Weak radiative decays of hyperons and charmed baryons in a quark model

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The weak radiative decays of $1/2^+$ baryons are studied in a quark model as arising through single-quark spectator and nonspectator quark transitions. These decays seem to occur predominantly through nonspectator processes, where the quarks may interact through the exchange of colored gauge bosons. We compare the decay-amplitude sum rules in left$\times$left and left$\times$right current-current interaction pictures.

I. INTRODUCTION

The weak radiative decays of hyperons, though naively expected to occur at a very low percentage in the branching fraction, have, nevertheless, attracted the attention of several authors,[1] as they can provide a means in deciding on the structure of the weak nonleptonic interactions. It was first shown by Hara[2] that with the usual assumption of octet dominance and CP invariance of the weak Hamiltonian for a current $\times$ current picture, the decays $\Sigma^+ \to p\gamma$ and $\Xi^- \to \Sigma^-\gamma$ vanish in the parity-violating mode. However, experimentally,[3] $\alpha(\Sigma^+ \to p\gamma)$ has been measured as $-1.03^{+0.53}_{-0.42}$, which demands the parity-violating (PV) amplitude for $\Sigma^+ \to p\gamma$ to be appreciable. This has been a source of considerable difficulty for the weak-interaction phenomenology. Most of the earlier attempts[4] to obtain nonzero PV amplitude for this decay have been in terms of the baryon-pole models, wherein the initial baryon is changed by the weak interaction into an intermediate baryon of increased strangeness, followed by the radiative emission of a photon, in that order, or vice versa. While the predicted rate for $\Sigma^+ \to p\gamma$ was more or less of the right order, the corresponding asymmetry parameter was more difficult to predict correctly. These models of the decay have been superceded in recent years by short-distance analysis,[5] but such an analysis does not allow the $\alpha(\Sigma^+ \to p\gamma)$ to be calculated correctly.[5] Another alternative, suggested to explain the observed asymmetry for $\Sigma^+ \to p\gamma$, has been to assume that the parity-conserving and parity-violating parts of the weak Hamiltonian transform as λ_6 and λ_7 components[6,7] (respectively) of an SU(3) octet, as in the quark-density model.[8] Inclusion of right-handed currents also suggest similar SU(3) structure[9] for the weak Hamiltonian, but Shifman *et al.*[10] argue that it fails to account for the observed asymmetry.

In the present work, we employ a quark model to study the weak radiative decays of uncharmed and charmed baryons and derive sum rules among the various decay amplitudes. In view of the nebulus state of knowledge about the radiative decays of charmed baryons, these relations may only be of theoretical interest for the time being. However, for the uncharmed baryons, our analysis leads to $\langle \Sigma^-\gamma | \Xi^- \rangle = 0$, while $\langle p\gamma | \Sigma^+ \rangle$ is nonzero. With the advent of high-energy hyperon beams at Fermilab and the CERN SPS, it may be possible to study these decays in detail and the sum rules may help in deciding on the structure of the weak Hamiltonian.

We do not attempt to explore any model of the underlying interactions responsible for radiative weak decays. We do not make the assumption that these decays arise only from a quark decaying into another quark with the emission of a photon, as in Fig. 1 (single-quark spectator transition). In such a transition, the weak decays of a hadron are usually assumed to proceed through the heavy quark, with the light quarks acting as spectators.[11] This suggests that the lifetimes of charm states $D^+(c\bar{d})$, $F^+(c\bar{s})$, $D^0(c\bar{u})$, and $\Lambda_c^+(cud)$ are equal, but the recent lifetime measurements[12] indicate that D^0 and Λ_c^+ lifetimes are significantly shorter than those of D^+ and F^+. The discrepancy suggests that interactions involving light quarks (the nonspectator interactions) may play a role,[13] as in Fig. 2. In the nonspectator transitions, quarks may interact through the exchange of colored gauge bosons (gluons) which may include, in some way, the quantum-chromodynamics (QCD) corrections. In recent papers,[14] Sharma and Kanwar have employed a quark model to calculate the radiative decay widths of vector mesons and magnetic moments of uncharmed baryons as arising through spectator and nonspectator quark processes, and shown that in the light of the latest data,[15] an acceptable

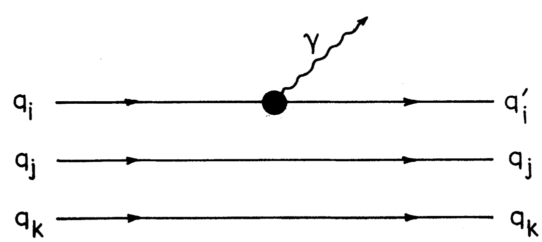

FIG. 1. Single-quark spectator transition.

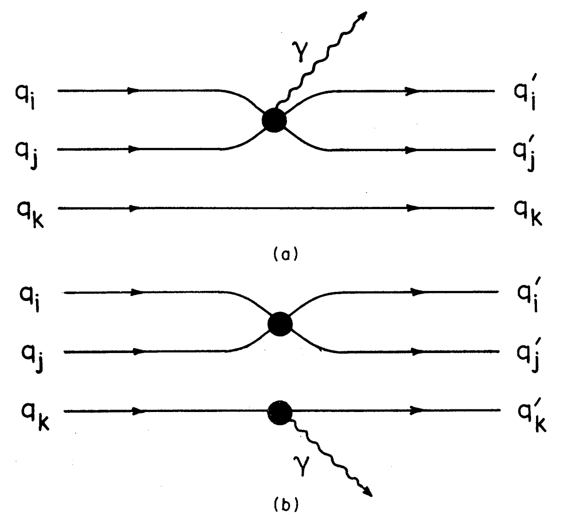

FIG. 2. Nonspectator transitions.

fit is possible if we add nonspectator contributions. In the present work, we analyze the possibility of the radiative weak decays of uncharmed and charmed baryons occurring through single-quark spectator transitions and nonspectator transitions.

II. PRELIMINARIES

We make the following assumptions for the calculations.[16]

(i) The weak Hamiltonian H_w (a) is of the current-current form, i.e., $H_w=\frac{1}{2}(JJ^\dagger+J^\dagger J)$, and (b) belongs, in general, to all the representations present in the direct product

$$15\otimes 15=1\oplus 15_S\oplus 15_A\oplus 20_S\oplus 45_A\oplus 45_A\oplus 84_S\,. \tag{2.1}$$

In the Glashow-Iliopoulos-Maiani (GIM) model,[17] H_w belongs only to $20''$ and 84. The other representations can appear through SU(4) breaking and/or unconventional currents,[9,18(a),18(b)] such as right-handed currents, second-class currents, etc. For the charm-changing decays, the 15 and the singlet representations do not contribute.

(ii) The baryons are nonrelativistic bound states of three quarks in the S state and are described by the spin-unitary-spin wave functions[19] belonging to 120 representation of SU(8).

(iii) The weak radiative decays occur through the emission of a real photon accompanied by the single-quark spectator transition[20] and the non-spectator transition. For the latter, the quarks may interact through the exchange of gluons and may include, in some way, the QCD corrections to these decays. The transition amplitude for the decay $B\to B'+\gamma$ is then written as

$$\langle\gamma B'|H_W|B\rangle=\left\langle\gamma B'\left|\sum_{i=1}^{3}H_1^{(i)}+\sum_{i\neq j=1}^{3}H_2^{(i,j)}\right|B\right\rangle, \tag{2.2}$$

where $H_1^{(i)}$ and $H_2^{(i,j)}$ are weak Hamiltonians for photon emission accompanied by the ith single-quark (Fig. 1) and (i,j)th nonspectator two-quark transitions (Fig. 2).

(iv) The current and constituent quarks are identical. It has been shown by Sebastian[21] that in the nonrelativistic quark model, this assumption is valid.

III. WEAK HAMILTONIAN IN THE GIM MODEL

A. Single-quark spectator interaction

The weak Hamiltonian for the single-quark spectator transition can be written as

$$H_1=a^n(\bar q^a F q_b)A+\text{H.c.}\,, \tag{3.1}$$

where A represents the photon wave function, n denotes the dimensionality of the representation to which H_w belongs, and F is a function of momenta of quarks a and b, and a vector.[16]

The GIM weak Hamiltonian has the following structure:

$$H_w^{20''}=a^{20''}(\bar q^a F q_c)A_d^b H^{[c,d]}_{[a,b]} \tag{3.2}$$

and

$$H_w^{84}=a^{84}(\bar q^a F q_c)A_d^b H^{\{c,d\}}_{\{a,b\}}\,, \tag{3.3}$$

where $H^{[c,d]}_{[a,b]}$ and $H^{\{c,d\}}_{\{a,b\}}$ represent the transformation properties of weak Hamiltonians $20''$ and 84, respectively. A^i_j is the photon belonging to the $15\oplus 1$ representation of SU(4). The component corresponding to the electromagnetic current is

$$A^i_j\sim A^1_1+A^4_4-\tfrac{1}{3}A^\alpha_\alpha\quad(\alpha=1,2,3,4)\,. \tag{3.4}$$

CP invariance leads to

$$a^{20''}=a^{84}=0\,. \tag{3.5}$$

The GIM weak Hamiltonian, therefore, gives no contribution to the weak radiative decays through the single-quark spectator process.

B. Nonspectator interaction

In the nonspectator transition, we will not consider explicit gluon corrections to the quark transitions. We simply assume that the corrections are absorbed in the effective Hamiltonian. The proper understanding of gluon corrections would require consideration of the dynamics of quark-quark interaction. Here we treat the nonspectator transitions in general symmetry arguments, without going into the detailed nature of the interaction between quarks. As the quarks have been assumed to be in the S state, the weak Hamiltonian causing this type of transition is written as

$$H_2=\{k^n[(\bar{q}^a q_c)(\bar{q}^b\vec{\sigma}q_d)+(\bar{q}^a\vec{\sigma}q_c)(\bar{q}^b q_d)]+l^n[(\bar{q}^a\vec{\sigma}q_c)\times(\bar{q}^b\vec{\sigma}q_d)]$$
$$+p^n[(\bar{q}^a q_c)(\bar{q}^b q_d)(\bar{q}^e\vec{\sigma}q_f)]+q^n[(\bar{q}^a\vec{\sigma}q_c)\cdot(\bar{q}^b\vec{\sigma}q_d)(\bar{q}^e\vec{\sigma}q_f)]\}\cdot\vec{A}+\text{H.c.} \tag{3.6}$$

The first two terms of the Hamiltonian corresponds to Fig. 2(a) and the next two correspond to Fig. 2(b). The σ's are so chosen as to ensure the vector nature of the photon.

Several arguments have been given in support of the enhancement of the $\Delta I=\frac{1}{2}$ amplitude (or octet dominance) in the nonleptonic decays of hyperons, and this point of view is further supported by a recent analysis[22] on the structure of the nonleptonic Hamiltonian in the framework of asymptotically free field theories of strong interactions. The extension of these ideas would suggest 20″ dominance in SU(4). The CP-invariant 20″-dominant GIM weak Hamiltonian for the PV weak radiative decays has the following components:

$$H_w^{20''}=+k_1^{20''}\{[(\bar{q}^f q_c)(\bar{q}^a\vec{\sigma}q_d)+(\bar{q}^f\vec{\sigma}q_c)(\bar{q}^a q_d)]\cdot\vec{A}_f^b H^{[c,d]}_{\{a,b\}}-[(\bar{q}^a q_f)(\bar{q}^b\vec{\sigma}q_c)+(\bar{q}^a\vec{\sigma}q_f)(\bar{q}^b q_c)]\cdot\vec{A}_d^f H^{[c,d]}_{[a,b]}\}$$
$$+l_1^{20''}\{[(\bar{q}^f\vec{\sigma}q_c)\times(\bar{q}^a\vec{\sigma}q_d)]\cdot\vec{A}_f^b H^{[c,d]}_{[a,b]}-[\bar{q}^a\vec{\sigma}q_f)\times(\bar{q}^b\vec{\sigma}q_c)]\cdot\vec{A}_d^f H^{[c,d]}_{[a,b]}\}$$
$$+p_1^{20''}\{[(\bar{q}^a q_c)(\bar{q}^b q_d)(\bar{q}^f\vec{\sigma}q_e)]\cdot\vec{A}_f^e H^{[c,d]}_{[a,b]}\}+q_1^{20''}\{[(\bar{q}^a\vec{\sigma}q_c)\cdot(\bar{q}^b\vec{\sigma}q_d)(\bar{q}^f\vec{\sigma}q_e)]\cdot\vec{A}_f^e H^{[c,d]}_{[a,b]}\}. \tag{3.7}$$

IV. DECAY-AMPLITUDE SUM RULES

Using (3.7), we obtain the following relations among the various decay amplitudes for the 20″-dominant GIM Hamiltonian.

(i) $\Delta C=0$, $\Delta S=-1$ decays. For the uncharmed baryons, we have

$$\langle\Sigma^-\gamma|\Xi^-\rangle=0\,, \tag{4.1}$$

$$\sqrt{3}\langle N\gamma|\Lambda\rangle=\langle N\gamma|\Lambda\rangle-6\langle\Sigma^0\gamma|\Xi^0\rangle\,, \tag{4.2}$$

$$\langle\Lambda\gamma|\Xi^0\rangle=\langle N\gamma|\Lambda\rangle+5\sqrt{3}\langle\Sigma^0\gamma|\Xi^0\rangle\,, \tag{4.3}$$

$$\langle P\gamma|\Sigma^+\rangle=\sqrt{2}(3\langle\Sigma^0\gamma|\Xi^0\rangle-\langle N\gamma|\Sigma^0\rangle)\,. \tag{4.4}$$

Notice that $\Sigma^+\to p\gamma$ is allowed, and $\Xi^-\to\Sigma^-\gamma$ is forbidden. For the charmed baryons, we get

$$\sqrt{2}\langle\Lambda_1'^+\gamma|\Xi_1'^+\rangle=\langle N\gamma|\Lambda\rangle+19\langle\Sigma^0\gamma|\Xi^0\rangle \tag{4.5}$$

$$\sqrt{3}\langle\Lambda_1'^+\gamma|\Xi_1^+\rangle=-\tfrac{3}{5}\langle\Sigma_1^+\gamma|\Xi_1^+\rangle=-(1/\sqrt{2})\langle\Sigma_1^0\gamma|\Xi_1^0\rangle=-\tfrac{1}{2}\langle\Xi_2^+\gamma|\Omega_2^+\rangle$$
$$=\sqrt{\tfrac{3}{2}}\langle\Xi_1'^0\gamma|\Omega_1^0\rangle=-(1/\sqrt{2})\langle\Xi_1^0\gamma|\Omega_1^0\rangle=-(3/\sqrt{2})\langle\Sigma^0\gamma|\Xi^0\rangle\,. \tag{4.6}$$

(ii) $\Delta C=\Delta S=-1$ decays In the Cabibbo-enhanced mode, we have

$$\langle\Sigma^+\gamma|\Sigma_1^+\rangle=-2\langle\Sigma^0\gamma|\Sigma_1^0\rangle\,, \tag{4.7}$$

$$2\langle\Sigma^0\gamma|\Sigma_1^0\rangle=\langle\Sigma^+\gamma|\Lambda_1'^+\rangle+2\langle\Xi^0\gamma|\Xi_1'^0\rangle+2\langle\Lambda\gamma|\Sigma_1^0\rangle\,, \tag{4.8}$$

$$4\langle\Xi^0\gamma|\Xi_1^0\rangle=\sqrt{3}\langle\Sigma^+\gamma|\Lambda_1'^+\rangle+2\sqrt{3}\langle\Xi^0\gamma|\Xi_1'^0\rangle+4\sqrt{3}\langle\Lambda\gamma|\Sigma_1^0\rangle\,, \tag{4.9}$$

$$4\sqrt{3}\langle\Xi_1^+\gamma|\Xi_2^+\rangle=\langle\Sigma^+\gamma|\Lambda_1'^+\rangle+2\langle\Xi^0\gamma|\Xi_1'^0\rangle\,, \tag{4.10}$$

$$\sqrt{3}\langle\Xi_1'^+\gamma|\Xi_2^+\rangle=-2\langle\Sigma^+\gamma|\Sigma_1^+\rangle-3\langle\Xi_1^+\gamma|\Xi_2^+\rangle\,. \tag{4.11}$$

V. MOST GENERAL HAMILTONIAN

Many authors[23] have pointed out that the antisymmetric representations $(45_A+45_A^*)$, present in the direct product $15\otimes 15$, give an important contribution to the nonleptonic decays. It has been suggested by Abe, Fujii, and Sato[9] that weak nonleptonic decays and weak radiative decays can simultaneously be well explained in the presence of left × right current-current interaction, which has a (45 + 45*) transformation property in the charm sector. In order to make our study most general, we include these representations without going into their detailed dynamical origin.

A. Single-quark spectator transition

For this, the weak Hamiltonian transforming as (45, 45*) has the form

$$H_W^{45}=a^{45}(\bar{q}^a F q_c)A_d^b H^{[c,d]}_{\{a,b\}}\,, \tag{5.1}$$

$$H_w^{45*}=a^{45*}(\bar{q}^a F q_c)A_d^b H^{\{c,d\}}_{[a,b]}\,, \tag{5.2}$$

where the tensors $H^{[c,d]}_{(a,b)}$ and $H^{(c,d)}_{[a,b]}$ represent the transformation properties of the weak Hamiltonian in 45 and 45*, respectively, of SU(4).

CP invariance implies

$$a^{45}=a^{45*}\,. \tag{5.3}$$

We notice that through single-quark transition, all the Cabibbo-enhanced as well as Cabibbo-suppressed decay modes are forbidden. This, together with the result (3.5), implies that the weak radiative decays may occur predominantly through nonspectator transitions.

B. Nonspectator interaction

The *CP*-invariant weak Hamiltonian causing the nonspectator contribution has the following components:

$$\begin{aligned}
H_w^{45+45^*}=\;& k_1^{45+45^*}\{[(\bar q^f q_c)(\bar q^a\vec\sigma q_f)+(\bar q^f\vec\sigma q_c)(\bar q^a q_f)]H^{[c,d]}_{(a,b)}+[(\bar q^f q_c)(\bar q^a\vec\sigma q_f)+(\bar q^f\vec\sigma q_c)(\bar q^a q_f)]H^{(c,d)}_{[a,b]}\}\cdot\vec\lambda^b_d\\
&+k_2^{45+45^*}\{[(\bar q^f q_c)(\bar q^a\vec\sigma q_d)+(\bar q^f\vec\sigma q_c)(\bar q^a q_d)]\cdot\vec\lambda^b_f H^{[c,d]}_{(a,b)}+[(\bar q^a q_f)(\bar q^b\vec\sigma q_c)+(\bar q^a\vec\sigma q_f)(\bar q^b_f q_c)]\cdot\vec\lambda^f_d H^{(c,d)}_{[a,b]}\}\\
&+k_3^{45+45^*}\{[(\bar q^a q_f)(\bar q^b\vec\sigma q_c)+(\bar q^a\vec\sigma q_f)(\bar q^b q_c)]\cdot\vec\lambda^f_d H^{[c,d]}_{(a,b)}+[(\bar q^f q_c)(\bar q^a\vec\sigma q_d)+(\bar q^f\vec\sigma q_c)(\bar q^a q_d)]\cdot\vec\lambda^b_f H^{(c,d)}_{[a,b]}\}\\
&+l_1^{45+45^*}\{[(\bar q^f\vec\sigma q_c)\times(\bar q^a\vec\sigma q_f)]H^{(c,d)}_{[a,b]}+[(\bar q^f\vec\sigma q_c)\times(\bar q^a\vec\sigma q_f)]H^{[c,d]}_{(a,b)}\}\cdot\vec\lambda^b_d\\
&+l_2^{45+45^*}\{[(\bar q^f\vec\sigma q_c)\times(\bar q^a\vec\sigma q_d)]\cdot\vec\lambda^b_f H^{[c,d]}_{(a,b)}+[(\bar q^a\vec\sigma q_f)\times(\bar q^b\vec\sigma q_c)]\cdot\vec\lambda^f_d H^{(c,d)}_{[a,b]}\\
&+l_3^{45+45^*}\{[(\bar q^a\vec\sigma q_f)\times(\bar q^b\vec\sigma q_c)]\cdot\vec\lambda^f_d H^{[c,d]}_{(a,b)}+[(\bar q^f\vec\sigma q_c)\times(\bar q^a\vec\sigma q_d)]\cdot\vec\lambda^b_f H^{(c,d)}_{[a,b]}\}\\
&+p^{45+45^*}\{(H^{[c,d]}_{(a,b)}+H^{(c,d)}_{[a,b]})\cdot\vec\lambda^e_f[(\bar q^f\vec\sigma q_e)(\bar q^a q_c)(\bar q^b q_d)]\}\\
&+q^{45+45^*}\{(H^{[c,d]}_{(a,b)}+H^{(c,d)}_{[a,b]})\cdot\vec\lambda^e_f[(\bar q^f\vec\sigma q_e)(\bar q^a\vec\sigma q_c)\cdot(\bar q^b\vec\sigma q_d)]\}\,.
\end{aligned}\tag{5.4}$$

The left × right current-current weak Hamiltonian, which has (45+45*) transformation properties, gives the following relations:

(i) $\Delta C=0$, $\Delta S=-1$ *decays*. In the uncharmed sector, we get

$$\langle\Lambda\gamma|\Xi^0\rangle=\langle\Sigma^-\gamma|\Xi^-\rangle=0\,, \tag{5.5}$$

$$2\sqrt2\langle N\gamma|\Lambda\rangle=2\sqrt3\langle p\gamma|\Sigma^+\rangle=-2\sqrt6\langle N\gamma|\Sigma^0\rangle=\sqrt6\langle\Sigma^0\gamma|\Xi^0\rangle\,. \tag{5.6}$$

For the charmed baryons, we have

$$2\langle\Lambda_1'^+\gamma|\Xi_1^+\rangle=2/\sqrt3\langle\Sigma_1^+\gamma|\Xi_1^+\rangle=2/\sqrt3\langle\Lambda_1'^+\gamma|\Xi_1'^+\rangle=-\sqrt6\langle\Sigma_1^0\gamma|\Xi_1^0\rangle=\sqrt2\langle\Xi_1'^0\gamma|\Omega_1^0\rangle=-\sqrt6\langle\Xi_1^0\gamma|\Omega_1^0\rangle=-\sqrt3\langle\Xi_2^+\gamma|\Omega_2^+\rangle\,. \tag{5.7}$$

(ii) $\Delta C=\Delta S=-1$ *decays*. In the Cabibbo-enhanced mode, we obtain the following sum rules:

$$\langle\Sigma^+\gamma|\Lambda_1'^+\rangle=\langle\Lambda\gamma|\Sigma_1^0\rangle\,, \tag{5.8}$$

$$\langle\Xi^0\gamma|\Xi_1'^0\rangle=\tfrac14\langle\Sigma^+\gamma|\Sigma_1^+\rangle=-\tfrac12\langle\Sigma^0\gamma|\Sigma_1^0\rangle=\tfrac15\langle\Xi^0\gamma|\Xi_1^0\rangle=-\langle\Xi_1^+\gamma|\Xi_2^+\rangle=-1/\sqrt3\langle\Xi_1'^+\gamma|\Xi_2^+\rangle\,. \tag{5.9}$$

VI. SUMMARY AND CONCLUSIONS

We have investigated the problem of weak radiative decays of hyperons and charmed baryons in a quark model. At this stage, it is difficult to make comments on the weak radiative decays of charmed baryons because of the nonavailability of the experimental data. However, we arrive at two interesting results: an important part of radiative weak decay amplitudes arises through nonspecator interactions and the single-quark specator interactions give null contribution to these decays. In the GIM scheme, all the possible decay modes are allowed to occur, except the mode $\Xi^-\to\Sigma^-\gamma$, for the charm-conserving case. However, the relation $\langle\Sigma^-\gamma|\Xi^-\rangle=0$ is experimentally invalid. As in the case of nonleptonic decays, a 15-dimensional component of the weak Hamiltonian seems to be required here, too; which may arise through incomplete cancellations[24] of the different contributions, owing to large mass difference between u and c quarks. Then, the amplitude for the decay $\Xi^-\to\Sigma^-\gamma$, is not zero.[25] The decay $\Sigma^0\to N\gamma$ should be swamped by the faster electromagnetic decay $\Sigma^0\to\Lambda\gamma$ and will probably be very difficult to detect experimentally.

Since the GIM model with 20″ dominance leads to several unsatisfactory features for both uncharmed and charmed hadrons, we consider the possibility of the antisymmetric representations present in the direct product 15 × 15. In a recent paper,[23] it has been shown, by using simple dynamical assumptions, that the GIM contribution (20″ + 84) to the charmed hadronic decays is small, and that the dominant contribution would come from 45 and 45* representations, which may arise through left × right current-current interaction. Inclusion of these antisymmetric representations forbids all the possible Cabibbo-enhanced and Cabibbo-suppressed radiative decays through single-quark transitions and it is found that the dominant contribution to these decays comes from nonspectator interactions.

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