n(a_{ij})\le M_n^{(S)}\ . \qquad (2.10)$$

The proof is absolutely similar to that of (2.7). Note that, since the Hamiltonian is a linear function of the a_{ij}'s, the ground-state energy is a concave function of these parameters,[11] so it is impossible to have any local minimum.

Intuitively, the generalization is that the less symmetric the a_{ij} distribution, the more deeply bound the ground state. Consider for instance the case $n=4$. It is easy to prove that, for any potential V_8, the distribution $\{a_{12}=6\bar{a}$, other $a_{ij}=0\}$ leads to a mass lower than that of two mesons $\{a_{12}=a_{34}=3\bar{a}$, other $a_{ij}=0\}$, which in turn lies below the symmetric ground state $\{a_{ij}=\bar{a}\forall i,j\}$. We now suppose for simplicity that only two values show up for the a_{ij}'s, say

$$a_{12}=a_{34}=a$$

and

$$a_{13}=a_{23}=a_{14}=a_{24}=a'\ .$$

Such a configuration appears generally in the problem of four colored quarks with the correspondence $a_{ij}=\langle\tilde{\lambda}_i\lambda_j\rangle$ if one considers simple color states. The two-meson case corresponds to $a=-\frac{16}{3}$ and $a'=0$, the true state to $a=-\frac{8}{3}$ and $a'=-\frac{4}{3}$, and the mock state to $a=\frac{4}{3}$ and $a'=-\frac{10}{3}$, as shown by Eq. (2.4).

If the two distributions (1) and (2) are such that $a^{(1)}<a^{(2)}<\bar{a}$, it follows from the above-mentioned concavity property of the lowest eigenvalue[11] that the masses satisfy $M^{(1)}<M^{(2)}<M^{(S)}$. In particular, this proves rigorously that with any additive potential (2.1), a true state lies higher than the threshold.

The comparison is, however, more difficult if the two distributions break the symmetry in a different manner, i.e., $a^{(1)}<\bar{a}<a^{(2)}$. This is the case if one wants to compare the mass of a mock state with the true one or with the dissociation threshold. Intuitively one expects

$$2M(Q\bar{Q})<M(6\text{-}\bar{6})<M(\bar{3}\text{-}3) \qquad (2.12)$$

since the spreads of the distribution satisfy

$$|a-a'|_{1\text{-}1}>|a-a'|_{6\text{-}\bar{6}}$$
$$>|a-a'|_{\bar{3}\text{-}3}\ . \qquad (2.13)$$

Indeed, since the mass M is maximum for a symmetric distribution ($a=a'$), there should follow rather naturally a behavior of the type

$$M-M^{(S)}\propto-(a-a')^2+\cdots\ . \qquad (2.14)$$

The hyperspherical formalism[12] sheds some light on this point. In the hyperscalar approximation ($L=0$), the effective hyperradial potential is

$$\begin{aligned}V_{\text{eff}}(\xi)&=\frac{12}{m\xi^2}+\left[\sum_{i<j}a_{ij}\right]\tilde{V}_0(\xi)\ ,\\ \tilde{V}_0(\xi)&=\int d\Omega_8V_0(r_{12})\Big/\int d\Omega_8\ ,\end{aligned} \qquad (2.15)$$

where ξ is the hyperradius and Ω_8 the set of eight angles describing the kinematics. One can see that, in this approximation, any a_{ij} distribution with a given sum leads to the same binding energy. The differences come from the nonhyperscalar corrections which, for the ground state, are given by the $L=2, 4, 6, \ldots,$ harmonics. In the case where all the strengths a_{ij} are equal, one needs at least $L=4$ to get again a completely symmetric harmonic, so the hyperscalar approximation is almost exact (as in the case of baryons[8,10]). For a nonsymmetric distribution, the ground state may contain some $L=2$ component. The negative shift due to the $L=2$ admixture to the dominant $L=0$ piece is roughly proportional to the square of the transition potential $\langle L=0 | V | L=2 \rangle$, i.e., essentially to the quadratic variance of the distribution of the a_{ij}'s. More detailed expressions can be easily written using the formulas given in Ref. 12. For a distribution of the type (2.11), there is only one $L=2$ harmonic directly connected to the $L=0$ one and one has $\langle L=0 | V | L=2 \rangle \propto (a-a')$. So, for small $(a-a')$ the solution of the coupled $L=0$ and $L=2$ equations leads to the behavior in the formula (2.14).

One should stress that dramatic effects can occur when high internal color excitations like sextet or octet are involved. As shown by Greenberg and Lipkin,[13] with a too sharp confinement like r^β with $\beta > 2$, the total Hamiltonian is unbounded below for 6-$\bar{6}$, and the inequality (2.12) can certainly be violated or become meaningless. Assuming a smooth behavior near the maximum, as in Eq. (2.14) or a saturation of the ground-state wave function by a few hyperspherical harmonics, as in the previous paragraph implies indeed implicitly that the potential is not too sharp. On the other hand, our rigorous result $2M(Q\bar{Q}) < M(\bar{3}\text{-}3)$ implies that there is no difficulty with the color $\bar{3}$-3, in agreement with Ref. 13.

To illustrate our inequalities (2.7) and (2.12) we choose two realistic phenomenological potentials, namely the popular "Coulomb-plus-linear"[14] model and a simple power law which has recently been shown to describe successfully all heavy mesons[15] as well as the Ω^-.[10] They are

$$V^{\mathrm{I}}_{Q\bar{Q}}(r) = -\tfrac{16}{3} V^{\mathrm{I}}_8(r) = -\tfrac{4}{3}\frac{\alpha_s}{r} + \lambda r \,, \tag{2.16}$$

$$V^{\mathrm{II}}_{Q\bar{Q}}(r) = -\tfrac{16}{3} V^{\mathrm{II}}_8(r) = A + B r^\beta \,. \tag{2.17}$$

In order to compute the masses of the $Q\bar{Q}$, QQQ, and $QQ\bar{Q}\bar{Q}$ states, we use a variational method which consists of determining the closest harmonic-oscillator wave function, as done by Gromes and Stamatescu, by Dias de Deus, Henriques, and Pulido,[16] and by many others. For the $Q\bar{Q}$ ground state, we use $\psi \propto \exp(-\tfrac{1}{2}\alpha r^2)$ and minimize

$$M^{\mathrm{I}}(\alpha) = 2m + \frac{3}{2}\frac{\alpha}{m} + \frac{2}{\sqrt{\pi}}\left[\frac{\lambda}{\sqrt{\alpha}} - \tfrac{4}{3}\alpha_s\sqrt{\alpha}\right]$$

or (2.18)

$$M^{\mathrm{II}}(\alpha) = 2m + \frac{3}{2}\frac{\alpha}{m} + A + B\frac{\Gamma(\tfrac{3}{2}+\tfrac{1}{2}\beta)}{\Gamma(\tfrac{3}{2})}\alpha^{-\beta/2}$$

with respect to α. Similar expressions exist for the P state. For a symmetric QQQ baryon, we use the trial wave function $\psi \propto \exp -\tfrac{1}{2}\alpha(\xi_1^2 + \xi_2^2)$ where $\vec{\xi}_1 = \vec{x}_2 - \vec{x}_1$ and $\vec{\xi}_2 = (2\vec{x}_3 - \vec{x}_1 - \vec{x}_2)/\sqrt{3}$. For a four-quark state, we introduce the Jacobi variables

$$\vec{\xi}_1 = \vec{x}_2 - \vec{x}_1 \,, \quad \vec{\xi}_2 = \vec{x}_3 - \vec{x}_4 \,,$$
$$\vec{\xi}_3 = (\vec{x}_3 + \vec{x}_4 - \vec{x}_1 - \vec{x}_2)/\sqrt{2} \tag{2.19}$$

and the trial wave function

$$\psi = N \exp(-\tfrac{1}{2}\alpha_1 \vec{\xi}_1^{\,2} - \tfrac{1}{2}\alpha_2 \vec{\xi}_2^{\,2} - \tfrac{1}{2}\alpha_3 \vec{\xi}_3^{\,2}) \tag{2.20}$$

allowing us to calculate analytically any matrix element of the type $\langle r_{ij}^{\beta} \rangle$. We recognize the crudeness of such an approximation, but we feel that this procedure is convenient for our purpose since the different configurations with $n=2$, 3, or 4 quarks are treated consistently. For instance, with the potential II, the rigorous solution of the Schrödinger equation (with unit mass and strength B) gives

$$\Delta M = M(QQQ) - \tfrac{3}{2} M(Q\bar{Q}) = 0.0266 \,. \tag{2.21}$$

Within the harmonic-oscillator approximation (HOA), the agreement is not impressive for both $M(QQQ)$ and $M(Q\bar{Q})$, but for ΔM we get the fairly good value $\Delta M = 0.0256$. Similarly a quantity like the mass gap between a state and its threshold is presumably well estimated with the HOA. Also, as it has been noted,[17] the HOA satisfies the virial theorem and the proper scaling properties[18] in case of power-law potentials. Moreover, the HOA preserves the inequalities (2.7) and (2.10) which we have derived for additive potentials of the type (2.7). Indeed our basic ansatz for the proof is the variational principle (2.9) and we now simply replace the Hilbert space $\mathscr{H}$ by the subset made of the Gaussian wave functions.

To fix the parameters of model I, we impose, to fit within the HOA, the $1S$ and $1P$ level of charmonium, i.e.,

$$M(1S)=\tfrac{3}{4}M(J/\psi)+\tfrac{1}{4}M(\eta_c)$$
$$=3.067 \text{ GeV} , \qquad (2.22)$$
$$M(1P)=[M(\chi_0)+3M(\chi_1)+5M(\chi_2)]/9$$
$$=3.523 \text{ GeV} .$$

We choose the same quark mass $m=1.35$ GeV as in the bag model we consider later (Sec. IV). This leads us to $\alpha_s=0.44$ and $\lambda=0.194$. If one compares it with standard fits, the latter value is slightly too high, due to the systematic error induced by the HOA. For the model II, we use the parameters and masses from a fit to quarkonium masses by Martin,[15] a fit which is a little distorted by the HOA. Inspection of our results in Table I shows that the four-quark states are rather far from the threshold. With such a large phase space, they will decay immediately by a "superallowed"[1] rearrangement into two mesons and never show up as structures in the spectrum.

To get some narrow heavy-multiquark states, one needs changes or improvements to the simple additive model (2.1). We shall discuss some possibilities in the following sections.

III. THE CASE OF UNEQUAL MASSES

So far, within the additive potential model (2.1), we were restricted to quarks or antiquarks having the same mass. The main result is that for any color wave function (assumed to be factorized), a four-quark state is above the threshold, namely $M(QQ\bar{Q}\bar{Q})\geq 2M(Q\bar{Q})$. We now show that this is not always true with unequal masses. We shall give some explicit examples and later discuss the problem more generally. Of course, we always assume that any mass is large enough to justify a nonrelativistic potential picture. We also restrict ourselves to central potentials which do not depend on the mass of the quarks. Such a property is expected in QCD, apart from small deviations due to recoil effects, as, e.g., in Darwin type of corrections to the static potential. Analyses based on explicit phenomenological models,[15,19] on general properties of the Schrödinger equation,[20] or on approximate solutions of the inverse problem[21] show that, indeed, the present data on heavy mesons do not require a mass dependence of the $Q\bar{Q}$ potential.

Let us first consider a system $QQ\bar{Q}'\bar{Q}'$ involving two masses m and m' and interacting through the chromoharmonic potential[22]

$$V_{ij}=-K\tilde{\lambda}_i\tilde{\lambda}_j r_{ij}^2 . \qquad (3.1)$$

Using the same Jacobi variables as previously [see Eq. (2.15)] and keeping to distributions of the type (2.11) with only two different color factors $a=\langle\tilde{\lambda}_1\tilde{\lambda}_2\rangle$ and $a'=\langle\tilde{\lambda}_1\lambda_3\rangle$, the Hamiltonian can be written

$$H=-\frac{\nabla_{\xi_1}^2}{m}-\frac{\nabla_{\xi_2}^2}{m'}-\frac{\nabla_{\xi_3}^2}{\mu}$$
$$+K(a+a')(\xi_1^2+\xi_2^2)+2Ka'\xi_3^2 \qquad (3.2)$$

with $\mu=2mm'/m+m'$. (As in Sec. II we disre-

TABLE I. $c\bar{c}$, ccc, and $cc\bar{c}\bar{c}$ masses with potentials I and II in the harmonic-oscillator approximation. Units are GeV and GeV^2.

			Model I	Model II
$c\bar{c}$	$1S$	M	3.067	3.082
		α	0.343	0.431
	$1P$	M	3.523	3.513
		α	0.223	0.273
ccc	$(1S)^2$	M	4.765	4.778
		α	0.259	0.328
$cc\bar{c}\bar{c}$	$\bar{3}$-3	M	6.437	6.450
$(1S)^3$		$\alpha_1=\alpha_2$	0.249	0.316
		α_3	0.195	0.247
	6-$\bar{6}$	M	6.383	6.400
		$\alpha_1=\alpha_2$	0.186	0.236
		α_3	0.319	0.404

gard the effect of color mixing, included in Ref. 22, for the case $m'=m$.) The binding energy of a four-quark state can be easily compared to that of the threshold. For a mock and a true state the relative differences are, respectively,

$$R_6=\frac{E(6\text{-}\bar{6})-E(1\text{-}1)}{E(1-1)}=\frac{\sqrt{3}(1+\sqrt{x})-(1+x)^{1/2}(4-\sqrt{5})}{4(1+x)^{1/2}} \tag{3.3}$$

and

$$R_3=\frac{E(\bar{3}\text{-}3)-E(1\text{-}1)}{E(1\text{-}1)}=\frac{3(1+\sqrt{x})-(2\sqrt{6}-\sqrt{3})(1+x)^{1/2}}{2\sqrt{6}(1+x)^{1/2}}, \tag{3.4}$$

where $x=m'/m$. The above expressions exhibit interesting patterns. At $x\sim 50$, the true state becomes lighter than the mock one. When x continues to increase, the true, and later the mock state go below the threshold $2Q\bar{Q}'$. This appears at rather large x, at $x\simeq 300$ and $x\simeq 3000$, respectively. For more reasonable values of $x\sim 5-20$, the states, although still above, are not very far from the threshold.

We have examined the effect of the mass differences with several other potentials, always using the HOA. From our investigations it appears that the ordering of the states at large x depends upon the shape of the potential. Consider for instance the case of power-law potentials r^β. For $\beta<0$ (nonconfining) or $0<\beta\lesssim 1.9$, the asymptotic ordering in the HOA is

$$M(\bar{3}\text{-}3)<2M(Q'\bar{Q})<M(6\text{-}\bar{6}) . \tag{3.5}$$

In particular, with the potential of Martin[15] ($\beta=0.1$), the true state is bound for $x\gtrsim 18$. So, within the present limit on the mass of the t quark a $tt\bar{s}\bar{s}$ would be found. For $\beta\sim 2$, the ordering becomes

$$M(\bar{3}\text{-}3)<M(6\text{-}\bar{6})<2M(Q'\bar{Q}) \tag{3.6}$$

as already mentioned. For large β, as $\beta=3$, one has

$$M(6\text{-}\bar{6})<2M(Q'\bar{Q})<M(\bar{3}\text{-}3) . \tag{3.7}$$

With our "Coulomb-plus-linear" potential (2.16), we also get the true state below the threshold at large x, say $x>x_0$. The value of x_0 as a function of the lightest quark mass m is displayed in Table II. Inspection of the results shows that a $tt\bar{c}\bar{c}$ state should be narrow, provided one takes seriously this simple potential and the color transformation (2.1).

Now it seems worthwhile to compare the effects due to unequal masses with our previous results for identical quarks. Remember that for equal masses the lowest $QQ\bar{Q}\bar{Q}$ state is obtained through the strongest asymmetry of the color factors giving the level ordering (2.12) $(1\text{-}1)\leq(6\text{-}\bar{6})\leq(\bar{3}\text{-}3)$. In the case of unequal masses, there is another type of asymmetry which consists of clustering the two heaviest quarks together with the maximal color interaction between them. For a $QQ\bar{Q}'\bar{Q}'$ configuration, the latter asymmetry favors the $(\bar{3}\text{-}3)$ state and, as shown by our above analysis, can become the leading pattern for some kind of interactions. For instance, a harmonic potential gives at large x the level ordering (3.6) in complete opposition to the result for equal masses.

Another interesting example is provided by the configuration $(QQ'\bar{Q}\bar{Q}')$ which possesses two thresholds $T_1=(Q\bar{Q})+(Q'\bar{Q}')$ and $T_2=(Q\bar{Q}')+(Q'\bar{Q})$. The general concavity property,[11] if applied to the inverse reduced mass, tells us that, for any flavor-independent potential, $T_1\leq T_2$. Again it appears more economical energetically to enhance the asymmetry when distributing the masses. In T_1, indeed, the two heaviest masses are clustered together, whereas in T_2, they are mixed with the lightest ones. It is clear that a $(QQ'\bar{Q}\bar{Q}')$ quark composite would hardly beat the T_1 threshold which cumulates the maximal asymmetry in the distribution of the masses and in the distribution of the color factors a_{ij}. We therefore conclude that the states which are the most likely to appear in the spectrum are $(QQ\bar{Q}'\bar{Q}')$ rather than $(Q\bar{Q}Q'\bar{Q}')$, i.e., precisely those whose exotic flavor quantum numbers will give an unambiguous signa-

TABLE II. Minimum value of the quark-mass ratio x_0 required to get the true state $QQ\text{-}\bar{Q}'\bar{Q}'$ below the threshold with the Coulomb-plus-linear potential (in the HOA).

m (GeV)	x_0
0.5	16
1	12
2	10
5	8

ture as multiquarks. We are of course aware that producing and identifying such an exotic heavy meson would not be an easy experimental task.

IV. THE BAG MODEL

The simple additive model (2.1) is far from being a completely satisfactory choice for the potential energy between quarks. Its main motivation is certainly simplicity. Now, QCD seems quite involved in the infrared region, so simplicity is not necessarily a good agreement in guessing the quark dynamics. Let us be more precise. One of our major arguments in Eq. (2.1) for the suppression of a sizable color-singlet exchange component is that it would lead to confining forces between hadrons,[22] or, say, between an antiquark and a baryon.[23] However, if the octet exchange potential (2.1) is taken seriously, it still produces disturbing van der Waals forces between hadrons[13,24,25] in contradiction with the standard Yukawa picture which works very well at large distances. It is precisely the bag model which eliminates the problem of spurious van der Waals forces, since the gluons which mediate the interaction cannot escape out of a bag and propagate in the medium to reach another bag. Some interesting attempts[26] have even been made to generate a coupling of the bag (in the case of ordinary hadrons) with the pion field and hence to recover, at large distances, the traditional one-pion exchange potential. To summarize, our impression is that the bag model is a powerful phenomenological tool, which keeps contact with the speculation on confinement in nonperturbative QCD (existence of two phases).

The MIT group has used the bag model in the fixed-cavity approximation and has obtained a good fit of the ordinary hadrons.[27] Later, Jaffe studied multiquark states made of light quarks.[1] For heavy quarks, the cavity approximation is inadequate. Another method was proposed in 1975.[6] It is called the adiabatic approximation and is very much reminiscent of the Born-Oppenheimer treatment of the molecular spectrum. In this method, the interquark potential is first deduced from the bag equations and then plugged into the Schrödinger equation. Such a model has been shown to provide a good fit of heavy quarkonia[7] (J/ψ and Υ families) and some speculations on heavy baryons such as ccc, ccb, . . . , have also been made.[8] An interesting property in the case of baryons is that the potential does not follow the additive rule (2.1) and contains manifestly three-body components. By accident, the QQQ potential does not differ too much numerically from the naive extrapolation (2.1) of the $Q\bar{Q}$ case. This does not mean, however, that the additive rule (2.1) will hold also for multiquarks.

We have repeated the calculation of Refs. 7 and 8 for a bag including four heavy quarks. For simplicity, we have retained only spherical bags centered at the center of mass. In this approximation, the energy of the whole system is

$$E(\vec{r}_i,R)=\tfrac{4}{3}\pi R^3\Lambda_B{}^4+\tfrac{2}{3}\alpha_s\sum_{i=1}^{4}\left[\frac{R}{R^2-\vec{r}_i{}^2}-\frac{1}{R}\ln\left[1-\frac{\vec{r}_i{}^2}{R^2}\right]\right]$$
$$+\alpha_s\sum_{i<j}\frac{\lambda_i\lambda_j}{4}\left\{\frac{1}{|\vec{r}_i-\vec{r}_j|}+\frac{R}{(R^4+\vec{r}_i{}^2\vec{r}_j{}^2-2R^2\vec{r}_i\cdot\vec{r}_j)^{1/2}}\right.$$
$$\left.-\frac{1}{R}\ln\left[\frac{1}{2}-\frac{\vec{r}_i\cdot\vec{r}_j}{2R^2}+\frac{(R^4+\vec{r}_i{}^2\vec{r}_j{}^2-2R^2\vec{r}_i\cdot\vec{r}_j)^{1/2}}{2R^2}\right]\right\}, \tag{4.1}$$

which gives the potential

$$V_4(\vec{r}_i)=\min\{E(\vec{r}_i,R);R\geq r_i\}\ . \tag{4.2}$$

We have used the same parameters as in Refs. 7 and 8, namely

$$\alpha_s=0.385\,,\quad \Lambda_B=0.235\text{ GeV}\,,$$
$$m_c=1.35\text{ GeV}\,,\quad m_b=4.75\text{ GeV}\,. \tag{4.3}$$

Using the HOA, i.e., the trial spatial wave function (2.20) and a frozen factorized color wave function, we get the masses displayed in Table III. The results are deceiving, i.e., states are above the threshold. Does this mean that the static potential derived from the bag model will never give narrow multiquarks? Probably not. Some improvements to our calculation indeed remain to be done.

TABLE III. $cc\bar{c}\bar{c}$ masses in the spherical bag model. Units are GeV and GeV^2.

Color state	M	$\alpha_1=\alpha_2$	α_3
$\bar{3}$-3	6.276	0.382	0.312
6-$\bar{6}$	6.252	0.310	0.372
1-1	6.221	0.374	0.255
8-8	6.260	0.312	0.362

(i) As in the case of empirical pairwise potentials, the use of unequal masses can help. For instance, a true state in the present calculation is found at $\Delta m_3=166$ MeV above the threshold for $cc\bar{c}\bar{c}$, $\Delta m_3=227$ MeV for $bb\bar{b}\bar{b}$, but only $\Delta m_3=97$ MeV for $cc\bar{b}\bar{b}$.

(ii) Departures from the spherical approximation could help to push some states below the threshold. Let us discuss this point in some detail. To get a narrow four-quark state, the potential energy $V_4(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4)$ should be "rather often" below $V_{\rm th}=V_{Q\bar{Q}}(\vec{r}_1-\vec{r}_2)+V_{Q\bar{Q}}(\vec{r}_3-\vec{r}_4)$ which has the threshold as ground state. Here $V_{Q\bar{Q}}$ means the $Q\bar{Q}$ potential in the same bag model. The potentials V_4 and $V_{\rm th}$ are shown in Fig. 1 in the case of a symmetric tetrahedral configuration. V_4 is more attractive than $V_{\rm th}$ for any interquark distances.

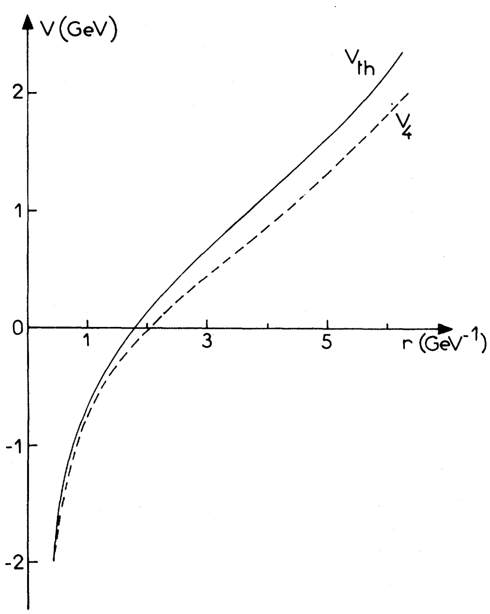

FIG. 1. Comparison of the $QQ\bar{Q}\bar{Q}$ potential V_4 with the potential $V_{\rm th}$ governing the threshold $Q\bar{Q}+Q\bar{Q}$. The quarks are in a symmetric tetrahedral configuration with interquark distance r.

Note that, when $r\to 0$, $V_4\approx V_{\rm th}$ since both reduce to the one-gluon exchange contribution which follows the additive rule (2.1). For finite r, the volume energy in Eq. (4.2) plays a role and is responsible for V_4 being smaller than $V_{\rm th}$. There are, however, cases where $V_4>V_{\rm th}$. Consider, for instance, a rectangular configuration of length L with, at each end, a color-singlet $Q\bar{Q}$ pair with interquark separation l [see Fig. 2(a)]. The quantity V_4-$V_{\rm th}$ is shown in Fig. 2(b) as a function of L for $l=1$ GeV^{-1}. It becomes positive for $L>L_0\sim 2.5l$. Clearly, for very large L, two small bags represent less potential energy because of the volume term. At intermediate distances $L\sim L_0$; however, the

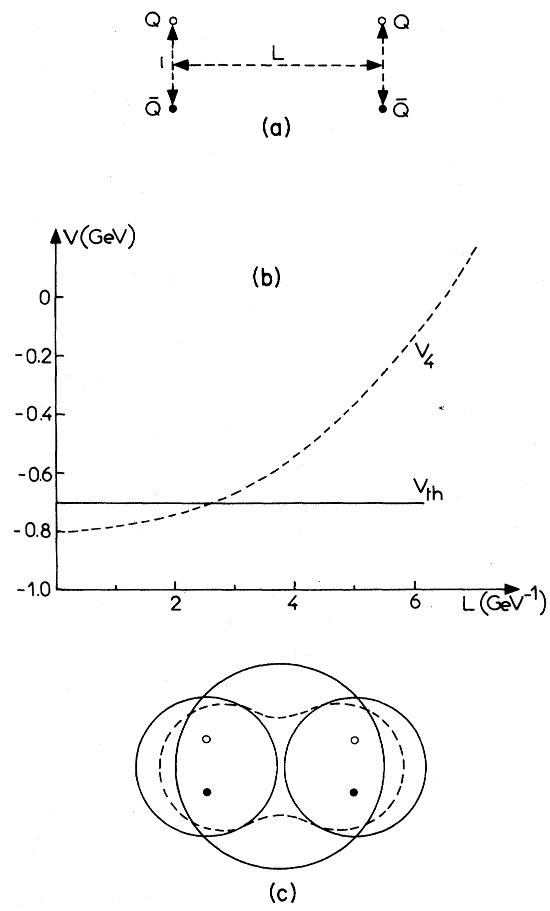

FIG. 2. (a) Rectangular configuration considered. The $Q\bar{Q}$ pair at each end is assumed to be a color singlet. (b) Comparison of the $QQ\bar{Q}\bar{Q}$ potential V_4 and the sum $V_{\rm th}$ of the two $Q\bar{Q}$ potentials, for $l=1$ GeV^{-1}. (c) Comparison of the bag shapes for the threshold $Q\bar{Q}+Q\bar{Q}$ and the quark composite $QQ\bar{Q}\bar{Q}$. The dashed curve corresponds to a tentative improvement of the spherical approximation.

ordering of V_4 and $V_{\rm th}$ depends crucially on the spherical approximation we used for the bag. In Fig. 2(c), we show for $l=1$ GeV^{-1} and $L=4$ GeV^{-1} the two $Q\bar{Q}$ bags with energy $V_{\rm th}$ and the single spherical $QQ\bar{Q}\bar{Q}$ bag leading to the potential $V_4 > V_{\rm th}$. By the dashed curve, we show an intuitive guess of an optimal shape which would tentatively lead to an improved potential $\tilde{V}_4 < V_{\rm th}$. A rigorous deformed-bag calculation of $\tilde{V}_4$ remains, however, to be done.

(ii) We considered the color wave function as being definitively frozen. In the absence of constraints from the Pauli principle, e.g., with quarks bearing different flavors, one may argue, in the spirit of the adiabatic approximation of Born and Oppenheimer, that the color wave function as well as the bag shape and gluon field can evolve when the quarks move. This means that, for any given set of quark coordinates, one has to consider the most general color-singlet wave function, e.g., $|\psi_c\rangle = \cos\theta\,|\bar{3}\text{-}3\rangle + \sin\theta\,|6\text{-}\bar{6}\rangle$ and, in computing the potential energy, minimize on θ as well as on the parameters describing the bag shape. Such a new freedom would, of course, lower the energy of the ground state.

As an illustration, we have considered the fictitious case of two quarks and two antiquarks with the same mass m_c (for simplicity) but without constraint of antisymmetrization. We have kept the spherical approximation for the bag. With a fixed color wave function, the ground state always lies above 6.22 GeV (see Table III). If one now minimized on the color angle θ at each point, the ground-state energy is lowered up to 6.075 GeV, which is actually below the threshold since the same bag model gives a quarkonium mass $M(c\bar{c})=3.055$ GeV (both values in the HOA).

The conclusion of this section is that the bag model seems to offer the possibility of binding several heavy quarks together, especially if those bear different flavors. This is, however, not achieved in the approximation of a spherical bag and a factorized color wave function. In line with the present study is the work by Chao[28] who considered two heavy quarks and two light quarks and in a Born-Oppenheimer-type treatment, derived an effective $Q\bar{Q}$ potential describing the $Q\bar{Q}q\bar{q}$ dynamics inside the bag. His estimate $M(Q\bar{Q}q\bar{q})-M(Q\bar{Q})\simeq 600$ MeV illustrates that, within the bag model, the multiquark states are not pushed very high in the spectrum. [Remember that, experimentally, the spin averaged $q\bar{q}$ mass is $\frac{1}{4}M(\pi)+\frac{3}{4}M(\rho)=610$ MeV.] This was already shown by Jaffe[11] when he estimated the masses of states made of several light quarks in the fixed-cavity approximation. The main reason, common to all those bag-model calculations, is that the volume energy increases less rapidly than the number of quarks enclosed into the bag.

V. HYPERFINE CORRECTIONS TO THE S STATE

Since the work of De Rújula, Georgi, and Glashow,[29] there has been a great deal of activity on the spin-dependent forces between quarks as a test or an application of QCD. In particular, the chromomagnetic interaction due to one-gluon exchange

$$V_{SS}(i,j) = -\frac{\pi\alpha_s}{6m_i m_j}\tilde{\lambda}_i\tilde{\lambda}_j\vec{\sigma}_i\cdot\vec{\sigma}_j\delta^3(r_{ij}) \tag{5.1}$$

had been recognized early as playing a crucial role in multiquark spectroscopy.[1,2] For consistency, the virtual annihilation into one gluon should also be accounted for,[30] as in the case of positronium in QED.

In early works on "color chemistry"[2-4] the masses were estimated rather crudely. First, the values before spin corrections were given by empirical mass formulas rather than computed from a specific interaction. Also, due to the lack of explicit wave functions, the matrix elements $\langle\delta^3(r_{ij})\rangle$ involved in the spin-spin term (5.1) were not calculated but simply taken from the meson or the baryon case and therefore overestimated (compare, e.g., the values of the α's in Table I for $Q\bar{Q}$, QQQ, and $QQ\bar{Q}\bar{Q}$). Here we shall try to calculate the hyperfine splittings in a more realistic way.

First, we rewrite (5.1) as (for equal masses)

$$V_{SS}(i,j) = -C\delta^3(r_{ij})\tilde{\lambda}_i\tilde{\lambda}_j\vec{\sigma}_i\cdot\vec{\sigma}_j \tag{5.2}$$

and we determine C by imposing the reproduction in the HOA of the experimental hyperfine splitting of charmonium

$$\begin{aligned}\Delta M &= M(J/\psi)-M(\eta_c)\\ &\simeq 0.112\ \text{GeV}\ .\end{aligned} \tag{5.3}$$

With $\langle\delta^3(r)\rangle = (\alpha/\pi)^{3/2}$ and the oscillator parameters in Table I, this gives

$$\begin{aligned}C^{\rm I} &= 0.146\ \text{GeV}^{-2}\ ,\\ C^{\rm II} &= 0.103\ \text{GeV}^{-2}\ .\end{aligned} \tag{5.4}$$

Note that with purely phenomenological potentials, the hyperfine constant C has no direct connection with the strength of the central force and can be treated as an independent parameter. Such a strategy was adopted for instance in Refs. 10 and 15. Here C^{II} is only slightly modified due to the use of HOA instead of exact wave functions. For a potential with an explicit Coulomb piece like (2.16), C can be in principle deduced from the central potential if the $1/r$ term is seriously understood as one-gluon exchange. This would give

$$C^{\mathrm{I}'}=\frac{\pi\alpha_s}{6m^2}=0.126\ \mathrm{GeV}^{-2} \tag{5.5}$$

in rather good agreement with the empirical determination (5.4).

It is easy to calculate the diagonal hyperfine corrections to the masses calculated in Sec. II, since in the HOA

$$\langle\delta(r_{12})\rangle=\left[\frac{\alpha_1}{\pi}\right]^{3/2},$$

$$\langle\delta(r_{13})\rangle=\left[\frac{2\alpha_1\alpha_3}{\pi(\alpha_1+\alpha_3)}\right]^{3/2}.$$

The results are displayed in Table IV for the $cc\bar{c}\bar{c}$ case and potential I. They are almost identical for potential II. This is not surprising as soon as both models are adjusted on the experimental J/ψ-η_c splitting. The hyperfine corrections are small and repulsive except for one of the 0^+ states. Since during the same time, the threshold has been lowered from the spin-averaged $2M(c\bar{c})$ to $2M(\eta_c)$—a negative shift of 168 MeV—it is clear that all $cc\bar{c}\bar{c}$ states remain far above the threshold in those potential models. The color mixing, which has been neglected here, does not seem sufficient to bring the 400 MeV of extra attraction which are necessary. Note also that the annihilation term, if included, would give a repulsive contribution

Our numerical study supports our statement in the Introduction that heavy multiquarks cannot rely upon chromomagnetic forces to achieve their binding below their dissociation threshold.

TABLE IV. Diagonal hyperfine corrections for the $cc\bar{c}\bar{c}$ S states in the case of potential I.

Color state	Unperturbed mass (GeV)	J^P	ΔM (GeV)
6-$\bar{6}$	6.383	0^+	0.017
$\bar{3}$-3	6.437	0^+	-0.011
		1^+	0.003
		2^+	0.032

VI. *P* STATES

We now turn to higher states with negative parity. For simplicity, we keep to the $cc\bar{c}\bar{c}$ case. Various types of P states can be considered, the true state $(QQ)_{\bar{3}}$-$(\bar{Q}\bar{Q})_3$, the mock one $(QQ)_6$-$(\bar{Q}\bar{Q})_{\bar{6}}$, where the diquarks and antiquarks are pure S waves, a state $(Q\bar{Q})_8$-$(Q\bar{Q})_8$, or more general superpositions. All are, of course, expected to lie above the absolute threshold for natural dissociation $2M(Q\bar{Q})_s$, made of two quarkonia in the ground state. One may argue, however, that the decay into the lowest channels is sometimes suppressed by the dynamics of the quark-rearrangement process. For instance, a pure (QQ)-$(\bar{Q}\bar{Q})$ P state with S-wave diquarks would not decay into two S-wave mesons, but should produce at least one P-wave meson. Such rules, which have been derived by Gavela *et al.* for the harmonic oscillator,[22] are expected to be almost exact within any potential model.

In a spontaneous dissociation, the quarks are simple spectators, so the spin and the total orbital momentum are separately conserved in the superallowed decays. This gives further interesting restrictions. Consider for instance a 1^{--} T state with total quark spin 2. It cannot decay easily into only J/ψ or η_c's, since the overlap integral of initial and final wave functions vanishes. Moreover, the $\eta_c\chi_0$ and $\eta_c\chi_1$ channels are excluded by total angular momentum and quark-spin conservation, respectively. So, it will decay mainly into $\psi\chi_0$, whose threshold is at around 6.51 GeV. The problem is now whether or not this 1^{--} (cc-$\bar{c}\bar{c}$) states lies below the $\psi\chi_0$ threshold. Some hope relies upon the spin-orbit and tensor interactions, which are not negligible. Remember, indeed, that those forces are responsible for the rather large splitting of the χ states of charmonium.

To estimate the mass of these P states, we use the following trial wave function:

$$\psi^m=N'\exp[-\tfrac{1}{2}\alpha_1(\vec{\xi}_1^{\,2}+\vec{\xi}_2^{\,2})-\tfrac{1}{2}\alpha_3\vec{\xi}_3^{\,2}]\xi_3^{\,m} \tag{6.1}$$

($m=\pm1$ or 0) which are analogous to those of Eq. (2.20) for S states. Here $\vec{\xi}_3$ joins the center of the diquark to that of the antidiquark. As for S states we neglect color mixing whose effect on orbital excitation has been studied by Gavela *et al.* in the case of harmonic forces.[22] After minimization on α_1 and α_3, we get the results displayed in Table IV. With both additive potential models I and II, the masses turn out to be rather high, above any $\psi\chi_J$ dissociation threshold.

We now work out the spin corrections in the case of the potential I. The effect of the spin-spin forces (5.2) is easily estimated for the wave function (6.1). The relevant matrix elements are $\langle\delta_{12}\rangle=(\alpha_1/\pi)^{3/2}$ and $\langle\delta_{13}\rangle=[\alpha_3/(\alpha_1+\alpha_3)][2\alpha_1\alpha_3/\pi(\alpha_1+\alpha_3)]^{3/2}$. The spin-spin splittings are dominated by the terms internal to the diquarks, which are always repulsive. The exact value is given in Table V.

In addition, we have spin-orbit and tensor components

$$V_{LS}(i,j)=-\frac{\tilde{\lambda}_i\tilde{\lambda}_j}{8m_c^2}\left[\frac{3\alpha_s}{r_{ij}^3}-\frac{3}{4}\frac{\lambda}{r_{ij}}\right]\vec{1}_{ij}\cdot\left[\frac{\vec{\sigma}_i+\vec{\sigma}_j}{2}\right], \tag{6.2}$$

$$V_T(i,j)=-\frac{\tilde{\lambda}_i\tilde{\lambda}_j}{16m_c^2r_{ij}^3}\alpha_s(3\vec{\sigma}_i\cdot\hat{r}_{ij}\vec{\sigma}_j\cdot\hat{r}_{ij}-\vec{\sigma}_i\cdot\vec{\sigma}_j)\ . \tag{6.3}$$

The linear confining term is assumed here to be a Lorentz scalar, as indicated by the phenomenological analysis of the spectrum of mesons and baryons.[31] Note that the expression (6.2) needs in principle some corrections when the (i,j) pair is not at rest.[32] Summing the contributions from all pairs involves some tedious algebraic manipulations. To get a first rough estimate of the splittings, we replace, as in previous works,[4,33] the operators $\vec{1}_{ij}$, $i=1,2$, $j=3,4$ by the classical value $\vec{1}/2$ where $\vec{1}$ is the angular momentum between the diquark and the antidiquark. Similarly each interquark distance r_{ij} is replaced by ξ_3. As a result, we get effective tensor and spin-orbit operators between the diquark and the antidiquark.[33] Their contributions are displayed in Table VI for the lightest P states. Also shown is the lowest superallowed threshold. We note that all the splittings are extremely small and the masses remain very high.

Summarizing, it seems hopeless to get narrow $cc\bar{c}\bar{c}$ P states within additive potential models. Of course, there are some uncertainties in our estimates. Our masses are, however, too high to be pushed down below the disintegration threshold by refining the calculation. In fact, drastic changes in the quark dynamics would be necessary to modify our conclusions.

VII. SUMMARY AND DISCUSSION

In this paper, we have investigated several ways of binding four heavy quarks together, in an attempt to get narrow states, i.e., lying below any threshold for spontaneous dissociation. The result depends on the assumptions made on the quark dynamics and on the flavor combination in which the quarks enter.

We first considered the very general class of additive models $V=\sum_{i<j}\tilde{\lambda}_i\tilde{\lambda}_jV_8(r_{ij})$. Such a choice is made rather often in the literature. By diagonalizing such a $QQ\bar{Q}\bar{Q}$ potential in the color space, Lipkin[34] accounted for the effect of color mixing, ignoring possible restrictions due to the Pauli principle in the case of identical quarks. He showed that in some cases of optimal attraction, the four-quark potential is above the potential governing the dissociation threshold, i.e.,

$$V(QQ\bar{Q}\bar{Q})\geq V(Q\bar{Q})+V(Q\bar{Q})\ . \tag{7.1}$$

This rather general result is violated only for potentials with unreasonably sharp variations.[35] It strongly suggests that the $QQ\bar{Q}\bar{Q}$ states are un-

TABLE V. cc-$\bar{c}\bar{c}$ P-state mass spectrum.

		Model I	Model II
$\bar{3}$-3	M	6.718	6.714
	$\alpha_1=\alpha_2$	0.224	0.279
	α_3	0.164	0.201
6-$\bar{6}$	M	6.832	6.822
	$\alpha_1=\alpha_2$	0.141	0.173
	α_3	0.270	0.332

TABLE VI. Spin effects of the lowest P states, in the case of potential I.

Color	J^P	Threshold	Unperturbed mass	Spin-spin shift	Spin-orbit and tensor shift
6-$\bar{6}$	1^{--}	$\eta_c{}^s\chi$ (6.4)	6.832	0.011	0
$\bar{3}$-3	0^{-+}	$\eta_c\chi_0$ (6.39)	6.718	0.010	−0.023
	1^{--}	$\psi\chi_0$ (6.51)	6.718	0.020	−0.024

stable, i.e.,

$$M(QQ\bar{Q}\bar{Q})\geq M(Q\bar{Q})+M(Q\bar{Q}) , \qquad (7.2)$$

since the configurations of optimal attraction are likely to attract a large amount of the wave function and hence to play the dominant role. There are, however, other configurations where the inequality (7.1) is reversed, casting some doubt on the ordering (7.2) of the masses. This is the case, for instance, when the two quarks are close to each other and form a color-$\bar{3}$ diquark spatially separated from the antidiquark. In fact, a rigorous proof of the inequality (7.2) requires a four-body calculation. This was done by Gavela *et al.*,[22] who computed the masses of the $QQ\bar{Q}\bar{Q}$ ground state and orbital excitations in the case of identical quarks interacting through a harmonic potential. They found all states rather heavy. In particular, their $QQ\bar{Q}\bar{Q}$ ground state lies above the dissociation threshold, i.e., satisfies the relation (7.2). In this paper, we simplified the treatment of color, as explained in Sec. II. Within this easier framework, we proved that in the case of identical quarks, there is no stable $QQ\bar{Q}\bar{Q}$ state, a result independent of the shape of the confining potential V_8. Using phenomenological interactions, we found for instance the first $cc\bar{c}\bar{c}$ state around 300 MeV above the threshold made of two charmonia, and the spin-independent corrections do not appreciably reduce this gap.

We also considered some examples of orbitally excited multiquarks, namely the $cc\bar{c}\bar{c}$ P states. Under some assumptions on the decay mechanism, one may argue that, for some of those states, the first threshold is $\psi\chi$ instead of $\eta_c\eta_c$. Even so, we did not find any narrow $cc\bar{c}\bar{c}$ P state emerging from our calculation.

There are alternatives to these simple additive models. Barbour and Ponting[36] for instance, did not assume that color-octet exchanges govern the long-range potential. Instead, they took seriously the rule given by the elongated bag model, which is $V_{AB}\propto\Gamma_{AB}/\sqrt{C}_A$, where $C_A=C_B$ is the SU(3) Casimir operator associated with the color charge born by the subsystems A and B at each end of the string.[36] Note that their potential is not of the two-body type any more. The work of Ref. 37 concerns baryons and multiquarks made of light quarks. For the case of heavy quarks, we know that the cylindrical-tube limit of the bag is rather elusive.[6,7] So, to derive the potential, it is more appropriate to start from a spherical bag, as done in Sec. IV. We got multibody forces which provide additional attraction at short distances. This seriously increases the chances of having narrow multiquarks. For configurations like $cc\bar{c}\bar{c}$ or $bb\bar{b}\bar{b}$, we indeed obtain masses not too far from the threshold. However, we can hardly draw conclusions on the stability of the states from our rough four-body calculation done with the spherical approximation for the bag and the usual simplifying hypotheses like the neglect of surface tension or the leading order in α_s inside the bag. It would be worthwhile, in our opinion, to investigate this bag model in more detail, even if the technology involved is not easy, to see whether it leads, contrary to the naïve additive model, to a proliferation of heavy multiquarks.

An important point concerns states made of quarks bearing different masses. Although for our calculations in Sec. III we used additive models, our qualitative conclusions are certainly rather general. The cryptoexotic configuration $QQ'\bar{Q}\bar{Q}'$, lies above its lowest dissociation threshold $Q\bar{Q}+Q'\bar{Q}'$. On the other hand, the genuine exotic $QQ\bar{Q}'\bar{Q}'$ can be stable against dissociation if the ratio of the quark masses is large enough. Our predictions concern states like $tt\bar{s}\bar{s}$ which are far from the present experimental possibilities of production and detection. Such exotic objects, however, could not be misinterpreted as orbital, radial, or gluonic excitations of ordinary mesons and would provide unambiguous signatures of their multiquark structure.

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