Nucleus-nucleus collisions and interpretation of cosmic-ray cascades above 100 TeV

T. K. Gaisser and Todor Stanev*
Bartol Research Foundation of The Franklin Institute, University of Delaware, Newark, Delaware 19711

Phyllis Freier and C. Jake Waddington
School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455
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We use low-energy data on pion production in collisions between nuclei to study some characteristics of high-energy cosmic-ray showers. The fact that pion production is suppressed in collisions initiated by heavy nuclei has the result that fluctuations in showers generated by such nuclei are larger than would be expected in some previously used models, though not so large as to impair use of air showers for studies of composition and cross section. We emphasize here implications for new cosmic-ray experiments around 10^9 GeV.

I. INTRODUCTION

Knowledge of the chemical composition is fundamental to understanding the origin, acceleration, and propagation of cosmic rays. At energies much above 10^{14} eV, however, the identification of single primary cosmic-ray particles is at present impossible because of their low flux. At such energies the only source of available information comes from the cascades initiated by energetic primary particles in the atmosphere—the extensive air showers (EAS). A similar situation exists for the study of hadronic interactions above 10^{15} eV.

Air-shower experiments such as the University of Utah's Fly's Eye,[1] the University of Durham's fast-timing Čerenkov array[2] and the Soviet air Čerenkov experiment[3] show great promise for obtaining rather direct information both about the properties of hadronic cross sections and about composition at $10^{17}-10^{18}$ eV (and even higher in the case of Fly's Eye). This is so because these techniques map longitudinal development, including the early stages, of individual showers, a capability that distinguishes them from conventional EAS experiments, which typically record densities, arrival times, and directions of charged particles at only one depth.

Even for these new experiments, however, interpretation is not straightforward because of the intermingling of effects due to primary composition and effects due to hadronic interactions. As a pertinent example, we note that a *pp* cross section that continued to increase with energy as observed below 50 TeV (Ref. 4) would lead to a much reduced hadron interaction length, increasing the difficulty of distinguishing among different primary nuclei on the basis of the locations of their early interactions in the atmosphere.

For these reasons, simulations will continue to play a fundamental role in interpretation of these and other EAS experiments, both with regard to their implications for composition and for the properties of particle interactions. A particularly significant ingredient of such simulations, in view of the possible importance of complex nuclei (i.e., those with atomic number $Z > 1$) is a proper treatment of nuclear breakup and especially of pion production in nucleus-nucleus collisions. Indeed, this is an essential element of the analysis of any air-shower experiment and is of general importance for all observations dealing with the propagation of cosmic rays in the atmosphere.

The purpose of this paper is to study the possible effects on interpretation of air-shower data of the treatment of fragmentation and pion production in energetic nucleus-nucleus collisions. In particular, we will determine under what circumstances the simple superposition model is adequate and when a more detailed account of nuclear breakup is required. We also will provide a simple algorithm for nucleus-nucleus collisions that incorporates as much information as possible from experimental observations of such collisions at lower energies, which can be used in simulations of EAS's.

Previous work along these lines has not emphasized direct use of experimental data on pion production in collisions between nuclei. To remedy this omission as far as possible is a main goal of this work. In their initial studies of nuclear collisions in the context of air showers Dixon, Turver, and Waddington[5] arbitrarily assumed that 25% of the released nucleons interacted in the target nucleus to produce pions. Tomaszewski and Wdowczyk later reported,[6] however, that 60–75 % of released nucleons interacted, and this last number was used in some later calculations.[7,8] In this paper we want to investigate the extent to which this discrepancy can be removed by examination of some data on meson production in nucleus-nucleus collisions.

The basic idea underlying our approach is to relate the observed multiplicity of pions to the number of interacting nucleons through the measured value of the corresponding multiplicity in proton-proton collisions. The use of the data on pion multiplicity in nucleus-nucleus collisions is unfortunately not as direct as one might hope because of cascading in the nucleus, which tends to increase the number of secondary mesons per interacting nucleon in the projectile, and because of coherent interactions of groups of nucleons, which have the opposite effect.

The plan of the paper is as follows. We first describe the experimental data used. We then describe in Sec. III the algorithm we use to construct histories of nuclear fragmentation and pion production for use in cascade calculations. Section IV illustrates the use of the algorithm for sample calculations of EAS longitudinal development. In the Appendix we check the consistency of our simple model of pion production with other data on nucleus-nucleus collisions.

II. NUCLEUS-NUCLEUS DATA

Our work is based on a sample of 549 interactions produced by nuclei with Z between 8 and 28 in nuclear emulsion.[9] The geomagnetic threshold energy at which these nuclei were collected was $\sim$7 GeV/nucleon, giving a mean primary energy of $\sim$20 GeV/nucleon and a median energy of 11 GeV/nucleon, which is about as high an energy as can be obtained for a selection that depends on the geomagnetic threshold. There is also some information, with much more limited statistics, on collisions of nuclei at energies up to several TeV per nucleon, which we refer to later.

For every interaction the following quantities were measured: $Z_{\rm inc}$, the charge of the incident nucleus; $Z^i_{\rm sec}$, charge(s) of fragment(s) with $Z \geq 3$; N_α, the number of fast α particles; N_h, the number of heavily ionizing tracks produced by slow particles; and N_s, the number of relativistic, singly charged shower particles. As it is not possible to distinguish between the minimum ionizing tracks due to protons and those due to mesons, the number of released protons was taken from charge conservation,

$$N_p = Z_{\rm inc} - \sum Z^i_{\rm sec} - 2N_\alpha \ . \tag{1}$$

The number of charged mesons produced is then $N_\pi = N_s - N_p$. The emission angles of all tracks except those of the N_h particles were also measured to an accuracy of better than 5 mrads.

We confine our attention here to those events from the sample with $2 \leq N_h \leq 7$, which are predominantly interactions of incident nuclei with target nuclei of the CNO group in the emulsion, and therefore similar to interactions on atmospheric nuclei. The proportion of such events in the

TABLE I. Interactions of $Z_{\rm inc}$ with CNO in emulsion ($2 \leq N_h \leq 7$). f = fraction without charged meson production. $\langle\ \rangle$ means average per event with charged-meson production.

$Z_{\rm inc}$	No. of events	No. of events with meson production	f	$\langle N_p \rangle$	$\langle N_\pi \rangle$	C	$C_{\rm geom}$
8–10	68	56	0.18	4.1	8.0	0.38 ± 0.05	0.57
11–14	46	31	0.33	6.1	9.9	0.31 ± 0.06	0.49
15–18	27	17	0.37	6.3	7.3	0.22 ± 0.05	0.54
19–22	30	23	0.23	8.2	7.0	0.16 ± 0.03	0.46
23–26	23	16	0.30	12.1	6.0	0.09 ± 0.03	0.37
All	194	143	0.26	6.4	7.9	0.24 ± 0.02	

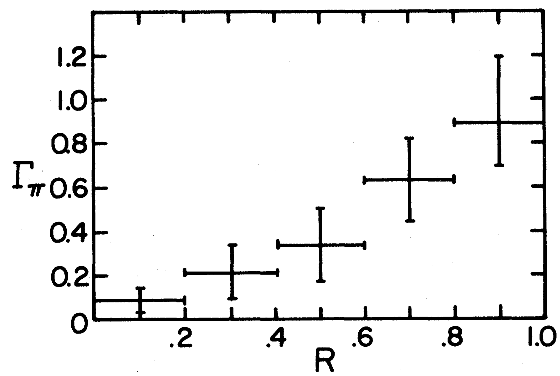

FIG. 1. Ratio Γ_π of events without charged-meson production to events with charged-meson production versus the ratio R of charge of fragment to incident nucleus. Most events with no pion production occur in peripheral collisions with large fragments.

sample was 36% of the total, which includes some interactions on Ag-Br nuclei. We estimate this contamination to be at most one-third.[10] The composition of the beam for the sample used is given in Table I, which shows that the charge selection was strongly (and deliberately) biased towards high charges and therefore does not reflect the normal cosmic-ray abundances.

One important feature of the fragmentation process, which has not been taken into account before, was obvious from the data—an appreciable fraction of inelastic nucleus-nucleus collisions has no appreciable charged-meson production, even at these energies where one should expect reasonably high multiplicity. There is no evidence from the data for a Z dependence of the fraction of interactions without charged-pion production, and the mean value of this fraction for the sample is 0.26 ± 0.04.[11]

However, the gross features of the interactions, such as the fragmentation probabilities and the number of protons freed, are quite different for the two types of events. For example, Fig. 1 shows the ratio Γ_π of the number of events with no charged-π production to those with charged-π production plotted as a function of the ratio R of fragment to primary charge. It is clear that most of the events with no charged-π production correspond to peripheral collisions characterized by relatively weak fragmentation, i.e., with $R \approx 1$.

Using that fraction of events with charged-π production together with the average multiplicity $\langle m \rangle$ found in p-p collisions, we can roughly estimate the fraction of freed nucleons which interact in the target nucleus to produce pions. We call this fraction C:

$$C = \frac{\langle N_\pi \rangle}{2\langle N_p \rangle} \frac{1}{(\langle m \rangle /2)(1+w)} , \tag{2}$$

where w is the average number of interactions of a wounded nucleon in the target nucleus. The numerator in Eq. (2) is just the mean charged-meson multiplicity per nucleus-nucleus collision. The denominator is the average number of released nucleons ($\cong 2\langle N_p \rangle$) multiplied by the average multiplicity in a collision between a nucleon and a target nucleus. Table I shows the values of C for different primary nuclei and for the whole sample for $w = 1.4$ and $\langle m \rangle = 2.15$.[12] For comparison, we show in the last column of Table I C_{geom}, which is a simple geometric estimate of C. Within the independent-nucleon picture one estimates the number of nucleons in a projectile of mass A that interact as[13]

$$W_A = A \sigma_{p-\text{air}} / \sigma_{A-\text{air}} , \tag{3}$$

so that $C_{\text{geom}} = \frac{1}{2} W_A / \langle N_p \rangle$. C is always less than C_{geom}, and this difference increases with increasing Z.

In contrast, the experiment of the ABBBBCD-MPSTTUVWY group[14] at JINR, Dubna finds agreement with the geometrical picture for d, He, and carbon on carbon targets. Evidently the simple geometrical estimate of W_A works for projectiles with $A \leq$ target mass, but not for those with $A >$ target mass, as suggested by the systematic trend of Table I. The Dubna experiment, at 4.2 GeV/nucleon, measures the momentum and charge of outgoing singly charged particles and so can define a noninteracting proton as a positive particle with more than 75% of the beam momentum per nucleon. Unfortunately, only data for d, He, and carbon on carbon and tantalum targets are available.

We can compare the experimental results here to a limited amount of data at higher energy, where $C = 0.45$ for 21 interactions with $6 \leq Z \leq 12$ ($\langle Z \rangle = 8$) and $C = 0.20$ for 12 interactions with $14 \leq Z \leq 23$ ($\langle Z \rangle = 19$). These are obtained from the data with $N_h \leq 7$ of Lohrmann *et al.*[15] ($\sim$200 GeV/A), Abraham *et al.*[16] (1–20 TeV/A) and Somogyi *et al.*[17] ($\sim$300 GeV/A), taking into account the energy dependence of $\langle n_{\text{ch}} \rangle_{pp}$ and assuming the same value for w. In view of large statistical uncertainties, these values are quite consistent with those obtained in Table I. In addition, the data of Refs. 15 and 16 confirm that the fraction of charge released as protons in events with

$N_h \leq 8$ remains constant at $\sim 40\%$ up to several TeV/A.

Interpretation of C as the fraction of released nucleons that interact assumes a model of nucleus-nucleus collisions in which nucleons interact independently to produce pions. The experimental fact is that the multiplicity of produced pions is surprisingly low in collisions induced by heavy nuclei. This fact has also been noted by Somogyi *et al.*,[17] who argue that the number of interacting nucleons is large (consistent with a geometrical picture of the nucleus-nucleus collision), but that the multiplicity per interacting nucleon is anomalously low because groups of nucleons in the projectile nucleus act coherently to produce pions. In either case, the crucial feature of the data for cascade development is that there are relatively few elementary interactions per nucleus-nucleus collision and, consequently, relatively large fluctuations in shower development. For simplicity here we assume the interactions are independent collisions of projectile nucleons rather than coherent interactions of groups of projectile nucleons.

III. FRAGMENTATION MODEL

Having made this simplifying assumption, we can then use any desired model of nucleon-nucleon collisions to extrapolate to ultra high energies. We assume here that the difference between *pp* and *p*-air collisions has negligible effect on cascade development.[18] The further assumption that fragmentation probabilities and the number of elementary interactions depend only on nuclear charges but not on energy per nucleon allows us to use the 194 observed interactions as a data bank for Monte Carlo simulation of nuclear collisions in cosmic-ray cascades. The data of Refs. 15–17 offer some support for this energy-independent extrapolation of fragmentation probabilities.

It is then natural to divide any cascade calculation into two parts: (1) a subprogram which constructs a fragmentation history of each incident nucleus and (2) a subprogram which calculates development of the component cascades initiated by nucleon-nucleon collisions along the shower core. The output of the first part is the location in the atmosphere of the point of first interaction of each nucleon in the incident nucleus.

The data file we use in the simulation contains fragmentation events of nuclei with Z from 8 to 26. It is therefore possible to pick randomly from the data itself a fragmentation history down to $Z_{\rm inc}=8$. To smooth the file and to account for the experimental fact that $Z_{\rm inc}$ is measured with error of ± 1, for every $Z_{\rm inc}$ we used a floating group including the events from $Z_{\rm inc}-2$ to $Z_{\rm inc}+2$. This allowed us to construct a large number of different fragmentation histories. For the fragmentation of nuclei with $Z<8$ the average fragmentation probabilities in air obtained by Freier and Waddington[19] were used as the basis of a random sampling. All He nuclei were assumed to be totally broken up in the interactions with air nuclei.

The points of interaction of the primary nuclei and all fragments were picked from exponential distributions with mean free path λ corresponding to a nucleus-nucleus cross section given by

$$\sigma_{A-\rm air}=\pi R_0^2(A^{1/3}+(14.5)^{1/3}-\delta)^2 \qquad (4)$$

with $\delta=1.12$ and $R_0=1.47$ fm.[20] If in the randomly picked event pion production had occurred, then one of the freed nucleons was forced to interact within the target nucleus. Each of the other freed nucleons was assigned a random probability to interact within the target nucleus in such a way that the mean fraction of released nucleons that interact was given by C in Table I. For $3\leq Z<7$ we assumed $C=0.37$. For helium nuclei the mean number of interacting nucleons was 1.35. Every released nucleon which did not interact immediately was followed until it interacted in the atmo-

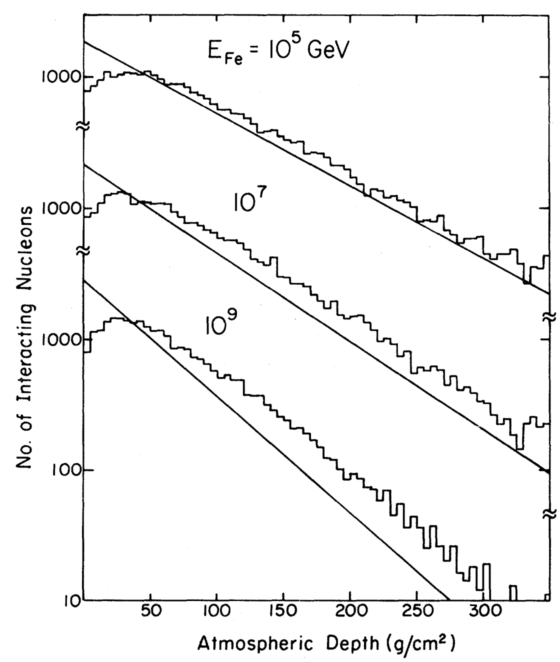

FIG. 2. Distribution of points of first interaction in the atmosphere of nucleons in incident iron nuclei.

TABLE II. Comparison of air-shower parameters using different models of nucleus-nucleus interactions. $Y_{1/4}$ and Y_{max} here are averages of depth of one-quarter maximum and depth of maximum in gm/cm^2. σ is standard deviation.

	Nonsuperposition				Superposition				$C=0.75$ Nonsuperposition			
A_{inc}	$Y_{1/4}$	$\sigma/Y_{1/4}$	Y_{max}	σ/Y_{max}	$Y_{1/4}$	$\sigma/Y_{1/4}$	Y_{max}	σ/Y_{max}	$Y_{1/4}$	$\sigma/Y_{1/4}$	Y_{max}	σ/Y_{max}
1	511	0.085	782	0.061	a	a	a	a	a	a	a	a
4	471	0.089	745	0.059	457	0.042	733	0.034	a	a	a	a
14	442	0.057	721	0.041	415	0.024	693	0.021	440	0.051	716	0.038
26	416	0.051	692	0.033	393	0.024	668	0.019	403	0.042	683	0.023
56	385	0.051	661	0.025	369	0.019	646	0.014	375	0.036	652	0.019
Low-energy composition	480	0.118	754	0.076	471	0.126	744	0.080				

[a]Same value as corresponding entry in the first section of the table.

sphere according to an energy dependent cross section.[21]

$$\sigma_{p-\mathrm{air}}=280+2.5\ln^{1.8}(E_p/100\text{ GeV}) . \quad (5)$$

As a check of the self-consistency of the model the number of nucleons, $N(x)$, still bound in fragments at atmospheric depth x was obtained for Fe nuclei penetrating the atmosphere. We found $N(x)=56e^{-x/\lambda}$ with $\lambda=44$ g/cm^2, as in the earlier work of Ref. 19, which used a different data set.

To illustrate the effects of a realistic treatment of fragmentation and pion production we show in Fig. 2 the distribution of points of first interactions of nucleons per 5 g/cm^2 interval of the atmosphere for 500 incident Fe nuclei at each of three energies. The nucleon interaction lengths are, respectively, 81, 64, and 48 g/cm^2 at 10^5, 10^7, and 10^9 GeV energy per nucleus. The straight lines in Fig. 2 show the same quantities for the superposition model. The superposition model assumes that the incident nucleus is equivalent to a beam of free nucleons that interact independently from the beginning with the nucleon interaction lengths stated above.

IV. AIR-SHOWER SIMULATIONS

Next we use this realistic algorithm for pion productions in collisions of energetic nuclei to calculate some simple features of air showers at 3×10^{17} eV. This is typical of energies of cascades observable in the new EAS experiments.[1–3] We choose to calculate features of longitudinal development that are accessible to these experiments and to some extent to older air-shower experiments as well. We have used the simple method described by Gaisser[22] to calculate subshowers initiated by the interactions of the individual nucleons with energy E_0/A at points as determined by the nuclear fragmentation algorithm for a primary nucleus of mass A and total energy E_0. The model for the nucleon showers as based on a scaling model of hadron interactions as described by Gaisser *et al.*[7,22]

In Table II we compare the results for atmospheric depth of shower maximum (y_{max}) and depth at which showers reach one-fourth maximum on the rising edge ($y_{1/4}$) for three sets of assumptions about collisions between nuclei. The model labeled "nonsuperposition" uses the values of C given in Table I. It is clear that a realistic treatment of fragmentation leads to significantly larger fluctuations than the superposition picture.

For comparison we also show in Table II the results of the assumption that $C=0.75$ for nuclei heavier than α particles. For heavy nuclei it gives results intermediate between superposition and the model of fragmentation and pion production developed here. The last row of the table shows overall results for a mixture of primary nuclei similar to the observed composition at low energy.

Results for the mixed composition within the simulation based on the data of Table I are illustrated in Fig. 3. (Note that this composition compares abundances at the same energy per nucleus, not energy per nucleon.)

V. CONCLUSION

The basic conclusion that we reach by comparing multiplicity of secondary mesons in the observed collisions between nuclei to the multiplicity in pp collisions at the same energy per nucleon is that the nuclear collisions produce relatively few pions. When we interpret this in terms of an

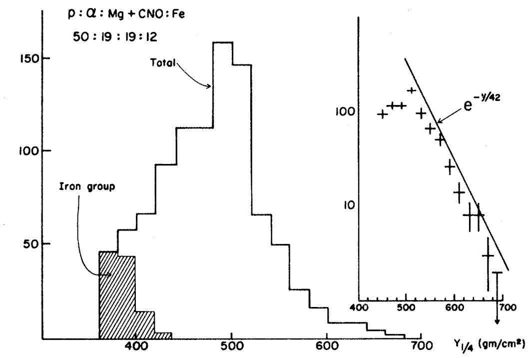

FIG. 3. Distribution of depths of $\frac{1}{4}$ maximum on the rising edge for about 1000 simulated showers of total energy per nucleus of 3×10^{17} eV. We used the library of real events as the basis for fragmentation and pion production in collisions between nuclei, as described in the text. The assumed primary composition (on an energy per nucleus basis) is noted, and the showers due to Fe primaries are shown by the shaded region. The inset shows the same histogram plotted logarithmically to indicate how it may be possible to use the tail of the distribution (which is due primarily to protons) to determine $\sigma_{p-\mathrm{air}}$.

equivalent number of nucleon-nucleon collisions we find that the fraction of nucleons that interact to produce pions is small. For example, on average only about 10% of nucleons released from an iron nucleus incident on a nitrogen target interact to produce pions in the first interaction.

The main consequence of the small number of equivalent interactions in nucleus-nucleus collisions for cascade development is that fluctuations in shower development are significantly larger than in the superposition picture. Relative fluctuations in depth of maximum and in depth at which showers reach one-quarter maximum are about twice as large as in the superposition picture. The average effect of the realistic picture of fragmentation is to make showers penetrate somewhat further into the atmosphere. We find $y_{\max}$ about 2% deeper and $y_{1/4}$ about 5% deeper on average than in the superposition picture.

These considerations are also relevant to experiments with thin calorimeters in which charges of both primary and fragment nuclei can be measured directly but in which not all energy is deposited in the calorimeter. Fluctuations in early cascade development, especially fluctuations in C, will contribute to fluctuations in the relation between E(total) and E(visible) for primaries with $Z>1$. An example of such an experiment is the Japanese-American emulsion-chamber collaboration.[23]

Since some previous air-shower calculations have assumed that the fraction of released nucleons that interact is $C=0.75$ we also checked the consequences of this assumption. The results for α particles are not affected since we have used the same picture of α fragmentation in both cases. Results for incident CNO are also very similar to the present nonsuperposition model, whereas those for the heavier nuclei are intermediate between nonsuperposition and superposition. In particular, relative fluctuations in $y_{\max}$ and $y_{1/4}$ of showers initiated by heavy nuclei are underestimated by about 25% in the model with $C=0.75$ instead of $C\cong0.1-0.2$. Elbert *et al.*[8] found, however, that different assumptions about pion production (ranging from $C\cong0.1$ to $C\cong0.6$ for iron) had a negligible effect on fluctuations in the muon to electron ratio at sea level.

The result that $C=0.75$ came from an analysis of interactions of nitrogen nuclei with airlike targets in emulsion.[6] That result depended on an analysis of angular distributions of secondary shower tracks and did not make use of the observed multiplicity as we do here. In the Appendix we show that the two sets of data are consistent and argue that the angular criterion is not adequate to infer the number of equivalent nucleon-nucleon collisions.

The actual numerical values obtained for C, the fraction of released nucleons that interact, depend sensitively on the values used for the average number of wounded nucleons in the projectile and for the average multiplicity of charged mesons per nucleon-nucleon collision [w and $\langle m\rangle$ in Eq. (2)]. The former is model dependent and the latter has uncertainties of at least 10% due to uncertainties in the basic data and in the cosmic-ray spectrum used to form the weighted average $\langle m\rangle$. Fortunately, the implications for cascade development discussed above are insensitive to uncertainties at the level of 10–20%. For example, a calculation in which we took $C=0.33$ instead of the values in Table I gave essentially the same results as shown in Table II.

We have shown results for depth of one-fourth maximum here because this quantity has been suggested[1] as a practical measure of depth of shower initiation to study nucleon cross sections and primary composition. We note that fluctuations for α-initiated showers are comparable to protons and that even heavier nuclei have significant fluctuations, though they are smaller than for protons and α's. The ease with which the new air-shower experiments[1-3] can simultaneously measure composition and cross section will thus depend on what the composition is as well as whether the cross section continues to increase to $10^8 - 10^9$ GeV. We also note from Table II that on average $y_{max} - y_{1/4} \cong 275$ g/cm^2 independent of the mass of the primary nucleus. It follows that the slope of the rising edge of showers will not be a good measure of primary mass. Some measure of shower origin, such as $y_{1/4}$ as originally suggested,[1] would thus appear to remain the best indicator of the mass of the nucleus that initiates an air shower.

The inset in Fig. 3 shows the distribution of $y_{1/4}$ on a logarithmic scale. Showers that start late are predominantly protons, so the tail of this distribution presumably reflects $\sigma_{p-\text{air}}$. (In the illustration, $\sigma_{p-\text{air}}$ corresponds to an interaction length of 42 g/cm^2 at 3×10^{17} eV.) Moreover, the shoulder in Fig. 3 at $\frac{1}{4}$ maximum <400 g/cm^2 is almost entirely due to the iron component, which is assumed to be only 12% in this illustration. We are therefore optimistic that the direct shower-observation methods[1-3] will be able simultaneously to obtain significant information about composition and about hadronic cross sections provided instrumental broadening can be reduced sufficiently.

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APPENDIX

In this appendix we compare the data used here with that of Ref. 6 in order to trace the source of the discrepancy between the results of Table I for C and the value of $C=0.75$ used in Ref. 6 for the fraction of released nucleons that interact. We then discuss a simple model of pion production in collisions between nuclei in order to find a geometrical interpretation of our results.

The data used in the work of Tomaszewski and Wdowczyk (TW)[6] are from emulsion exposures made during the International Collaborative Emulsion Flights (ICEF).[24] Their data for incident N are compared with the Freier and Waddington (FW) data in Tables III and IV. If we take $E_{min}=5$ GeV/nucleon for the ICEF data, then the corresponding multiplicity of charged pions per pp

TABLE III. FW data (incident oxygen nuclei).

	I	No. of bound p	No. of released p	$N_{\pi^\pm}$	$N_{\pi^\pm}$[a]/released N	N_s/event	C
All	119	420	532	1304	1.23	15.4	0.48
$N_h \le 8$	72	337	239	435	0.91	9.4	0.35

[a]Assume No. of released neutrons = No. of released protons.

TABLE IV. TW data (incident nitrogen nuclei).

	I	No. of bound p	No. of released p	$N_{\pi^\pm}$	$N_{\pi^\pm}$[a]/released$N$	N_s/event	C
All	90	198	432	972	1.13	15.6	0.57
$N_h \le 8$	61	170	257	377	0.73	10.4	0.37

[a]Assume No. of released neutrons = No. of released protons.

collision is 1.66.[12] TW give $N_s = 10.4$ per event for 61 events with $N_h \leq 8$. We assume N_s is the number of singly charged, minimum-ionizing particles. The number of protons among these is $(61\times 7-170)/61=4.2$ per event; thus $N_{\pi^\pm}=6.2$ per event. With $\langle m \rangle = 1.66$ and $w=1.4$ in Eq. (2), we thus estimate $C=0.37$, in good agreement with 0.35 of FW. The results for all interactions in emulsion are also comparable.

There are, however, some noticeable differences between the two data sets, especially the ratio of released to bound protons, which is significantly larger (2.2) for TW than for FW (1.2). Some of the differences could be due to an increased tendency for α emission for oxygen as compared to nitrogen projectiles. If so, however, one might expect a smaller value of C for the nitrogen data than for the oxygen data. The actual value obtained for C is quite sensitive to the value used for $\langle m \rangle$, which in turn depends sensitively on the cosmic-ray spectrum and cutoff energy. These are not well known for the ICEF experiment. In any case, there does not appear to be a serious discrepancy in the data, and it seems reasonable to conclude that $C \sim 40\%$ in collisions between light nuclei.

This differs from the conclusion of TW that $C=0.72$, which was based on angular distributions of the released protons.[25] This brings into question the use of that criterion to distinguish wounded protons. The actual angular criterion used to separate wounded from unscathed protons in TW was not stated. If it is assumed that the noninteracting protons follow an evaporation spectrum[26] then the appropriate dividing point would be about 1.2° for the FW data and about 1.8° for ICEF. If we use the 1.2° criterion on the $Z=8$ FW data described above we get $C=0.87$, as compared to $C=0.81$ obtained from the uncorrected data of TW. Evidently, the experimental angular distributions themselves are in agreement with each other.

There are two possibilities. Either the angular distributions of noninteracting, released protons are much broader than predicted by evaporation theory or groups of nucleons in the projectile have interacted coherently as suggested by Somogyi *et al.*[17] In the latter case the effective energy of the interaction would be increased because of the mass of the cluster. This effect has been discussed by Afek *et al.*[27]

It is interesting to note that the α's from the projectile also have a broader angular distribution than would be expected from evaporation theory. As an alternative we therefore tried using the angle of emitted α's in each event as a criterion to divide wounded from unscathed nucleons. We took the dividing angle as twice the angle of the widest α in each event with an emitted α. In events without α emission we used twice the angle of the proton with the smallest angle. This criterion gives $C=0.34$, in rough agreement with the result based on pion multiplicity.

A preliminary analysis of data at somewhat lower energy obtained from a balloon flight at Palestine, Texas[28] suggests that that data is also generally consistent with the data we use here. A similar conclusion concerning the number of interacting nucleons has also been reached by Kalmykov and Kulikov[29] on the basis of low-energy (average ~ 3.5 GeV/nucleon) interactions in a satellite emulsion exposure. Their result is equivalent to $C \simeq 0.43$ for $6 \leq Z \leq 9$ and $C \simeq 0.35$ for $Z \leq 10$, comparable to our values in the first two rows of Table I.

The qualitative features of the data in Table I of the text can be interpreted geometrically using a picture discussed earlier.[30] The basic assumption is that pion production in a nucleus-nucleus interaction is proportional to the fraction of the total volume of the nuclear matter involved in the collision. This fraction is represented by the overlap parameter X in the interaction, which depends on the strength of the fragmentation. Successively lighter fragments of the beam nucleus are produced as X is incrementally increased so that $X=\delta+d_i$, where d_i is the overlap parameter associated with a fragment of charge Z_i. $d_i = mxZ_i + q$, where both m and q depend on the mass of the beam and target nuclei. However this dependence is not very strong.

We have estimated the values of m and q from the partial fragmentation cross section for Fe on carbon and on polyethylene,[20,31] both of which represent targets similar to air. Both sets of data agree, with d_i represented as

$$d_i = 5.13 - 18xZ_i \ . \qquad \text{(A1)}$$

These values of m and q were used to obtain the impact parameter of each collision as follows. For every interaction the fraction of primary charge released (f) was determined as

$$f = \frac{Z_{\rm inc} - Z_i}{Z_{\rm inc}} \qquad \text{(A2)}$$

and d_i obtained as $d_i = f(-mZ_{\rm inc}) + (q + mZ_{\rm inc})$. Thus the impact parameter of the interaction b

TABLE V. Average values of the impact parameter.

Z_{inc}	$\langle b \rangle$ (fm)
8–10	1.2
11–14	1.6
15–18	2.0
19–22	2.3
23–26	2.9

was determined as

$$b = R_0 \left[A_{inc}{}^{1/3} + (14.5)^{1/3} - \delta \frac{d_i}{R_0} \right] \quad (A3)$$

with $R_0 = 1.35$ fm and $\delta = 0.83$ fm as in Westfall *et al.*[20] for charge changing interactions. Average values of b for each charge group are shown in Table V.

The physical picture that emerges is that, for a given target, the nucleons in heavy projectiles are on average more peripheral and hence less likely to interact than those in light projectiles. In contrast, the fraction of released nucleons appears to remain constant at roughly 50%, independent of projectile mass (see columns 1 and 5 of Table I). In the simple geometrical picture that leads to Eq. (3) one would expect $\langle b \rangle \sim A^{1/3} + \text{const}$, which would correspond to less than a 50% increase between $A = 8$ and $A = 26$. Table V shows more than a 100% increase in $\langle b \rangle$. Thus two approaches to the data [(through Eq. (2) and through Eq. (A2)] both suggest that the fraction of interacting nucleons for heavy projectiles on light targets is significantly less than the conventional estimate of Eq. (3).

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*On leave from Institute for Nuclear Research and Nuclear Energy, Sofia, Bulgaria.

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[10]This estimate is based on an extrapolation of the prong distributions from $N_h > 8$ (which can only be interactions on Ag-Br) to lower values, together with a knowledge of relevant interaction cross sections and the composition of nuclear emulsion. Similarly, we estimate that a comparable proportion of interactions on CNO targets are peripheral interactions with $N_h \le 1$. It is therefore difficult to estimate whether the $2 \le N_h \le 7$ group is more or less peripheral than the group of true interactions on CNO targets in the emulsion. It is important to remark that the prong distribution does not vary significantly with projectile charge for $8 \le 7 \le 26$. We therefore believe that the use of the group with $2 \le N_h \le 7$ to represent collisions on atmospheric target nuclei is not responsible for the systematic decrease of multiplicity per released nucleon with increasing projectile charge that is observed in the data. (See discussion below and Table I.)

[11]At this energy we would expect only $\sim 5\%$ of nucleon-nucleon interactions to involve inelastic collisions of nucleons without pion production. We therefore treat all nucleus-nucleus interactions without pion production as fragmentation without inelastic interactions of nucleons.

[12]A better value of w is probably about 1.8 [G. Berlad, A. Dar, and G. Eilam, Phys. Rev. D 13, 161 (1976)], but this would lead to even smaller values of C. The average multiplicity of charged mesons $\langle m \rangle$ is obtained by taking the average of $\langle n_{ch} \rangle_{pp}$ weighted by the cosmic-ray energy spectrum then subtracting the average multiplicity of protons, which is ~ 1.5 for pp collisions. We have used the parametrization of E. Albini *et al.*, Nuovo Cimento 32A, 102 (1976) for $\langle n_{ch} \rangle_{pp}$ and results given by H. Boggild *et al.*, Nucl. Phys. B27, 285 (1971) and M. Antinucci *et al.*, Lett. Nuovo Cimento 6, 121 (1973) for the multiplicity of protons. We estimate the uncertainty in $\langle m \rangle$ to be $\pm 5\%$.

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