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Parity-violating effects in electroproduction of the $\Delta(1232)$ by polarized electrons

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Parity-violating effects in the (quasi)elastic scattering of longitudinally polarized electrons from spin-$\frac{1}{2}$ targets are discussed in the framework of the unified gauge theories of weak and electromagnetic interactions. In particular, the R-L asymmetry in electroproduction of the $\Delta(1232)$ is studied in considerable detail, taking into account the full spin-$\frac{3}{2}$ structure of this resonance. Our results are valid for all values of E and the entire range of Q^2. Numerical predictions are given for the standard Weinberg-Salam model at several energies of the electron beam.

I. INTRODUCTION

There is mounting evidence that the simplest unified model of weak and electromagnetic interactions,[1] the Weinberg-Salam (WS) model, agrees remarkably well with experiment. Recent analyses[2] have centered on determining the coupling parameters of the weak gauge boson(s) Z to the various fermions of the theory. The coupling of the Z boson to neutrinos has been tested in a number of different experiments. The coupling to the electron, in contrast, has so far only been determined in one noncontroversial experiment in which an asymmetry for right-handed (R) and left-handed (L) electrons was observed in deep-inelastic scattering.[3] Since the interpretation of this experiment depends on the parton model, other less model-dependent experiments on the e-Z coupling are desirable. It has therefore been proposed[4] some time ago to measure weak and electromagnetic interference via the R-L asymmetry in elastic electron-nucleon scattering. Such experiments are currently being undertaken. Another feasible experiment, first suggested by Cahn and Gilman,[5] is to measure R-L asymmetry in Δ production by polarized electrons. These two experiments are in a way complementary: in Δ production the (strong) isovector part of the neutral current is isolated, while in elastic e-N scattering both isovector and isoscalar parts contribute (which can only be separated in a model-dependent way). In Ref. 5 an approximate expression, valid for $Q^2 << 2ME$, for the R-L asymmetry in Δ production is given. In this paper we reexamine this problem and present detailed calculations that take account of the full spin-$\frac{3}{2}$ structure of the Δ resonance and are valid for all Q^2. We clarify the assumptions necessary to arrive at our result and that of Ref. 5. Detailed predictions are given for the WS model for various electron energies.

In Sec. II we present the general formalism leading to an expression for the R-L asymmetry in the elastic as well as inelastic scattering of longitudinally polarized electrons by hadronic systems. Section III contains our detailed calculations for the asymmetry in electroproduction of the $\Delta(1232)$ by polarized electrons. The implications of our results are discussed in Sec. IV.

II. GENERAL FORMALISM

We consider the (quasi)elastic scattering of a longitudinally polarized electron of momentum k, whose mass we neglect, on a target (proton, parton, etc.) of momentum p and mass M. The momentum of the outgoing electron is k' and that of the other outgoing particle (of mass M') is p'. A parity-violating R-L asymmetry arises from the interference of the photon- and Z-exchange diagrams. In analogy to deep-inelastic scattering, we define structure functions for the interference term,

$$W_{\mu\nu} = (2\pi)^3 \overline{\sum} \delta^4(p+q-p')\langle p\,|\,J_\mu^{\mathrm{EM}}(0)\,|\,p'\rangle\langle p'\,|\,J_\nu^{Z}(0)\,|\,p\rangle$$
$$= -g_{\mu\nu}W_1 + \frac{p_\mu p_\nu}{M^2}W_2 - i\epsilon_{\mu\nu\alpha\beta}\frac{p^\alpha q^\beta}{M^2}W_3\,, \qquad (2.1)$$

where $J_\mu^{\rm EM}$ is the electromagnetic and J_ν^Z the weak neutral current, and $\overline{\sum}$ implies summation and average over the polarizations of the outgoing hadron and the target, respectively. The electron couples to the Z boson according to

$$\langle k' \,|\, J_\mu^Z \,|\, k \rangle = \bar{u}(k')(g_{V,e}\gamma_\mu + g_{A,e}\gamma_\mu\gamma_5)u(k) \,. \tag{2.2}$$

The vector and axial-vector coupling constants are, e.g., in SU(2) $\times$ U(1) models given in terms of the mixing angle θ_W by

$$g_{V,e} = \frac{e}{\sin\theta_W \cos\theta_W}\frac{1}{4}(-1+4\sin^2\theta_W + 2T_{3R}^e),\;\; g_{A,e} = \frac{e}{\sin\theta_W\cos\theta_W}\frac{1}{4}(1+2T_{3R}^e)\,, \tag{2.3}$$

where T_{3R}^e is the third component of the weak isospin of the right-handed electrons which is zero in the original Weinberg-Salam model.

From Eqs. (1) and (2) we obtain for the asymmetry A_{RL}, with the abbreviation $d\sigma_{R,L} = d^2\sigma_{R,L}/dQ^2dW^2$,

$$A_{RL} = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L} = -\frac{2Q^2}{(Q^2+M_Z{}^2)}\frac{1}{e^2}\left[g_{A,e}\left[2W_1\sin^2\frac{\theta}{2} + W_2\cos^2\frac{\theta}{2}\right] + g_{V,e}\frac{2(E+E')}{M}W_3\sin^2\frac{\theta}{2}\right]$$
$$\times \left[2W_1^{\rm EM}\sin^2\frac{\theta}{2} + W_2^{\rm EM}\cos^2\frac{\theta}{2}\right]^{-1}, \tag{2.4}$$

where E and E' are, respectively, the energies of the incoming and outgoing electron and θ is the scattering angle—all in the laboratory system. Further, $Q^2 = -(k-k')^2$ and $W^2 = (p+k-k')^2$ with $\sin^2(\theta/2) = Q^2/4EE'$, and $E' = E - (W^2 - M^2 + Q^2)/2M$. The structure functions $W_{1,2}^{\rm EM}$ are defined as usual in analogy to Eq. (1) with J_ν^Z replaced by $J_\nu^{\rm EM}$.

As a simple application we consider the well-known case of elastic ep scattering. If we define form factors as usual by

$$\langle p' \,|\, J_\mu^{\rm EM} \,|\, p \rangle = \bar{u}(p')\left[\gamma_\mu F_1^\gamma + i\frac{\sigma_{\mu\nu}q^\nu}{2M}F_2^\gamma\right]u(p)$$

and (2.5)

$$\langle p' \,|\, J_\mu^Z \,|\, p \rangle = \bar{u}(p')\left[\gamma_\mu F_1^Z + i\frac{\sigma_{\mu\nu}q^\nu}{2M}F_2^Z + \gamma_\mu\gamma_5 G_A^Z\right]u(p)\,,$$

the structure functions are calculated to be

$$W_1 = G_M^\gamma G_M^Z Q^2 \delta(W^2-M^2),\;\; W_2 = 4M^2\left[F_1^\gamma F_1^Z + F_2^\gamma F_2^Z\frac{Q^2}{4M^2}\right]\delta(W^2-M^2),\;\; W_3 = 2M^2 G_A^Z G_M^\gamma \delta(W^2-M^2)\,, \tag{2.6}$$

where

$$G_M = F_1 + F_2 \,. \tag{2.7}$$

For deep-inelastic scattering, where γ and Z scatter off pointlike constituents, we replace

$$F_1^\gamma \to Q_i^{\rm EM},\;\; F_2^\gamma \to 0,\;\; F_1^Z \to \tfrac{1}{2}(Q_i^L + Q_i^R),\;\; F_2^Z \to 0,\;\; G_A^Z \to \tfrac{1}{2}(Q_i^R - Q_i^L)\,, \tag{2.8}$$

where $Q_i^{\rm EM}$ is the electric charge and $Q_i^{L(R)}$ are the left- (right-) handed weak charges of the ith quark. In addition, one sums over quarks and multiplies with parton distributions as usual.

III. ASYMMETRY IN ELECTROPRODUCTION OF THE $\Delta(1232)$

Using the formalism presented in the previous section, we evaluate now the parity-violating asymmetry effects in the electroproduction of the $\Delta(1232)$ by polarized electrons. The form factors associated with the electromagnetic current and the weak neutral current of the hadrons in the process, $ep \to e\Delta^+$, are defined by[6]

$$\langle p' | J_\mu^{\rm EM} | p \rangle = \bar{u}^\lambda(p') \left[\left[\frac{C_3^\gamma}{M}\gamma^\nu + \frac{C_4^\gamma}{M^2}p'^\nu + \frac{C_5^\gamma}{M^2}p^\nu \right] (g_{\lambda\mu}g_{\rho\nu} - g_{\lambda\rho}g_{\mu\nu})q^\rho\gamma_5 \right] u(p) , \tag{3.1}$$

$$\begin{aligned}\langle p' | J_\mu^{Z} | p \rangle = \bar{u}^\lambda(p') \Bigg[& \left[\frac{C_{3V}^Z}{M}\gamma^\nu + \frac{C_{4V}^Z}{M^2}p'^\nu + \frac{C_{5V}^Z}{M^2}p^\nu \right] (g_{\lambda\mu}g_{\rho\nu} - g_{\lambda\rho}g_{\mu\nu})q^\rho\gamma_5 + C_{6V}^Z g_{\lambda\mu}\gamma_5 \\ & + \left[\frac{C_{3A}^Z}{M}\gamma^\nu + \frac{C_{4A}^Z}{M^2}p'^\nu \right] (g_{\lambda\mu}g_{\rho\nu} - g_{\lambda\rho}g_{\mu\nu})q^\rho + C_{5A}^Z g_{\lambda\mu} + \frac{C_{6A}^Z}{M^2}p_\lambda q_\mu \Bigg] u(p) ,\end{aligned} \tag{3.2}$$

where p and p' are, respectively, the momenta of the nucleon and the $\Delta(1232)$, and $q = p' - p$.

In the (strong) isospace, the weak neutral current can be decomposed into two parts: a vector and a scalar. More explicitly,

$$J_\mu^Z = \alpha V_\mu^3 + \beta A_\mu^3 + \text{isoscalar terms} . \tag{3.3}$$

For the SU(2) $\times$ U(1) models, α and β are given by

$$\begin{aligned}\alpha &= \frac{e}{\sin\theta_W \cos\theta_W}\frac{1}{2}(1 - 2\sin^2\theta_W + T_{3R}^u - T_{3R}^d) , \\ \beta &= -\frac{e}{\sin\theta_W \cos\theta_W}\frac{1}{2}(1 - T_{3R}^u + T_{3R}^d) ,\end{aligned} \tag{3.4}$$

where T_{3R} is the third component of the weak isospin of the right-handed fermions. As the isosinglets in Eq. (3.3) do not contribute to Δ production, the experimental measurement of the asymmetry in this process is expected to yield information directly on the (strong) isovector part of the neutral current.

The form factors in Eq. (3.2) can be related to those for the electroproduction and the charged-current production of the $\Delta(1232)$ by performing a rotation in isospace and making use of CVC (conserved-vector-current hypothesis). Thus, we find, in the notation of Llewellyn Smith,[6]

$$\begin{aligned}& C_{iV}^Z = \alpha C_i^\gamma, \quad i = 3,4,5 , \\ & C_{6V}^Z = 0 , \\ & C_{iA}^Z = -\beta C_i^A, \quad i = 3,4,5,6 .\end{aligned} \tag{3.5}$$

The electroproduction form factors are, as usual, denoted by C_i^γ and the quantities C_i^A are $-1/\sqrt{3}$ times the charged-current axial-vector form factors. In the limit of zero electron mass, C_{6A}^Z does not enter into our calculation. Further, the photoproduction and the electroproduction data can be explained adequately with $C_4^\gamma = C_5^\gamma = 0$ or $C_5^\gamma = 0$, $C_4^\gamma = -C_3^\gamma M/(M+M')$. It may be noted here that the theory of the spin-$\frac{3}{2}$ field also requires that C_4^γ and C_5^γ vanish.[7] Because of these reasons, we take the coupling parameter $C_5^\gamma = 0$ and our results remain still sufficiently general.

Now, using the spin-$\frac{3}{2}$ projection operator and Eqs. (3.1) and (3.2), the structure functions can be calculated. The result can be expressed in a very concise form after introducing linear combinations D_3, D_4 of the electromagnetic form factors by

$$\begin{aligned}& C_3^\gamma(Q^2) = -\frac{M'}{M}D_3(Q^2) , \\ & C_4^\gamma(Q^2) = D_3(Q^2) + D_4(Q^2)\end{aligned} \tag{3.6}$$

and kinematical factors

$$\begin{aligned}& a(Q^2) = (M+M')^2 + Q^2 , \\ & b(Q^2) = (M+M')(M-M') + Q^2 , \\ & c(Q^2) = (M-M')^2 + Q^2 .\end{aligned} \tag{3.7}$$

For the structure functions one obtains

$$\begin{aligned}& W_1 = \alpha\delta(W^2 - M'^2)\frac{c}{6M^4}(a^2 D_3{}^2 + b^2 D_4{}^2 + abD_3D_4) , \\ & W_2 = \alpha\delta(W^2 - M'^2)\frac{2Q^2}{3M^2}(aD_3{}^2 + cD_4{}^2 + bD_3D_4) , \\ & W_3 = \beta\delta(W^2 - M'^2)\frac{1}{3M^2}(2aD_3 + bD_4)\left[(b - 2c)\frac{M}{2M'}C_3^A + \tfrac{1}{2}bC_4^A - M^2 C_5^A \right] .\end{aligned} \tag{3.8}$$

As a consequence of the fact that only the (strong) isovector part of the current contributes to the production of the $\Delta(1232)$, Eq. (3.3) implies $W_1=\alpha W_1^{\rm EM}$, $W_2=\alpha W_2^{\rm EM}$. The expression for the asymmetry thus assumes the simpler form

$$A_{RL}=-\frac{2Q^2}{e^2(Q^2+M_Z{}^2)}\left\{\alpha g_{A,e}+g_{V,e}\left[\frac{2(E+E')}{M}W_3\sin^2\frac{\theta}{2}\right]\left[2W_1^{\rm EM}\sin^2\frac{\theta}{2}+W_2^{\rm EM}\cos^2\frac{\theta}{2}\right]^{-1}\right\}. \quad (3.9)$$

Numerical predictions are given in Figs. 1–4 for the standard Weinberg-Salam model,[1] assuming that the Q^2 dependence is approximately the same for all form factors and using the following as input:[8]

$C_3^\gamma(0)=1.85,\ \ C_4^\gamma(0)=-0.89$,

$C_3^A(0)=0,\ \ C_4^A(0)=+0.35,\ \ C_5^A(0)=+1.2$

$\sin^2\theta_W=0.23$ (Ref. 2) .

The implications of our results are discussed in the following section.

IV. DISCUSSION

Investigation of the parity-violating effects in electroproduction of the $\Delta(1232)$ by polarized electrons is an alternate and, in certain respects, a complementary way to test various gauge theories. Measurements of the asymmetry in this process, as in the case of elastic and deep-inelastic scattering of polarized electrons by nucleons, will yield additional information on the coupling of the Z boson to the electron. Furthermore, as the isospin changes in the $N\rightarrow\Delta$ transition, electroproduction of the $\Delta(1232)$ by polarized electrons is particularly suitable for the determination of the coupling parameters of the (strong) isovector part of the hadronic neutral currents.

In this paper we have considered, in the framework of the unified gauge theories of weak and electromagnetic interactions, the parity-violating effects when longitudinally polarized electrons scatter off spin-$\frac{1}{2}$ targets leading to arbitrary final states. The expression for the R-L asymmetry in the case of polarized-electron elastic scattering can be obtained easily from our general formalism, as is shown in Sec. II. Then we have evaluated explicitly the parity-violating effects in electroproduction of the $\Delta(1232)$ by polarized electrons, taking the spin-$\frac{3}{2}$ structure fully into account. Our results are valid for all values of E and Q^2. The expression for the asymmetry consists of two terms, the strengths of which are determined, for arbitrary gauge models, by the coupling-constant combinations

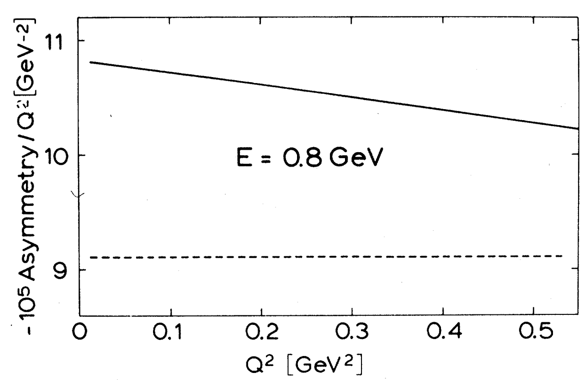

FIG. 1. Prediction of the standard Weinberg-Salam model with $\sin^2\theta_W=0.23$ for the asymmetry in electroproduction of the $\Delta(1232)$ off protons by longitudinally polarized electrons. The solid line represents the prediction of Eq. (3.9) for $-10^5 A_{RL}/Q^2$; the dashed line shows the contribution of only the first term in Eq. (3.9) (Cahn-Gilman result). The electron beam energy is 0.8 GeV.

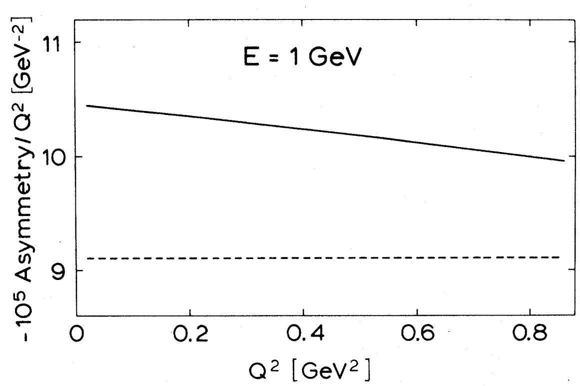

FIG. 2. Same as Fig. 1, but for electron beam energy 1 GeV.

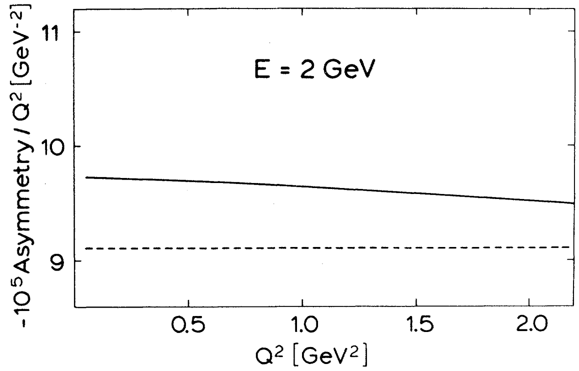

FIG. 3. Same as Fig. 1, but for electron beam energy 2 GeV.

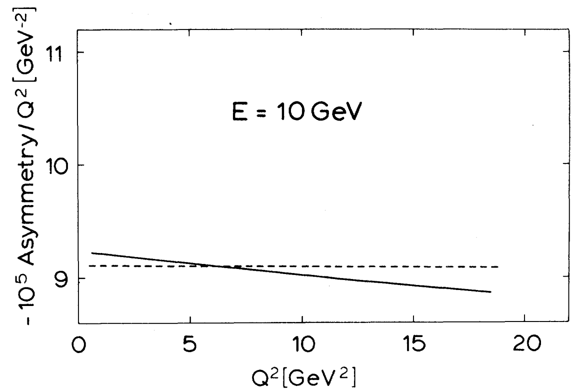

FIG. 4. Same as Fig. 1, but for electron beam energy 10 GeV.

$$\frac{\alpha g_{A,e}}{e^2}=(1-2\sin^2\theta_W+T^u_{3R}-T^d_{3R})(1+2T^e_{3R})(8\sin^2\theta_W\cos^2\theta_W)^{-1}\ , \tag{4.1}$$

$$\frac{\beta g_{V,e}}{e^2}=(1-4\sin^2\theta_W-2T^e_{3R})(1-T^u_{3R}+T^d_{3R})(8\sin^2\theta_W\cos^2\theta_W)^{-1}\ . \tag{4.2}$$

The asymmetry thus depends strongly on the assignments of weak isospin to right-handed particles. For the standard model, $T^e_{3R}=T^u_{3R}=T^d_{3R}=0$, with $\sin^2\theta_W=0.23$, the right-hand sides of Eqs. (4.1) and (4.2) assume the values 0.38 and 0.056, respectively, so that the first term is the dominant one. This term, which has already been given by Cahn and Gilman,[5] happens to be completely independent of the electromagnetic and weak N-Δ form factors. Thus for the standard model, we can expect the asymmetry to be given by the Cahn-Gilman result, with corrections of up to 20% due to the second term of Eq. (3.9), depending on the kinematical region and the input of N-Δ form factors. On the other hand, with the assignment $T^e_{3R}=-\frac{1}{2}$, the right-hand side of Eq. (4.1) vanishes, so that only the second term in the expression for the asymmetry contributes. The present experimental situation regarding assignment of weak isospin to right-handed particles has been thoroughly discussed by Kim, Langacker, Levine, and Williams.[2] From a fit to all present neutral-current data, using T^u_{3R}, T^d_{3R}, T^e_{3R} as free parameters, they obtain values close to zero for these three quantities.

The measurement of the asymmetry is a difficult experiment. Currently available electron machines operate with pulsed currents and small duty cycles. Such machines will at most permit, e.g., by an experiment in the resonance region detecting the $\pi^+(\mu^+)$ in a 4π geometry, the demonstration of the existence of the asymmetry, but not the measurement of its dependence on E and Q^2. In the near future, however, the situation may change. In various laboratories a new generation of electron accelerators is under discussion, operating with high constant currents and 100% duty cycle (as, for example, the 800-MeV Mainz Mikrotron project MAMI, under development at this universty). Such machines would permit $e^-\pi^+$ or e^-p coincidence experiments, from which clean samples of $e^-p\to e^-\Delta^+$ events with completely determined kinematics and high statistics could be obtained.[12]

Note added. After this paper had been submitted for publication, we became aware that independently and simultaneously D.R.T. Jones and S.T. Petcov [Phys. Lett. 91B, 137 (1980)] had investigated the same problem. Our results are more general in that the dependence of the asymmetry A on the form factors $C^\gamma_3,C^\gamma_4,C^A_3,C^A_4,C^A_5$ is displayed in full generality and the dependence of the coupling constants on the assumptions about the right-handed weak isospin of u,d,e is shown. When specialized according to the assumptions of Jones and Petcov, our formula (3.9) for the asymmetry A becomes equivalent to their result.

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