Implications of lepton-flavor mixing for neutral-current phenomenology

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Neutral-current data are reanalyzed incorporating arbitrary mixings among an arbitrary number of lepton generations. Within experimental uncertainties, the determination of electron neutral-current couplings is unaltered. For inelastic ν_μ ($\bar{\nu}_\mu$) scattering to give a value of $\sin^2\theta$ consistent with this, μ flavor must remain isolated from all others.

It is now widely accepted that the standard model of weak and electromagnetic interactions describes neutral-current (NC) phenomena satisfactorily (a recent statistical fit to all NC parameters is that of Langacker *et al.*[1]; see also the reviews of Baltay[2] and Winter[3]). All analyses of NC data, however, assume that neutrinos are massless, thus precluding any possibility of lepton-flavor mixing. Flavor mixing is inevitable (it is "natural") in a gauge-theory setting[4] once the neutrinos are nondegenerate and, no matter how small the mass differences (and masses themselves) are, can be substantial. Since the standard model can accommodate massive neutrinos, it is important to see whether the presently accepted determination of NC parameters is invalidated by possible substantial flavor mixing—or, conversely, accepting the correctness of the standard model, whether the presence or absence of flavor mixing can be established from such analyses. This paper reports the results of a fairly comprehensive reexamination of NC data from this point of view. Such a study is especially worthwhile now because of recent indications of anomalous flavor properties of weak leptonic currents[5,6] and even of nonzero neutrino masses.[7]

The (diagonal) neutral currents themselves are of course unaffected in the lowest order by flavor mixing induced by lepton mass-matrix diagonalization. Nevertheless, the determination of "NC" parameters is affected, for two reasons. (*i*) Neutrino reactions which are taken to be solely given by NC matrix elements in the absence of flavor mixing (e.g., "ν_μ" e scattering) have, in their presence, contributions from charged-current (CC) amplitudes (and the CC contribution to "$\bar{\nu}_e$" e scattering is strongly modified). (ii) CC parameters provide the normalization standard for NC observables. Point (ii) is easily incorporated but nevertheless causes essential modification to the results of conventional analyses. It is, however, point (i) which makes our analysis of the data somewhat nontrivial. It is therefore the main focus of attention here (we have considered other aspects of lepton-flavor mixing elsewhere[8]).

Assume an arbitrary number n of lepton doublets and denote by ν_i $(i=1,\ldots,n)$ the mass eigenstates of neutrinos. These are the true asymptotic fields defined on the Fock space of free particles and have nonzero right-handed projections (it is to be noted here that even though our discussion is framed in the language of massive Dirac neutrinos, all results are equally valid for Majorana neutrinos). We make two hypotheses, both well supported by experiment:

(A) The neutrino masses m_i are small enough not to change phase-space factors in the cross sections of interest (this follows from the known universality properties[8]).

(B) The right-handed projections of ν_i do not contribute to the charged currents. Since all neutrino beams arise from CC decays, we may then ignore possible contributions to the NC Lagrangian from right-handed neutrinos—such contributions are effectively proportional to the neutrino masses. It is thus sufficient to generalize the charged current to

$$J^{+}_{\alpha}=\bar{l}^{-}\gamma_\alpha(1+\gamma_5)Cl^0\,, \tag{1}$$

where $l^-=(e,\mu,\tau,\ldots)$, $l^0=(\nu_1,\nu_2,\ldots,\nu_n)$, and C is a unitary $n\times n$ matrix. The neutral current remains

$$J^{0}_{\alpha}=\bar{l}^{0}\gamma_\alpha(1+\gamma_5)l^0+\bar{l}^{-}\gamma_\alpha(g_V+g_A\gamma_5)l^-\,, \tag{2}$$

where g_V and g_A are relative coupling constants, modulo an overall scale. Specializing to neutrino-electron scattering, the effective Lagrangian is ($\sum\equiv\sum_{i=1}^{n}$ throughout this paper)

$$\begin{aligned}\mathcal{L}=-\frac{G}{\sqrt{2}}\Bigg\{&\Big[\sum\bar{\nu}_i\gamma_\alpha(1+\gamma_5)\nu_i\Big]\bar{e}\gamma_\alpha(g_V+g_A\gamma_5)e\\&+\Big[\sum C^{*}_{ei}\bar{\nu}_i\gamma_\alpha(1+\gamma_5)e\Big]\\&\times\Big[\sum C_{ei}\bar{e}\gamma_\alpha(1+\gamma_5)\nu_i\Big]\Bigg\}\,.\end{aligned} \tag{3}$$

Consider an antineutrino beam in which the relative intensity of $\bar{\nu}_i$ is ρ_i, with $\sum\rho_i=1$. The presumed elastic cross section for this beam on electrons is now actually the incoherent sum of cross sections for $\bar{\nu}_i e\to\bar{\nu}_j e$ with relative weights ρ_i:

$$\sigma_\rho(\bar{\nu}e)=\sum_{i,j=1}^{n}\rho_i\sigma(\bar{\nu}_i e\to\nu_j e) \tag{4}$$

with both NC and CC contributions for $i=j$ and only CC contributions for $i\neq j$. The usual calculation of the differential cross section is easily modified to accommodate the Lagrangian (3) and gives ($y=E_e/E_\nu$)

$$\frac{d\sigma_\rho(\bar{\nu}e)}{dy}=\left\{(g_V-g_A)^2+(1-y)^2\left[(g_V+g_A)^2+4\sum\rho_i|C_{ei}|^2(1+g_V+g_A)\right]\right.$$
$$\left.-\frac{ym_e}{E_\nu}(g_V-g_A)\left(g_V+g_A+2\sum\rho_i|C_{ei}|^2\right)\right\}\frac{G^2m_eE_\nu}{2\pi}\,. \tag{5}$$

This expression is general and includes as special cases the conventional "$\bar{\nu}_e$"e and "$\bar{\nu}_\mu$"e scattering; these correspond to $\sum\rho_i|C_{ei}|^2=1$ and 0, respectively. When flavor-mixing is allowed for, a "$\bar{\nu}_\mu$" beam in fact has $\rho_i^{(\mu)}=|C_{\mu i}|^2$ so that flavor-mixing corrections are proportional to

$$\alpha_{\mu e}\equiv\sum\rho_i^{(\mu)}|C_{ei}|^2=\sum|C_{\mu i}|^2|C_{ei}|^2.$$

This last quantity is known[8] to be negligible ($\lesssim 10^{-3}$) from data[9] on e production in "$\bar{\nu}_\mu$" beams. So the description of "$\bar{\nu}_\mu$"e (and also of "ν_μ"e) scattering is unchanged to an excellent approximation by flavor mixing, a conclusion independent of how many other leptonic flavors may exist and mix with μ flavor.

On the other hand, for reactor antineutrino ("$\bar{\nu}_e$") beams, $\rho_i^{(e)}=|C_{ei}|^2$ and the flavor-mixing corrections depend on the parameter $\alpha_{ee}\equiv\sum\rho_i^{(e)}|C_{ei}|^2=\sum|C_{ei}|^4$. Here recent indications[5,6] are that[8] $\alpha_{ee}\simeq 0.5$. It is therefore necessary to redetermine the allowed ranges of g_V and g_A for a given measured value of the cross section, taking the deviation of α_{ee} from 1 into account.

We have done this for various values of α_{ee} between 1 (conventional, no mixing) and 0 (maximal mixing of an infinite number of flavors) in the usual way. Equation (5), when integrated over the neutrino energy spectrum and the electron-detection efficiency, can be written in the form

$$A(g_V-g_A)^2+B[(g_V+g_A)^2+4\alpha_{ee}(1+g_V+g_A)]$$
$$+C(g_V-g_A)(g_V+g_A+2\alpha_{ee})=1\,, \tag{6}$$

where A, B, and C depend on the data.[10] The solutions of this equation taking account of errors in A, B, and C cover a family (parametrized by α_{ee}) of annular elliptic bands in the g_V-g_A plane. These bands as a whole are shifted substantially by changing α_{ee}. But the remarkable result is that their intersections with the corresponding bands determined by "ν_μ"e and "$\bar{\nu}_\mu$"e data (which, as we have seen above, are insensitive to flavor mixing) are little affected (Fig. 1 shows the intersections for $\alpha_{ee}=0.5$, the favored value). For the g_A-dominant solution we find

$$-0.17<g_V<0.22,\quad -0.70<g_A<-0.45\quad\text{for }\alpha_{ee}=0.5 \tag{7}$$

to be compared with

$$-0.19<g_V<0.17,\quad -0.70<g_A<-0.45\quad\text{for }\alpha_{ee}=1 \tag{8}$$

from the boundaries of the intersection region.

The empirical reason for this insensitivity is that the family of ellipses (6) have the common points of intersection $g_V+g_A=-1\pm C/(2B\sqrt{A})\simeq-1$, $g_V-g_A=\mp 1/\sqrt{A}$, independent of α_{ee} if $C^2\ll 4AB$, a condition which is met by the data (for

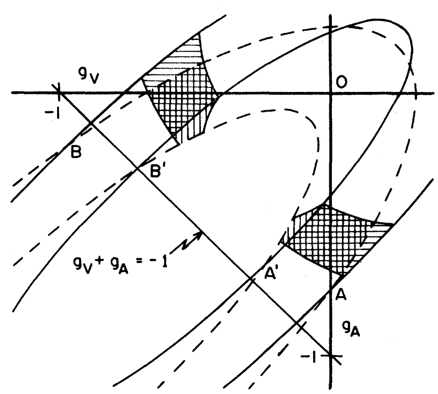

FIG. 1. Solutions for g_V,g_A for $\alpha_{ee}=1$, no flavor mixing (vertical hatching) and $\alpha_{ee}=0.5$, maximal mixing with one other flavor (horizontal hatching). A,B,A',B' are α_{ee}-independent points of intersection.

the inner family, e.g., $A \simeq 4.0$, $B \simeq 0.3$, $C \simeq -0.1$). As can be seen from Fig. 1, these points happen to be quite close to the allowed region of g_V and g_A; because of this the effect of varying α_{ee} is small in comparison to the errors on the solutions for g_V and g_A. More precise data will of course change this situation, but for the present our result is that the support provided to the standard model by, and the consequent determination of $\sin^2\theta$ from, νe scattering is unaffected by lepton-flavor mixing.[11]

We turn now to other NC phenomena. In polarized *ed* scattering, the asymmetry itself is independent of flavor mixing. So the use of the factorization hypothesis to remove the g_V-g_A ambiguity goes through as before.[12] Given also our conclusion above concerning νe scattering, there is nothing to add to the analysis as given, e.g., in Ref. 1. In particular, the result[1] of a one-parameter fit to the *ed* data[13] assuming the correctness of the standard model,

$$\sin^2\theta = 0.223 \pm 0.015\,, \tag{9}$$

remains essentially unaffected by flavor mixing.

The situation is entirely different for inelastic "ν_μ" and "$\bar{\nu}_\mu$" scattering on quarks. To illustrate the point, it is sufficient to consider the Paschos-Wolfenstein[14] ratio which, in the presence of flavor mixing, is (again in the standard model)

$$\begin{aligned} R &= \frac{\sigma_{\rm NC}(\text{“}\nu_\mu\text{”}) - \sigma_{\rm NC}(\text{“}\bar{\nu}_\mu\text{”})}{\sigma_{\rm CC}(\text{“}\nu_\mu\text{”}) - \sigma_{\rm CC}(\text{“}\bar{\nu}_\mu\text{”})} \\ &= \left(\sum |C_{\mu i}|^4\right)^{-1} \left(\tfrac{1}{2} - \sin^2\theta\right). \end{aligned} \tag{10}$$

The first factor on the right is a measure of the mixing of μ flavor with any other (not just e flavor). Since its value is unknown, a measurement of R does not determine $\sin^2\theta$. But we may use the experimental value[15]

$$R = 0.272 \pm 0.018 \tag{11}$$

and the mixing-independent determination [Eq. (9)] of $\sin^2\theta$ to conclude

$$\sum_i |C_{\mu i}|^4 = \frac{0.277 \pm 0.015}{0.272 \pm 0.018} \simeq 1\ . \tag{12}$$

We have thus arrived at the result that μ flavor does not mix *at all* (it is, of course, independently known[9,16] from the lack of e and τ production in "ν_μ" beams that it remains unmixed with e flavor and τ flavor). In the framework of the oscillation phenomenology, this result would imply that μ flavor does not oscillate into any other flavors at least in the region of L/E to which the value $R = 0.272 \pm 0.018$ corresponds.

Note added. After this work was done we have become aware of a number of other papers concerned with related questions.[17]

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[11]We have not discussed the phenomenology of neutrino oscillations. We only note here that in the presence of oscillations the flavor-mixing correction parameters α occurring in the cross-section formula would be replaced by time-dependent probability factors and the data of Ref. 9 would give a measure of the smallness of the probability factor $P(\mu \to e)$, rather than $\alpha_{\mu e}$, for a given L/E determined by the experimental setup. (Here L is the distance the neutrino has traveled, and E is its energy.) The neutral-current data on "$\bar{\nu}_\mu$"e and "ν_μ"e, used to determine neutral-current couplings, are also for the same value of L/E. Therefore, our conclusion that the description of "$\bar{\nu}_\mu$"e (and also of "ν_μ"e) remains unchanged holds good in the oscillation picture for arbitrary values of neutrino masses. It is to be noted further that to the extent that g_V and g_A are unchanged by the variation of α_{ee} between 1 and 0, they are also unaltered by oscillations among an arbitrary number of flavors for the entire range of oscillation periods and neutrino energies. We are especially grateful to K. V. L. Sarma for discussions on oscillations.

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