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Spin-dependent quark-quark interaction and baryon magnetic moments

M. Böhm
Physikalisches Institut der Universität Würzburg, Würzburg, Federal Republic of Germany

R. Huerta and A. Zepeda
Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional, Apartado Postal 14-740, Mexico 14, D.F., Mexico
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The magnetic moments of the baryons belonging to the low-lying SU(3) octet are computed using second-order perturbation theory taking into account the effect of the spin-dependent interactions that are expected to arise in quantum chromodynamics. The unperturbed eigenfunctions of the confining Hamiltonian are approximated by harmonic-oscillator wave functions, and the flavor $\times$ spin parts belong, in the limit of equal quark masses, to irreducible representations of SU(6). In this basis we then calculate the mixing of the $(56, 0^+)$ ground-state wave function with the orbital and radial excitations labeled by $(56, 0_R^+)$, $(70, 0^+)$, $(20, 1^+)$, and $(70, 2^+)$. This mixing arises not only from the spin-dependent interactions but also from the differences among quark masses. Finally we comment on the contributions to the magnetic moments not taken into account in this work.

I. INTRODUCTION

The magnetic dipole moments of the lowest-lying baryons have provided a useful ground to test ideas about symmetries such as (flavor) SU(3) and (nonrelativistic) SU(6). In the limit of largest symmetry, i.e., SU(6), one can express in terms of the proton (or any other) magnetic moment the magnetic moments of all baryon-octet members, those of the spin-$\frac{3}{2}$ decuplet, and all the allowed transition moments between octet and decuplet.[1] In the following we will limit ourselves to the octet where the SU(6) relations are $\mu_p/\mu_n = -\frac{3}{2}$ plus those which are obtained[2] in the limit of SU(3): $\mu_\Lambda = \frac{1}{2}\mu_n$, $\mu_{\Sigma^+} = \mu_p$, $\mu_{\Xi^-} = \mu_{\Sigma^-} = -\mu_p - \mu_n$, $\mu_{\Xi^0} = \mu_n$, $\mu_{\Sigma^0} = -\frac{1}{2}\mu_n$, and $\mu_{\Sigma\Lambda} = \frac{1}{2}\sqrt{3}\,\mu_n$. In other words, in the SU(3) limit one needs two data to know all the octet magnetic moments, while in the SU(6) limit one datum is enough.

Less constraining relations are obtained when one takes into account the breaking of the above symmetries. Let us first mention the results of the symmetric quark model where it is assumed that the quarks are in an S wave. Assuming further that when two quarks are identical they are in a spin-1 state then one gets the well known relations between the quark magnetic moments μ_q and the baryon magnetic moments[3]

$$\mu_p = \tfrac{1}{3}(4\mu_u - \mu_d), \quad \mu_n = \tfrac{1}{3}(4\mu_d - \mu_u),$$
$$\mu_{\Sigma^+} = \tfrac{1}{3}(4\mu_u - \mu_s), \quad \mu_{\Sigma^-} = \tfrac{1}{3}(4\mu_d - \mu_s),$$
$$\mu_{\Xi^0} = \tfrac{1}{3}(4\mu_s - \mu_u), \quad \mu_{\Xi^-} = \tfrac{1}{3}(4\mu_s - \mu_d),$$
$$\mu_{\Sigma^0} = \tfrac{1}{3}(2\mu_u + 2\mu_d - \mu_s), \quad \mu_\Lambda = \mu_s, \quad \mu_{\Sigma\Lambda} = \frac{1}{\sqrt{3}}(\mu_d - \mu_u). \tag{1}$$

To obtain these relations it is not necessary to assume that the quark magnetic moments are Dirac moments or that the quark masses are equal.[3] In fact the assumptions that lead to (1) are equivalent to the assumption that SU(6) is broken only by quark-mass differences as will be clear from the discussion in Sec. III. According to Eqs. (1), if one knows three baryon magnetic moments one can then predict the others.

On the other hand, if all one knows about breaking of SU(3) is that the symmetry of the strong-interaction Hamiltonian is broken by an octet operator that transforms as the hypercharge, then the above relations reduce to two sum rules[4]

$$\mu_{\Sigma^0} = \tfrac{1}{2}(\mu_{\Sigma^+} + \mu_{\Sigma^-}) + O(\epsilon_2) \tag{2a}$$

$$\mu_{\Sigma\Lambda} = -\frac{1}{\sqrt{3}}\left(\tfrac{1}{2}\mu_{\Sigma^0} + \tfrac{3}{2}\mu_\Lambda - \mu_{\Xi^0} - \mu_n\right) + O(\epsilon_3^2), \tag{2b}$$

where ϵ_i parametrizes the intensity of the breaking of (flavor) SU(i).

One can say, however, that we know more about the breaking of SU(3) [and of SU(6)]. In fact, during the last few years evidence has been accumulated in favor of the colored-gluon quark-quark interaction hypothesis embraced in the quantum-chromodynamics theory[5] and in particular it has been shown that the spin-dependent interaction (color fine and hyperfine interaction) put forward by De Rujula, Georgi, and Glashow[6] leads to a better understanding of the spectroscopy of hadrons, as well as of various decay peculiarities.[6,7] It is precisely the aim of the present paper to estimate the effects of this spin-dependent interaction on the baryon magnetic moments. We will approach this problem perturbatively and we will approximate the spin-independent Hamiltonian by that of a harmonic oscillator. We find, by inspection, that our results satisfy the relation (2b) up to corrections of order ϵ_2^2 and ϵ_3^2 [that is Eq. (2b) holds also when SU(2) is broken in the way described in this paper].

Before comparing our predictions with the experimental results one should remember that while the SU(3) relations seem to be too far[8,9] from the present experimental values, a χ^2 fit[9,10] to the quark-model relations (1) shows deviations of the order of 10 to 20%. This indicates that it makes sense to apply perturbation theory, as we do, to estimate corrections to the relations (1). Indeed, the numerical values that we obtain for the perturbative corrections to the magnetic moments come out to be of the order of at most 11% and, in general, in the correct direction. The effects considered in this work, however, are not enough to claim a complete understanding of the baryon magnetic moments.

The plan of the paper is the following. In Sec. II we present in detail the kinematics of the three-body problem for the general case of three different masses. This description allows us to make a consistent treatment of both SU(2) and SU(3) breaking. In this section we also describe the general nonrelativistic form of the magnetic moment operator. In Sec. III we recollect the structure of the baryon wave functions classified according to irreducible representations of SU(6) and we discuss the modifications of the wave functions when SU(6) is broken only by quark mass differences. Section IV is concerned with dynamics. Here we describe the spin-dependent interaction as well as the perturbative procedure around the harmonic-oscillator approximation to the confining (spin-independent) Hamiltonian. We also outline the derivation of the formula for the expansion of matrix elements up to second order in the perturbation for the magnetic-moment operator. In Sec. V we compute the matrix-elements of the perturbating Hamiltonian between the ground state and the excited states and evaluate numerically the corresponding first-order mixing in the ground state. Section VI is devoted to the calculation of the magnetic moments in the excited states. The most important results are listed in Tables II and III. Our conclusions and the discussion of our results can be found in Sec. VII. We have tried to make this article self-contained. A brief account of our main results has been given elsewhere.[11]

II. KINEMATICS OF THE THREE-BODY PROBLEM

Let $\vec{x}_i$ and m_i be the position coordinate and mass, respectively, of quark i. Let us define two relative coordinates $\vec{\rho}$ and $\vec{\lambda}$, related to the coordinates $\vec{x}_i$ by the following transformation:

$$\vec{X}_i = \mathfrak{M}_{ij}\vec{x}_j\,, \tag{3a}$$

where $\vec{X}_i = \{\vec{X}, \vec{\lambda}, \vec{\rho}\}$, $\vec{X}/\sqrt{3}$ is the center-of-mass coordinate and $M = m_1 + m_2 + m_3$:

$$\mathfrak{M} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & \sin\varphi \\ 0 & -\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} \sqrt{3}\,\frac{m_1}{M} & \sqrt{3}\,\frac{m_2}{M} & \sqrt{3}\,\frac{m_3}{M} \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{pmatrix} \tag{3b}$$

with $\tan\varphi = \delta - (1+\delta^2)^{1/2}$, $\delta = (2m_3^{-1} - m_1^{-1} - m_2^{-1})/(m_1^{-1} - m_2^{-1})\sqrt{3}$. The differences of coordinates $\vec{x}_i$ are then given in terms of $\vec{\rho}$ and $\vec{\lambda}$,

$$\vec{x}_1 - \vec{x}_2 = \sqrt{2}\,(\vec{\lambda}\sin\varphi + \vec{\rho}\cos\varphi)\,,$$

$$\vec{x}_2 - \vec{x}_3 = [\sqrt{3}\,(\vec{\lambda}\cos\varphi - \vec{\rho}\sin\varphi) - \vec{\lambda}\sin\varphi - \vec{\rho}\cos\varphi]/\sqrt{2}\,, \tag{4a}$$

$$\vec{x}_3 - \vec{x}_1 = [-\sqrt{3}\,(\vec{\lambda}\cos\varphi - \vec{\rho}\sin\varphi) - \vec{\lambda}\sin\varphi - \vec{\rho}\cos\varphi]/\sqrt{2}\,,$$

so that

$$(\vec{x}_1 - \vec{x}_2)^2 + (\vec{x}_2 - \vec{x}_3)^2 + (\vec{x}_3 - \vec{x}_1)^2 = 3(\vec{\rho}^2 + \vec{\lambda}^2)\,. \tag{4b}$$

Notice that

$$\vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{x}_1 + \vec{x}_2 - 2\vec{x}_3)\cos\varphi + \frac{1}{\sqrt{2}}(\vec{x}_1 - \vec{x}_2)\sin\varphi\,,$$

$$\vec{\rho} = \frac{-1}{\sqrt{2}}(\vec{x}_1 - \vec{x}_2)\sin\varphi + \frac{1}{\sqrt{6}}(\vec{x}_1 + \vec{x}_2 - 2\vec{x}_3)\cos\varphi\,. \tag{4c}$$

Let $\vec{P}_i = \{\vec{P}_X, \vec{P}_\lambda, \vec{P}_\rho\}$ and $\vec{p}_i = \{\vec{p}_1, \vec{p}_2, \vec{p}_3\}$ be the variables (momenta) canonically conjugated to the variables $\vec{X}_i$ and $\vec{x}_i$, respectively. They are related by

$$\vec{P}_i = (\mathfrak{M}^{-1})_{ji}\vec{p}_j \tag{5}$$

and lead to the following expression for the kinetic energy:

$$\sum_i \vec{P}_i^{\,2}/2m_i = 3\vec{P}_X^{\,2}/2M + \vec{P}_\lambda^{\,2}/2m_\lambda + \vec{P}_\rho^{\,2}/2m_\rho\,, \tag{6}$$

where

$$m_\rho^{-1} = (m_1^{-1} + m_2^{-1} + 4m_3^{-1})\cos^2\varphi/6 + \tfrac{1}{2}(m_1^{-1} + m_2^{-1})\sin^2\varphi + (m_1^{-1} - m_2^{-1})\cos\varphi\sin\varphi/\sqrt{3} \tag{7a}$$

$$m_\lambda^{-1} = (m_1^{-1} + m_2^{-1} + 4m_3^{-1})\sin^2\varphi/6 + \tfrac{1}{2}(m_1^{-1} + m_2^{-1})\cos^2\varphi - (m_1^{-1} - m_2^{-1})\cos\varphi\sin\varphi/\sqrt{3}\,. \tag{7b}$$

The total angular momentum $\vec{L}$ and the orbital part of the magnetic-moment operator $\vec{\mu}_L$ may be also expressed in terms of the relative variables if we consider the combination

$$\vec{c}=\sum_i g_i \vec{x}_i \times \vec{p}_i = \vec{X}_j \times \vec{P}_{j'} G_{j'j}, \tag{8a}$$

where

$$G_{j'j}=\sum_i \mathfrak{M}_{j'i} g_i \mathfrak{M}^{-1}{}_{ij}. \tag{8b}$$

$\vec{c}=\vec{L}$ ($\vec{\mu}_L$) if $g_i=1$ (μ_q). Since we are interested only in the intrinsic properties of the three-body system (baryon) we evaluate (8a) at $\vec{X}=0$, $\vec{P}_X=0$ and we obtain

$$\vec{L}=\vec{L}_\rho+\vec{L}_\lambda,$$

where

$$\vec{L}_\rho=\vec{\rho}\times\vec{P}_\rho,\quad \vec{L}_\lambda=\vec{\lambda}\times\vec{P}_\lambda, \tag{9}$$

$$\vec{\mu}_L=g_{\lambda\lambda}\vec{L}_\lambda+g_{\rho\rho}\vec{L}_\rho+g_{\rho\lambda}\vec{\rho}\times\vec{P}_\lambda+g_{\lambda\rho}\vec{\lambda}\times\vec{P}_\rho,$$

$$g_{\lambda\lambda}=\bar{g}_{\lambda\lambda}\cos^2\varphi+(\bar{g}_{\rho\lambda}+\bar{g}_{\lambda\rho})\cos\varphi\sin\varphi+\bar{g}_{\rho\rho}\sin^2\varphi,$$
$$g_{\rho\rho}=\bar{g}_{\lambda\lambda}\sin^2\varphi-(\bar{g}_{\rho\lambda}+\bar{g}_{\lambda\rho})\cos\varphi\sin\varphi+\bar{g}_{\rho\rho}\cos^2\varphi, \tag{10a}$$
$$g_{\rho\lambda}=(\bar{g}_{\rho\rho}-\bar{g}_{\lambda\lambda})\cos\varphi\sin\varphi-\bar{g}_{\lambda\rho}\sin^2\varphi+\bar{g}_{\rho\lambda}\cos^2\varphi,$$
$$g_{\lambda\rho}=(\bar{g}_{\rho\rho}-\bar{g}_{\lambda\lambda})\cos\varphi\sin\varphi+\bar{g}_{\lambda\rho}\cos^2\varphi-\bar{g}_{\rho\lambda}\sin^2\varphi,$$

with

$$\begin{aligned}\bar{g}_{\lambda\lambda}&=[(\mu_1+\mu_2-2\mu_3)m_3+2\mu_3 M]/2M,\\ \bar{g}_{\rho\rho}&=[(\mu_1+\mu_2)M-(\mu_1-\mu_2)(m_1-m_2)]/2M,\\ \bar{g}_{\rho\lambda}&=[(\mu_1-\mu_2)M\\ &\quad-(\mu_1+\mu_2-2\mu_3)(m_1-m_3)]/M\sqrt{12},\\ \bar{g}_{\lambda\rho}&=\sqrt{3}m_3(\mu_1-\mu_2)/2M.\end{aligned} \tag{10b}$$

We recall that the total magnetic moment is given by $\vec{\mu}=\vec{\mu}_S+\vec{\mu}_L$, where $\vec{\mu}_S=\sum\mu_i\vec{\sigma}_i$.

III. SU(6) BASIS AND BREAKING BY QUARK MASS DIFFERENCES

In the limit of SU(6) the baryon wave functions are completely symmetric under the simultaneous interchange of space, flavor, and spin variables and the flavor × spin part of the wave functions are classified[12] according to irreducible representations of SU(6). For the cases of interest in this paper the baryon wave functions, written in the order space × flavor × spin are of the form

$$\begin{aligned}|56,L,10_4\rangle&=(L)_{123}(10)_{123}(4)_{123},\\ |56,L,8_2\rangle&=\frac{1}{\sqrt{2}}(L)_{123}\{(8)_{12}(2)_{12}+[8]_{12}[2]_{12}\};\end{aligned} \tag{11a}$$

$$\begin{aligned}|20,L,8_2\rangle&=\frac{1}{\sqrt{2}}[L]_{123}\{(8)_{12}[2]_{12}-[8]_{12}(2)_{12}\},\\ |20,L,1_4\rangle&=[L]_{123}[1]_{123}(4)_{123};\end{aligned} \tag{11b}$$

$$\begin{aligned}|70,L,8_4\rangle&=\frac{1}{\sqrt{2}}\{(L)_{12}(8)_{12}+[L]_{12}[8]_{12}\}(4)_{123},\\ |70,L,10_2\rangle&=\frac{1}{\sqrt{2}}\{(L)_{12}(10)_{123}(2)_{12}+[L]_{12}(10)_{123}[2]_{12}\},\\ |70,L,8_2\rangle&=\tfrac{1}{2}\{[L]_{12}(8)_{12}[2]_{12}+[L]_{12}[8]_{12}(2)_{12}\\ &\quad+(L)_{12}[8]_{12}[2]_{12}-(L)_{12}(8)_{12}(2)_{12}\},\\ |70,L,1_2\rangle&=\frac{1}{\sqrt{2}}\{(L)_{12}[1]_{123}[2]_{12}-[L]_{12}[1]_{123}(2)_{12}\};\end{aligned} \tag{11c}$$

where L is the orbital angular momentum and where Δ, D, and d in $|\Delta,L,D_d\rangle$ denote, respectively, the dimension of the irreducible representation of SU(6), (flavor) SU(3), and (spin) SU(2). The permutation symmetry of each wave function in the corresponding variables is indicated by brackets: $(\ )_{123}$ and $[\ ]_{123}$ mean complete symmetry and antisymmetry; $(\ )_{12}$ and $[\ ]_{12}$ mean, respectively, symmetry and antisymmetry under the interchange of the first two variables. Thus, for example, the flavor part of the proton wave function is

$$(8,p)_{12}=\frac{1}{\sqrt{6}}(2|uud\rangle-|udu\rangle-|duu\rangle), \tag{12a}$$

$$[8,p]_{12}=\frac{1}{\sqrt{2}}(|udu\rangle-|duu\rangle). \tag{12b}$$

Wave functions of the form similar to (12) in which the SU(3) content is explicit may also be written in the flavor-product basis, in which

$$\begin{aligned}|p\rangle&=\frac{1}{\sqrt{3}}[|uud\rangle(0)_{123}(2)_{12}+|duu\rangle(0)_{231}(2)_{23}\\ &\quad+|udu\rangle(0)_{312}(2)_{31}]\\ &=\frac{1}{\sqrt{3}}[|uud\rangle(0)_{123}(2)_{12}+\text{c.p.}],\end{aligned} \tag{13}$$

where c.p. indicates cyclic permutations.

The space part of the wave functions in (11) needs in principle more labels to specify the radial as well the internal orbital (in l_ρ and l_λ) excitations. We will amend this omission later. For the moment it is enough to indicate that for the ground-state wave functions belonging to the 56-plet we are assuming that $l_\rho=l_\lambda=0$. This assumption guarantees that for the lowest-lying SU(3) octet and decuplet $\langle\mu_L\rangle=0$ and that the magnetic moments of the octet are those given by (1) (with $m_u=m_d=m_s$) as can be easily calculated.

To discuss the breaking of SU(6) by quark mass differences the wave functions written in the flavor-product basis are convenient. Let us assume that the quark-quark interaction is mass, flavor, and spin independent so that the breaking of SU(6) stems only from the kinetic energy

$$E_{\rm kin}=\vec{p}_\lambda^{\,2}/2m_\lambda+\vec{p}_\rho^{\,2}/2m_\rho, \tag{14}$$

where m_ρ and m_λ are given by Eqs. (7). It follows that (a) only the space part of the wave functions are affected by the differences in quark masses and (b) to specify the space wave functions one should indicate the values of the masses and the order in which the coordinate labels are chosen. Let then $(0;i,m_i;j,m_j;k,m_k)$ be the space wave function of the ground state in the unequal-mass case and let

$$|p\rangle = \frac{1}{\sqrt{3}}[|uud\rangle(0;1,m_u;2,m_u;3,m_d)(2)_{12} + |duu\rangle(0;2,m_u;3,m_u;1m_d)(2)_{23} + |udu\rangle(0;3,m_u;1;m_u;2,m_d)(2)_{31}] = \frac{1}{\sqrt{3}}[|uud\rangle(0;1,m_u;2,m_u;3,m_d)(2)_{12} + \text{c.p.}]\,. \tag{15}$$

Evidently it is still true that $l_\rho = l_\lambda = \langle \mu_L \rangle = 0$. Thus the results (1) for the magnetic moments are recovered.

Notice that the proton wave function (and the wave functions of all the baryons with two identical quarks) is completely symmetric in the variables corresponding to two identical quarks. To derive the results (1), however, it is enough to consider, as noticed by Franklin,[3,9] the simplified wave functions obtained by ignoring in (15) the terms denoted by c.p. (and the normalizing factor $1/\sqrt{3}$). On the other hand the wave function described in (15) evidently does not belong to one irreducible representation of SU(6); it may be decomposed, to first order in the quark mass differences, into a sum of the wave functions $|56,0,8_2\rangle$, $|70,0,8_2\rangle$, $|70,0,10_2\rangle$, and $|70,0,1_2\rangle$.

IV. SPIN-DEPENDENT INTERACTION AND PERTURBATION THEORY

It is generally believed that in the nonrelativistic approximation the Hamiltonian that describes the internal structure of the baryons is of the form[6,7]

$$H = \sum_{i=1} (m_i + \vec{p}_i^{\,2}/2m_i) + H_{\text{conf}} + H_S\,, \tag{16}$$

where H_{conf} is a confining potential which is assumed to be a flavor-independent function of the relative coordinates and H_S is the spin-dependent interaction which arises from the interaction of the color magnetic moments of the quarks. These two terms are given by

$$H_{\text{conf}} = \sum_{i<j}\left(-\frac{2\alpha_s}{3}|\vec{r}_{ij}|^{-1} + a\,|\vec{r}_{ij}|\right)\,, \tag{17}$$

$$H_S = H_{\text{SS}} + H_{\text{SO}} + H_T\,, \tag{18}$$

$$H_{\text{SS}} = \sum_{i<j}\frac{2\alpha_s}{3m_im_j}\frac{8\pi}{3}\vec{S}_i\cdot\vec{S}_j\delta^3(\vec{r}_{ij})\,, \tag{19a}$$

$$H_{\text{SO}} = \sum_{i<j}\frac{\alpha_s}{3}\frac{1}{r_{ij}^{\,3}}[\vec{S}_i\cdot\vec{r}_{ij}\times\vec{p}_i/m_i^2 - \vec{S}_j\cdot\vec{r}_{ij}\times\vec{p}_j/m_j^2 - 2(\vec{S}_i\cdot\vec{r}_{ij}\times\vec{p}_j - \vec{S}_j\cdot\vec{r}_{ij}\times\vec{p}_i)/m_im_j] - \frac{k}{2}\sum_{i<j}(\vec{S}_i\cdot\vec{r}_{ij}\times\vec{p}_i/m_i^2 - \vec{S}_j\cdot\vec{r}_{ij}\times\vec{p}_j/m_j^2)\,, \tag{19b}$$

$$H_T = \sum_{i<j}\frac{2\alpha_s}{3m_im_j}\frac{1}{r_{ij}^{\,3}}(3\vec{S}_i\cdot\hat{r}_{ij}\vec{S}_{ij}\cdot\hat{r}_{ij} - \vec{S}_i\cdot\vec{S}_j)\,, \tag{19c}$$

where α_s is the strong fine-structure constant, a is the slope of the linear potential, $\vec{S}_i$ is the spin of the ith quark, and $\vec{r}_{ij}$ is the separation between a pair of quarks. We have incorporated in the last part of the spin-orbit interaction a term which arises from the long-range potential in (17). The meaning of k in (19b) will be explained below. We are assuming that the Coulomb term in (17) arises from the exchange of a vector object (gluon) while the linear term is scalar in origin.[13] Although it is usually argued[13,14] that the resonance spectrum does not show traces of the spin-orbit interaction, we shall keep it and we will discuss in the last chapter the numerical results obtained with and without it for the baryon magnetic moments.

Our aim is to make a perturbative estimation of the effects of H_S on the baryon magnetic moments. However, the structure of H_{conf} given in Eq. (17) does not allow an analytic solution of the unperturbed problem. For this reason we will approximate H_{conf} by a harmonic-oscillator potential

$$H_0 = \sum_i (m_i + \vec{p}_i^{\,2}/2m_i) + \frac{k}{2}\sum_{i<j} r_{ij}^{\,2}\,. \tag{20}$$

The oscillator constant k in Eq. (20), which is the same as that in (19b) will be assumed to be flavor independent. In terms of the relative coordinates, H_0 becomes

$$H_0 = M + \vec{p}_\lambda^{\,2}/2m_\lambda + \vec{p}_\rho^{\,2}/2m_\rho + 3k(\rho^2+\lambda^2)/2 \tag{21}$$

and its eigenvalues are given by $M_n = M + \omega_\lambda(l_\lambda + 2r_\lambda + \frac{3}{2}) + \omega_\rho(l_\rho + 2r_\rho + \frac{3}{2})$, where the oscillator frequencies are

$$\omega_\lambda = (3k/m_\lambda)^{1/2}\,,\quad \omega_\rho = (3k/m_\rho)^{1/2}\,. \tag{22}$$

The ground-state eigenfunction of this Hamiltonian is[13]

$$(0^+;1,m_1;2,m_2,3,m_3) = (a_\rho/\pi)^{3/4}(a_\lambda/\pi)^{3/4}\exp[-(a_\rho\rho^2 + a_\lambda\lambda^2)/2]\,, \tag{23}$$

where $\rho_\rho = m_\rho\omega_\rho$, $a_\lambda = m_\lambda\omega_\lambda$.
In the limit of equal masses H_0 and the ground-state eigenfunctions become

$$\bar{H}_0 = M + \vec{p}_\lambda^{\,2}/2m + \vec{p}_\rho^{\,2}/2m + 3k(\rho^2+\lambda^2)/2\,, \tag{24}$$

$$(0_0^+)_{123}=\left(\frac{\alpha}{\pi}\right)^{3/2}\exp[-\alpha(\rho^2+\lambda^2)/2]\,. \qquad (25)$$

This eigenfunction should be combined with the flavor × spin wave functions of the 56-dimensional representation of SU(6) as indicated in Eq. (11). The internal quantum numbers of $(0_0^+)_{123}$ are $l_\rho=l_\lambda=r_\rho=r_\lambda=0$, thus the energy eigenvalue is $M+3\omega$. The wave functions of the first positive-parity excited states, whose energy is $M+5\omega$, are given, for example, in Ref. 13.

The complete Hamiltonian has become the sum of H_S given in Eq. (18) and H_0 given in Eq. (21). We are going to make a perturbative expansion around $\overline{H}_0$ defined as H_0 evaluated at $m_1=m_2=m_3=m$. For this purpose let us introduce quark-mass shifts by

$$m_i=m+\Delta_i\,,\ \sum_i \Delta_i=0\,. \qquad (26)$$

Evidently m varies from baryon to baryon so that we are not perturbing around an SU(3) limit. In terms of m and Δ_i, H_0 becomes

$$H_0=\overline{H}_0-\sum_i p_i{}^2\Delta_i/2m^2+\sum_i p_i{}^2\Delta_i{}^2/2m^3+\cdots\,. \qquad (27)$$

Since the mass differences inside the octet are of the same order as those between the octet and the decuplet we will consider

$$H_\Delta=-\sum_i p_i{}^2\Delta_i/2m^2 \qquad (28)$$

and $\overline{H}_S$, defined as H_S evaluated at $m_i=m$, as first-order perturbative terms, and $\sum_i p_i{}^2\Delta_i{}^2/2m^3$ and

$$H_S(\Delta)=\sum_i \frac{\partial H_S}{\partial m_i}\bigg|_{m_j=m}\Delta_i \qquad (29)$$

as second-order perturbative terms and so on. Thus $H=\overline{H}_0+H_1+H_2$, where

$$H_1=H_\Delta+\overline{H}_S,\quad H_2=\sum_i p_i{}^2\Delta_i{}^2/2m^3+H_S(\Delta)\,. \qquad (30)$$

Evidently the eigenfunctions of $\overline{H}_0$ do not lead, when combined with flavor and spin wave functions to irreducible representations of SU(6). Nevertheless, to facilitate notation we will keep attaching to the eigenfunctions of $\overline{H}_0$ the dimensionality of the SU(6) representation to which they will be associated when $m_u=m_d=m_s$.

A mass-dependent operator A, such as the magnetic-moment operator given in Eqs. (10), can also be expanded around the operator A_0 obtained in the limit $m_1=m_2=m_3=m$ as $A=A_0+A_1+A_2+\cdots$, where A_n is proportional to $(\Delta_j)^n$. In this way the expansion of the matrix elements of A is calculated from the expansion of A itself and from the expansion of the eigenfunctions $|\psi\rangle$ of H in terms of the eigenfunctions $|i\rangle$ of $\overline{H}_0$. This expansion simplifies considerably in the case of the magnetic moment because $\langle 0|\vec{\mu}_0|i\rangle=0$, where $|i\rangle$ is an eigenstate of $\overline{H}_0$. Thus we may write, to second order,

$$\langle\psi|\vec{\mu}|\psi\rangle=\langle 0|\vec{\mu}|0\rangle\left(1-\sum_i\frac{|\langle 0|H_1|i\rangle|^2}{(\epsilon_0-\epsilon_i)^2}\right)+2\,\mathrm{Re}\sum_i\frac{\langle 0|\vec{\mu}_1|i\rangle\langle i|H_1|0\rangle}{\epsilon_0-\epsilon_i}+\sum_{i,j}\frac{\langle 0|H_1|i\rangle\langle i|\vec{\mu}_0|j\rangle\langle j|H_1|0\rangle}{(\epsilon_0-\epsilon_i)(\epsilon_0-\epsilon_j)}\,, \qquad (31)$$

where $|0\rangle=|56,0_0^+,8_2,1/2\rangle$ and $|i\rangle\in\{|56,0_R^+,8_2,\frac{1}{2}\rangle$, $|70,0_2^+,10_2,\frac{1}{2}\rangle$, $|70,0_2^+,8_2,\frac{1}{2}\rangle$, $|70,0_2^+,1_2,\frac{1}{2}\rangle$, $|20,1_2^+,1_4,\frac{1}{2}\rangle$, $|20,1_2^+,8_2,\frac{1}{2}\rangle$, $|56,2_2^+,10_4,\frac{1}{2}\rangle$, $|70,2_2^+,8_4,\frac{1}{2}\rangle\}$. The last number in each ket $|i\rangle$ is the eigenvalue of the total angular momentum. Since $\langle 0|\vec{\mu}|0\rangle$ contains already all the breaking that arises from the quark mass differences in the kinetic energy, the effect of H_Δ in Eq. (31) cancels (as can be verified from the calculations of the next two sections) and thus $H_1=\overline{H}_S$ in Eq. (31). This cancellation will of course not occur in other perturbative effects such as the first-order mixing in the ground-state wave function given by

$$|\psi\rangle=|0\rangle+\sum_i|i\rangle\frac{\langle i|H_1|0\rangle}{\epsilon_0-\epsilon_i}\,, \qquad (32)$$

where $H_1=H_\Delta+\overline{H}_S$.

V. MIXING WITH THE EXCITED STATES

In this section we compute the matrix elements $\langle 0|H_1|i\rangle$ which contribute in the right-hand-side of Eqs. (31) and (32), that is $\langle 0|H_\Delta|i\rangle$, $\langle 0|\overline{H}_{SS}|i\rangle$, $\langle 0|\overline{H}_{SO}|i\rangle$ and $\langle 0|\overline{H}_T|i\rangle$ where the ket $|i\rangle$ stands for all the excited states, which we have mentioned previously and $|0\rangle$ for the ground state $|56,0_0^+\rangle$.

The only nonvanishing matrix elements are $\langle 0|H_\Delta|70,0_2^+\rangle$, $\langle 0|\overline{H}_{SS}|56,0_R^+\rangle$, $\langle 0|\overline{H}_{SS}|70,0_2^+\rangle$, $\langle 0|\overline{H}_{SO}|20,1_2^+\rangle$, and $\langle 0|\overline{H}_T|70,2_2^+\rangle$. A few remarks about the matrix elements are in order. H_Δ and $\overline{H}_{SS}$ mix the ground state only with $|56,0_R^+\rangle$ and $|70,0_2^+\rangle$. If one applies the transformation (3) and (5), for the equal-mass case, to H_Δ then one gets

$$H_\Delta=-\frac{1}{2m^2}\left[-\tfrac{1}{2}(\Delta_1+\Delta_2)(\vec{p}_\lambda{}^2-\vec{p}_\rho{}^2)+\frac{1}{\sqrt{3}}(\Delta_1-\Delta_2)\vec{p}_\lambda\cdot\vec{p}_\rho\right], \qquad (33)$$

which clearly shows that H_Δ is of mixed symmetry in the space variables and thus mixes $|56,0_0^+\rangle$ only with $|70,0_2^+\rangle$. In the same way we conclude that $\overline{H}_{SO}$ can mix the ground state only with the $|20,1_2^+\rangle$ because it is an operator with orbital angular momentum equal to one. On the other hand $\overline{H}_T$ being an operator of $\vec{L}=2$ can mix the ground state only with $|56,2_2^+\rangle$ and $|70,2_2^+\rangle$. However since $\bar{H}_T$ is of mixed symmetry in the space variables

$$\bar{H}_T=\left(\frac{\alpha_s}{\sqrt{2}m^2}\right)[3\vec{S}_1\cdot\hat{\rho}\vec{S}_2\cdot\hat{\rho}-\vec{S}_1\cdot\vec{S}_2]\rho^{-3}\,, \tag{34}$$

it gives no mixing with the completely symmetric state $|56,2_2^+\rangle$.

The calculation of these matrix elements is straightforward; the results are

$$H_\Delta|56,0_0^+,8_2\rangle=-\left(\frac{3}{2}\right)^{1/2}\frac{\omega}{2m}\Delta_q(|70,0_2^+,8_2\rangle\mp|70,0_2^+,10_2\rangle)\,, \tag{35a}$$

where $q=u$ for the proton, $q=d$ for the neutron and so on. The relative phase in the right-hand side is (-1) for p, Σ^+, and Σ^-, and $(+1)$ for n, Ξ^0, and Ξ^-. For the Σ^0 and Λ the results are the same, with $\Delta_q=\frac{1}{2}(\Delta_u+\Delta_d)$ and with a negative relative phase. A similar approach for the Λ state leads to

$$H_\Delta|56,0_0^+,8_2,\Lambda\rangle=\left(\frac{3}{2}\right)^{1/2}\frac{\omega}{2m}\frac{1}{2}(\Delta_u+\Delta_d)(|70,0_2^+,8_2,\Lambda\rangle+|70,0_2^+,1_2,\Lambda\rangle$$
$$+\frac{1}{\sqrt{2}}\frac{\omega}{2m}\frac{1}{2}(\Delta_u-\Delta_d)(|70,0_2^+,8_2,\Sigma^0\rangle+|70,0_2^+,10_2,\Sigma^0\rangle)\,, \tag{35b}$$

$$H_\Delta|56,0_0^+,8_2,\Sigma^0\rangle=\left(\frac{3}{2}\right)^{1/2}\frac{\omega}{2m}\frac{1}{2}(\Delta_u+\Delta_d)(-|70,0_2^+,8_2,\Sigma^0\rangle+|70,0_2^+,10_2,\Sigma^0\rangle)$$
$$+\frac{1}{\sqrt{2}}\frac{\omega}{2m}\frac{1}{2}(\Delta_u-\Delta_d)(|70,0_2^+,8_2,\Lambda\rangle-|70,0_2^+,1,\Lambda\rangle)\,. \tag{35c}$$

On the other hand, the nonzero matrix elements of $\bar{H}_{SS}$, $\bar{H}_{SO}$, and $\bar{H}_T$ are

$$\langle 56,0_R^+,8_2,\tfrac{1}{2}|\bar{H}_{SS}|56,0_0^+,8_2,\tfrac{1}{2}\rangle=-\alpha_s\omega(\omega/6\pi m)^{1/2}\,, \tag{36a}$$

$$\langle 70,0_2^+,8_2,\tfrac{1}{2}|\bar{H}_{SS}|56,0_0^+,8_2,\tfrac{1}{2}\rangle=\alpha_s\omega(\omega/3\pi m)^{1/2}\,, \tag{36b}$$

$$\langle 56,0_0^+,8_2,\tfrac{1}{2}|\bar{H}_{SO}|20,1_2^+,8_2,\tfrac{1}{2}\rangle=-\frac{\omega}{\sqrt{6}}[\alpha_s(\omega/9\pi m)^{1/2}+\omega/2m]\,, \tag{36c}$$

$$\langle 56,0_0^+,8_2,\tfrac{1}{2}|\bar{H}_T|70,2_2^+,8_2,\tfrac{1}{2}\rangle=-\alpha_s\omega(\omega/30\pi m)^{1/2}\,. \tag{36d}$$

In short, H_Δ and $\bar{H}_S$ produce, to first order, the following admixture in the ground state[15]:

$$|\psi\rangle=|56,0_0^+,8_2,\tfrac{1}{2}\rangle+C_1|56,0_R^+,8_2,\tfrac{1}{2}\rangle$$
$$+C_2|70,0_2^+,10_2,\tfrac{1}{2}\rangle+(C_3+C_4)|70,0_2^+,8_2,\tfrac{1}{2}\rangle$$
$$+C_5|20,1_2^+,8_2,\tfrac{1}{2}\rangle+C_6|70,2_2^+,8_4,\tfrac{1}{2}\rangle\,, \tag{37}$$

where C_1 and C_4 arise from H_{SS}, C_2 and C_3 from H_Δ, C_5 from $\bar{H}_{SO}$, and C_6 from $\bar{H}_T$. When the baryon in question is the Λ one has to substitute $|70,0_2^+,10_2,\frac{1}{2}\rangle$ by $|70,0_2^+,1_2,\frac{1}{2}\rangle$. In Table I we give the numerical values of the coefficients C_i estimated using $m_u=311$ MeV, $m_d=280$ MeV, $m_s=469$ MeV, $\alpha_s=1$ and $\omega_p=$ the oscillator frequency for the proton $=500$ MeV.

It is interesting to realize that the value of C_5 implies that the state $|20,1_2^+\rangle$ can be produced in πN scattering at the level of 4%, the reason being that the 20-dimensional representation of SU(6) is contained in the product 35×20.

VI. THE MATRIX ELEMENTS $\langle i|\vec{\mu}|j\rangle$

Since the spin part $\vec{\mu}_S$ of the magnetic-moment operator $\vec{\mu}$ is spin and flavor dependent while the orbital part $\vec{\mu}_L$ is flavor and space dependent, their matrix elements should be calculated separately. In both cases the computation can be approached in two different ways depending on whether the SU(6) wave functions are written down according to their SU(3) content or in the flavor-product basis. We have chosen to make the calculation of $\langle i|\vec{\mu}_S|j\rangle$ with the first type of wave functions and that of $\langle i|\vec{\mu}_L|j\rangle$ with the second type.

The matrix elements $\langle i|\vec{\mu}_S|j\rangle$

First of all we should notice that $\vec{\mu}_S$ can be expressed in terms of SU(6) generators. Indeed,

TABLE I. Values of the mixing coefficients in the wave functions of the baryons of the octet according to Eq. (37). This estimation was made using $m_u=311$ MeV, $m_d=280$ MeV, $m_s=469$ MeV, $\alpha_s=1$, and $\omega_p=500$ MeV.

	C_1	C_2	C_3	C_4	C_5	C_6
p	−0.15	0.01	−0.01	0.21	−0.22	−0.06
n	−0.15	0.01	0.01	0.21	−0.23	−0.07
Σ^+	−0.13	−0.04	0.04	0.18	−0.17	−0.06
Σ^0	−0.13	−0.05	0.05	0.19	−0.18	−0.06
Λ	−0.13	0.05	0.05	0.19	−0.18	−0.06
Σ^-	−0.13	−0.06	0.06	0.19	−0.18	−0.06
Ξ^0	−0.12	−0.04	−0.04	0.16	−0.14	−0.05
Ξ^-	−0.12	−0.05	−0.05	0.17	−0.15	−0.05

defining

$$N=\tfrac{1}{3}(\mu_u+\mu_d+\mu_s)\underline{1}+(\mu_u-\mu_d)I_3 +\tfrac{1}{2}(\mu_u+\mu_d-2\mu_s)Y\ , \qquad (38)$$

where $\underline{1}$, I_3, and Y are, respectively, the unit matrix and the generators corresponding to the third component of isospin and hypercharge in the fundamental representation of SU(3), then $\vec{\mu}_S$ becomes

$$\vec{\mu}_S=\mu_1\vec{\sigma}_1+\mu_2\vec{\sigma}_2+\mu_3\vec{\sigma}_3 =N\vec{\sigma}\otimes\underline{1}\otimes\underline{1}+\underline{1}\otimes N\vec{\sigma}\otimes\underline{1}+\underline{1}\otimes\underline{1}\otimes N\vec{\sigma}\ , \qquad (39)$$

where $\underline{1}$ is a 6×6 unit matrix. This property guarantees that $\vec{\mu}_s$ does not mix SU(6) irreducible representations, that it commutes with (total) hypercharge and third components of (total) isospin and that $\mu_S^{(3)}$ commutes with the third component of total spin.

To compute $\langle i|\vec{\mu}_S|j\rangle$ we will separate the case of baryons with two identical quarks from the case of baryons with three different quarks. In the first case two quark flavors, a and b, with one of them repeated, quark a for example, make up a three-dimensional space the basis of which is either of the product type, $|f_1\rangle=|aab\rangle$, $f_2=|aba\rangle$, $f_3=|baa\rangle$, or of the SU(3) type (reduced according to the permutation group S_3), $F_1=(10)$, $F_2=(8)$, $F_3=[8]$. Since $F=Uf$, where

$$U=\begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 2/\sqrt{6} & -1/\sqrt{6} & -1/\sqrt{6} \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}, \qquad (40)$$

then $\vec{\mu}_S=U\underline{\vec{\mu}}_S U^T$, where $\underline{\vec{\mu}}_S$ is evaluated in the flavor-product basis

$$\underline{\vec{\mu}}_S=\begin{pmatrix} \tfrac{1}{3}(2\mu_a+\mu_b)(\vec{\sigma}_1+\vec{\sigma}_2+\vec{\sigma}_3) & \frac{1}{\sqrt{18}}(\mu_a-\mu_b)(\vec{\sigma}_1+\vec{\sigma}_2-2\vec{\sigma}_3) & \frac{1}{\sqrt{6}}(\mu_a-\mu_b)(\vec{\sigma}_1-\vec{\sigma}_2) \\ \frac{1}{\sqrt{18}}(\mu_a-\mu_b)(\vec{\sigma}_1+\vec{\sigma}_2-2\vec{\sigma}_3) & \tfrac{1}{6}(5\mu_a+\mu_b)(\vec{\sigma}_1+\vec{\sigma}_2)+\tfrac{1}{6}(4\mu_b+2\mu_a)\vec{\sigma}_3 & -\frac{1}{\sqrt{12}}(\mu_a-\mu_b)(\vec{\sigma}_1-\vec{\sigma}_2) \\ \frac{1}{\sqrt{6}}(\mu_a-\mu_b)(\vec{\sigma}_1-\vec{\sigma}_2) & -\frac{1}{\sqrt{12}}(\mu_a-\mu_b)(\vec{\sigma}_1-\vec{\sigma}_2) & \tfrac{1}{2}(\mu_a+\mu_b)(\vec{\sigma}_1+\vec{\sigma}_2)+\mu_a\vec{\sigma}_3 \end{pmatrix}. \qquad (41)$$

To calculate the matrix elements of the third component of the spin operators in the states with total spin projection $=\frac{1}{2}$ we use the same procedure, that is, we first work in the product basis $|s_\alpha\rangle=|\uparrow\uparrow\downarrow\rangle, |\uparrow\downarrow\uparrow\rangle, |\downarrow\uparrow\uparrow\rangle$, where the matrix elements are diagonal and then we use the fact that the product basis and the basis formed with irreducible representations of SU(2), $|S_\alpha\rangle=(4),(2),[2]$, are related by the same U matrix given in Eq. (40).

The next step is to arrange the SU(3) flavor wave functions and SU(2) spin functions into irreducible representations of SU(6) and to include in the elements involving the $|20,1_2^+,8_2,\frac{1}{2}\rangle$ and $|70,2_2^+,8_4,\frac{1}{2}\rangle$ wave functions the Clebsch-Gordan coefficients that are necessary to make up total angular momentum $=\frac{1}{2}$ with projection $=\frac{1}{2}$. Since our interest is restricted to the states $|0\rangle=|56,0_0^+,8_2,\frac{1}{2}\rangle$ and $|i\rangle\in\{|70,0_2^+,10_2,\frac{1}{2}\rangle, |70,0_2^+,1_2,\frac{1}{2}\rangle, |56,0_R^+,8_2,\frac{1}{2}\rangle, |70,0_2^+,8_2,\frac{1}{2}\rangle, |20,1_2^+,8_2,\frac{1}{2}\rangle, |70,2_2^+,8_4,\frac{1}{2}\rangle\}$ the nonvanishing matrix elements of $\mu_S^{(3)}$ reduce to those given in Table II (the entries under Σ^0, Λ, and

TABLE II. Matrix elements of $\mu_S^{(3)}$ in states with total angular momentum $\frac{1}{2}$ and projection $\frac{1}{2}$.

	$(56,0^+,8_2,\frac{1}{2})$	$(70,0_2^+,8_2,\frac{1}{2}$	$(70,0_2^+,10_2,\frac{1}{2})$	$(70,0_2^+,10_2,\frac{1}{2})-(70,0_2^+,8_2,\frac{1}{2})$	$(20,1_2^+,8_2,\frac{1}{2})$	$(70,2_2^+,8_4,\frac{1}{2})$
p	$\frac{1}{3}(4\mu_u-\mu_d)$	$\frac{1}{3}(2\mu_u+\mu_d)$	$\frac{1}{3}(2\mu_u+\mu_d)$	$-\frac{2}{3}(\mu_u-\mu_d)$	$-\frac{1}{3}\mu_d$	$-\frac{1}{3}(2\mu_u+\mu_d)$
n	$\frac{1}{3}(4\mu_d-\mu_u)$	$\frac{1}{3}(2\mu_d+\mu_u)$	$\frac{1}{3}(2\mu_d+\mu_u)$	$-\frac{2}{3}(\mu_u-\mu_d)$	$-\frac{1}{3}\mu_u$	$-\frac{1}{3}(2\mu_d+\mu_u)$
Σ^+	$\frac{1}{3}(4\mu_u-\mu_s)$	$\frac{1}{3}(2\mu_u+\mu_s)$	$\frac{1}{3}(2\mu_u+\mu_s)$	$-\frac{2}{3}(\mu_u-\mu_s)$	$-\frac{1}{3}\mu_s$	$-\frac{1}{3}(2\mu_u+\mu_s)$
Σ^0	$\frac{1}{3}(2\mu_u+2\mu_d-\mu_s)$	$\frac{1}{3}(\mu_u+\mu_d+\mu_s)$	$\frac{1}{3}(\mu_u+\mu_d+\mu_s)$	$-\frac{1}{3}(\mu_u+\mu_d-2\mu_s)$	$-\frac{1}{3}\mu_s$	$-\frac{1}{3}(\mu_u+\mu_d+\mu_s)$
Σ^-	$\frac{1}{3}(4\mu_d-\mu_s)$	$\frac{1}{3}(2\mu_d+\mu_s)$	$\frac{1}{3}(2\mu_d+\mu_s)$	$-\frac{2}{3}(\mu_d-\mu_s)$	$-\frac{1}{3}\mu_s$	$-\frac{1}{3}(2\mu_d+\mu_s)$
Ξ^0	$\frac{1}{3}(4\mu_s-\mu_u)$	$\frac{1}{3}(2\mu_s+\mu_u)$	$\frac{1}{3}(2\mu_s+\mu_u)$	$-\frac{2}{3}(\mu_u-\mu_s)$	$-\frac{1}{3}\mu_u$	$-\frac{1}{3}(2\mu_s+\mu_u)$
Ξ^-	$\frac{1}{3}(4\mu_s-\mu_d)$	$\frac{1}{3}(2\mu_s+\mu_d)$	$\frac{1}{3}(2\mu_s+\mu_d)$	$-\frac{2}{3}(\mu_d-\mu_s)$	$-\frac{1}{3}\mu_d$	$-\frac{1}{3}(2\mu_s+\mu_d)$
Λ	μ_s	$\frac{1}{3}(\mu_u+\mu_d+\mu_s)$	$\frac{1}{3}(\mu_u+\mu_d+\mu_s)$	$-\frac{1}{3}(\mu_u+\mu_d-2\mu_s)$	$-\frac{1}{9}(2\mu_u+2\mu_d-\mu_s)$	$-\frac{1}{3}(\mu_u+\mu_d+\mu_s)$
$\Sigma^0\Lambda$	$-(\mu_u-\mu_d)/\sqrt{3}$	0	0	—	$-\frac{1}{3\sqrt{3}}(\mu_u-\mu_d)$	0

$\Sigma\Lambda$ will be explained shortly).

Now we turn our attention to the baryons which contain three different valence quarks. This case differs from the case of baryons with two identical quarks in that the flavor basis is six-dimensional. However the relation between the product basis and the SU(3) basis can be expressed also in terms of the matrix U defined in Eq. (40):

$$\begin{pmatrix} \Sigma^{*0} \\ (\Sigma^0) \\ -(\Lambda) \\ \Lambda^* \\ [\Lambda^0] \\ [\Sigma^0] \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} U & U \\ U & -U \end{pmatrix} \begin{pmatrix} |uds\rangle \\ |dsu\rangle \\ |sud\rangle \\ |dus\rangle \\ |sdu\rangle \\ |usd\rangle \end{pmatrix} . \tag{42}$$

This allows a straightforward computation of $\vec{\mu}_s$ in the SU(3) basis. On the other hand the matrix elements of the spin operators can be handled as in the previous case. After transforming to the SU(6) basis and taking into account, in the $(20, 1_2^+)$ and $(70, 2_2^+)$ cases, the corresponding Clebsch-Gordan coefficients, we obtain the results which complete Table II.

The matrix elements $\langle i|\vec{\mu}_L|j\rangle$

In order to minimize the flavor dependence of the $\vec{\mu}_L$ operator we will write the SU(6)× O(3) wave functions in the flavor-product basis. Let us consider first the case of baryons with two identical quarks, where we can always arrange the labels 1 and 2 so that they correspond to the identical quarks (for which $\mu_1=\mu_2$ and $m_1=m_2$). Thus $\vec{\mu}_L$ becomes effectively

$$\vec{\mu}_L = (m_3\mu_1/M + 2m_1\mu_3/M)\vec{L}_\lambda + \mu_1\vec{L}_\rho \tag{43}$$

and we immediately obtain $\vec{\mu}_L|56, 0^+\rangle = 0$, while for the states with $l_\rho \neq 0$ and $l_\lambda \neq 0$ we see that $\vec{\mu}_L$ mixes only those with the same l_ρ and the same l_λ. Since, furthermore, $\vec{\mu}_L$ is a vector operator it has zero matrix elements between states with $L=0$, and the nonzero nondiagonal matrix elements involve states either with the same total orbital angular momentum L or with $\Delta L = \pm 1$. In the set of states under consideration these nondiagonal matrix elements are the $[0_2^+]_{12}$ (in the 70), $[1_2^+]_{123}$ (in the 20) and the $[1_2^+]_{123}$ (in the 20), $[2_2^+]_{12}$ (in the 70). However, since the spin wave functions in $|20, 1_2^+, 8_2\rangle$ and $|70, 2_2^+, 8_4\rangle$ are orthogonal, only the former survives and we obtain, after combining all the appropriate Clebsch-Gordan coefficients,

$$\langle 20, 1_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, p|\mu_L^{(3)}|70, 0_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, p\rangle = \langle 20, 1_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, p|\mu_L^{(3)}|70, 0_2^+, 10_2, \tfrac{1}{2}, \tfrac{1}{2}, \Delta^+\rangle$$
$$= 2m_u(\mu_u - \mu_d)/3M . \tag{44}$$

On the other hand we can have nonzero diagonal matrix elements of $\vec{\mu}_L$ only in the state $(20, 1_2^+)$ and in the state $(70, 2_2^+)$. The results are

$$\langle 20, 1_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, p|\mu_L^{(3)}|20, 1_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, p\rangle = [\mu_u(1+m_d/M) + \mu_d 2m_u/M]/3 , \tag{45}$$

$$\langle 70, 2_2^+, 8_4, \tfrac{1}{2}, \tfrac{1}{2}, p|\mu_L^{(3)}|70, 2_2^+, 8_4, \tfrac{1}{2}, \tfrac{1}{2}, p\rangle = [\mu_u(1+m_d/M) + \mu_d 2m_u/M]/2 . \tag{46}$$

Next let us consider the baryons with three different quarks. In these cases we cannot simplify $\vec{\mu}_L$ but we will arrange the labels 1, 2, and 3 in the order in which the flavor labels appear in each part of the wave functions. In this way all the arguments used in the previous case apply also here for that part of $\vec{\mu}_L$ which is proportional to $\vec{L}_\rho$ and $\vec{L}_\lambda$. Since this part is symmetric under the interchange of the flavor labels 1 and 2 then it does not give rise to Σ^0-Λ transitions. On the other hand the coefficients of the three-body operators $\vec{\rho}\times\vec{p}_\lambda$ and $\vec{\lambda}\times\vec{p}_\rho$ are antisymmetric in the labels 1 and 2 and thus contribute only to the Σ^0-Λ transition magnetic moments. For the Σ^0-Σ^0 and Λ-Λ magnetic moments the results are (g_{ij}^{uds} means g_{ij} evaluated with $1=u$, $2=d$, $3=s$).

$$\langle 20, 1_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, \Sigma^0|\mu_L^{(3)}|70, 0_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, \Sigma^0\rangle = -\langle 20, 1_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, \Lambda|\mu_L^{(3)}|70, 0_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, \Lambda\rangle$$
$$= \langle 20, 1_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, \Lambda|\mu_L^{(3)}|70, 0_2^+, 1_2, \tfrac{1}{2}, \tfrac{1}{2}, \Lambda^*\rangle$$
$$= \langle 20, 1_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, \Sigma^0|\mu_L^{(3)}|70, 0_2^+, 10_2, \tfrac{1}{2}, \tfrac{1}{2}, \Sigma^{0*}\rangle$$
$$= (g_{\rho\rho}^{uds} - g_{\lambda\lambda}^{uds})/3 , \tag{47}$$

$$\langle 20, 1_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, \Sigma^0|\mu_L^{(3)}|20, 1_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, \Sigma^0\rangle = \langle 20, 1_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, \Lambda|\mu_L^{(3)}|20, 1_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, \Lambda\rangle$$
$$= (g_{\rho\rho}^{uds} + g_{\lambda\lambda}^{uds})/3 , \tag{48}$$

$$\langle 70, 2_2^+, 8_4, \tfrac{1}{2}, \tfrac{1}{2}, \Sigma^0|\mu_L^{(3)}|70, 2_2^+, 8_4, \tfrac{1}{2}, \tfrac{1}{2}, \Sigma^0\rangle = \langle 70, 2_2^+, 8_4, \tfrac{1}{2}, \tfrac{1}{2}, \Lambda|\mu_L^{(3)}|70, 2_2^+, 8_4, \tfrac{1}{2}, \tfrac{1}{2}, \Lambda\rangle$$
$$= (g_{\rho\rho}^{uds} + g_{\lambda\lambda}^{uds})/2 . \tag{49}$$

The three-body operators mix the ground state only with the $|20, 1_2^+, 8_2\rangle$ state. However, according to our phase conventions,

$$\langle 20, 1_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, \Lambda\,|\mu_L^{(3)}|56, 0_0^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, \Sigma^0\rangle = -\langle 20, 1_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, \Sigma^0\,|\mu_L^{(3)}|56, 0_0^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, \Lambda\rangle\,, \tag{50}$$

which implies that the term containing $\langle 0|\vec{\mu}_1|i\rangle$ in Eq. (31) cancels (the same phenomenon occurs in the Σ^0-Λ transition between the $|20, 1_2^+, 8_2\rangle$ and the $|70, 0_2^+, 10_2\rangle$). Because of the orthogonality of the spin wave functions, the matrix element $\langle 20, 1_2^+, 8_2, \Sigma^0\,|\mu_L^{(3)}|20, 1_2^+, 8_2, \Lambda\rangle$ vanishes. The remaining matrix elements, in the set of states under consideration, are

$$\langle 70, 2_2^+, 8_4, \tfrac{1}{2}, \tfrac{1}{2}, \Sigma^0\,|\mu_L^{(3)}|70, 2_2^+, 8_4, \tfrac{1}{2}, \tfrac{1}{2}, \Lambda\rangle = 0\,, \tag{51}$$

$$\begin{aligned}\langle 20, 1_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, \Lambda^0\,|\mu_L^{(3)}|70, 0_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, \Sigma^0\rangle &= \langle 20, 1_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, \Sigma^0\,|\mu_L^{(3)}|70, 0_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, \Lambda)\\ &= -(g_{\rho\lambda}^{uds} + g_{\lambda\rho}^{uds})/3\,.\end{aligned} \tag{52}$$

Since the explicit mass dependence in the right-hand side of Eqs. (44)–(49) and (52) arises only from the explicit mass dependence of $\vec{\mu}_L$ given in Eq. (10) and since in Eq. (31) we need only the zeroth-order term in the expansion of $\vec{\mu}$, we have listed in Table III only the limit $m_1 = m_2 = m_3 = m$ of the results given in Eqs. (44)–(49) and (52). The complete matrix elements of $\vec{\mu}$ are obtained adding up the results listed in Tables II and III. Whenever in these tables the Λ appears associated to an SU(3) decuplet one should understand instead an SU(3) singlet. The formulas not listed in these tables are those which involve two opposite signs:

$$\begin{aligned}\langle 70, 0_2^+, 10_2, \tfrac{1}{2}, \tfrac{1}{2}, \Sigma^0\,|\mu_S^{(3)}|70, 0_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, \Lambda\rangle &= -\langle 70, 0_2^+, 1_2, \tfrac{1}{2}, \tfrac{1}{2}, \Lambda\,|\mu_S^{(3)}|70, 0_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, \Sigma^0\rangle\\ &= (\mu_d - \mu_u)/\sqrt{3}\,,\end{aligned} \tag{53}$$

$$\begin{aligned}\langle 70, 0_2^+, 10_2, \tfrac{1}{2}, \tfrac{1}{2}, \Sigma^0\,|\mu_L^{(3)}|20, 1_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, \Lambda\rangle &= -\langle 70, 0_2^+, 1_2, \tfrac{1}{2}, \tfrac{1}{2}, \Lambda\,|\mu_L^{(3)}|20, 1_2^+, 8_2, \tfrac{1}{2}, \tfrac{1}{2}, \Sigma^0\rangle\\ &= (g_{\rho\lambda}^{uds} + g_{\lambda\rho}^{uds})/3\,.\end{aligned} \tag{54}$$

Taking into account the phases in Eqs. (35a) and (35b) as well as the formulas of Tables II and III and Eqs. (53) and (54) one can verify that the quadratic and linear effects of H_Δ in Eq. (31) cancel. Considering furthermore, that

$$\langle 56, 0_R^+, 8_2\,|\vec{\mu}\,|56, 0_R^+, 8_2\rangle = \langle 0\,|\vec{\mu}\,|0\rangle\,, \tag{55}$$

one concludes that the terms with $\langle 0\,|\overline{H}_{ss}\,|56, 0_R^+\rangle$ in Eq. (31) cancel among themselves. Thus the calculation of the corrected magnetic moments should be done according to the equation

$$\langle\psi\,|\vec{\mu}\,|\psi\rangle = \langle 0\,|\vec{\mu}\,|0\rangle\left(1 - \sum_i \frac{|\langle 0\,|\overline{H}_S|i\rangle\,|^2}{(\epsilon_0 - \epsilon_i)^2}\right) + \sum_{i,j}\frac{\langle 0\,|\overline{H}_S\,|i\rangle\langle i\,|\vec{\mu}_0\,|j\rangle\langle j\,|\overline{H}_S\,|0\rangle}{(\epsilon_0 - \epsilon_i)(\epsilon_0 - \epsilon_j)}\,, \tag{56}$$

where $|i\rangle \in \{|70, 0_2^+, 8_2\rangle, |20, 1_2^+, 8_2\rangle, |70, 2_2^+, 8_2\rangle\}$. The matrix elements of $\overline{H}_S$ are given in Eqs. (36).

It is interesting to notice that our expression

TABLE III. Matrix elements of $\mu_L^{(3)}$.

	$(70, 0_2^+, 8_2, \frac{1}{2}) - (20, 1_2^+, 8_2, \frac{1}{2})$	$(70, 0_2^+, 10_2, \frac{1}{2}) - (20, 1_2^+, 8_2, \frac{1}{2})$	$(20, 1_2^+, 8_2, \frac{1}{2})$	$(70, 2_2^+, 8_4, \frac{1}{2})$
p	$\frac{2}{9}(\mu_u - \mu_d)$	$\frac{2}{9}(\mu_u - \mu_d)$	$\frac{2}{9}(2\mu_u + \mu_d)$	$\frac{1}{3}(2\mu_u + \mu_d)$
n	$\frac{2}{9}(\mu_d - \mu_u)$	$\frac{2}{9}(\mu_u - \mu_d)$	$\frac{2}{9}(2\mu_d + \mu_u)$	$\frac{1}{3}(2\mu_d + \mu_u)$
Σ^+	$\frac{2}{9}(\mu_u - \mu_s)$	$\frac{2}{9}(\mu_u - \mu_s)$	$\frac{2}{9}(2\mu_u + \mu_s)$	$\frac{1}{3}(2\mu_u + \mu_s)$
Σ^0	$\frac{1}{9}(\mu_u + \mu_d - 2\mu_s)$	$\frac{1}{9}(\mu_u + \mu_d - 2\mu_s)$	$\frac{2}{9}(\mu_u + \mu_d + \mu_s)$	$\frac{1}{3}(\mu_u + \mu_d + \mu_s)$
Σ^-	$\frac{2}{9}(\mu_d - \mu_s)$	$\frac{2}{9}(\mu_d - \mu_s)$	$\frac{2}{9}(2\mu_d + \mu_s)$	$\frac{1}{3}(2\mu_d + \mu_s)$
Ξ^0	$\frac{2}{9}(\mu_s - \mu_u)$	$\frac{2}{9}(\mu_u - \mu_s)$	$\frac{2}{9}(2\mu_s + \mu_u)$	$\frac{1}{3}(2\mu_s + \mu_u)$
Ξ^-	$\frac{2}{9}(\mu_s - \mu_d)$	$\frac{2}{9}(\mu_d - \mu_s)$	$\frac{2}{9}(2\mu_s + \mu_d)$	$\frac{1}{3}(2\mu_s + \mu_d)$
Λ	$-\frac{1}{9}(\mu_u + \mu_d - 2\mu_s)$	$\frac{1}{9}(\mu_u + \mu_d - 2\mu_s)$	$\frac{2}{9}(\mu_u + \mu_d + \mu_s)$	$\frac{1}{3}(\mu_u + \mu_d + \mu_s)$
$\Sigma^0\Lambda$	$-(\mu_u - \mu_d)/3\sqrt{3}$		0	0

(56) for the baryon magnetic moments satisfies the first-order SU(3) sum rule (2b). To prove this statement it is enough to take into account that this sum rule is satisfied by the zeroth-order magnetic moments given in Eq. (1) as well as by the relevant magnetic moments of the excited states $\langle 70, 0_2^+, 8_2|\vec{\mu}|70, 0_2^+, 8_2\rangle$, $\langle 70, 0_2^+, 8_2|\vec{\mu}|20, 1_2^+, 8_2\rangle$, $\langle 20, 1_2^+, 8_2|\vec{\mu}|20, 1_2^+, 8_2\rangle$, and $\langle 70, 2_2^+, 8_4|\vec{\mu}|70, 2_2^+, 8_4\rangle$ given in Tables II and III and in Eqs. (44)–(49) and (52). Indeed, if we write the sum rule (2b) as $\sum_B C_B\mu_B = O(\epsilon_3^2)$, where C_B is the corresponding coefficient, and if we write Eq. (56) as

$$\mu_B = \mu_B^0\Big(1-\sum_i (f_i^B)^2\Big) + \sum_{i,j} f_i^B f_j^B \mu_{ij}^B ,$$

where $f_i^B = f_i(m=\overline{m}_B)$ and $\overline{m}_B$ is the average quark mass in the corresponding baryon, i.e., $\overline{m}_n = (2m_d + m_u)/3$, etc., then using

$$\sum_B C_B\mu_B^0 = 0\,, \quad \sum_B C_B \mu_{ij}^B = O(\epsilon_3^2)$$

one obtains, after expanding $f_i(m_B)$ around the SU(3) limit $m_B = \overline{m}$

$$\sum_B C_B\mu_B = \sum_{i,j}[f_i(\overline{m})df_j(\overline{m})/d\overline{m} + (i \leftrightarrow j)]\sum_B(\overline{m}_B - \overline{m})C_B\mu_{ij}^B - \sum_i 2f_i(\overline{m})[df_i(\overline{m})/d\overline{m}]\sum_B(\overline{m}_B - \overline{m})C_B\mu_B^0 + O(\epsilon_3^2).$$

Finally, using $\overline{m}_{\Sigma^0} = \overline{m}_\Lambda = \overline{m}_{\Sigma\Lambda} = \overline{m}$ and $\overline{m}_n - \overline{m} = -(\overline{m}_{\Xi^0} - \overline{m})$, which imply

$$\sum_B(\overline{m}_B - \overline{m})C_B\mu_B = (\overline{m}_n - \overline{m})\sum_B C_B\mu_B\,,$$

we get $\sum_B C_B\mu_B = O(\epsilon_3^2)$.

VII. CONCLUSIONS

First we would like to mention that, up to second order in the spin-dependent interaction and mass differences, only three types of excited states contribute to the magnetic moments of the low-lying baryons. These excited states are those which in the limit $m_u = m_d = m_s$ reduce to $|70, 0_2^+, 8_2\rangle$, $|20, 1_2^+, 8_2\rangle$, and $|70, 2_2^+, 8_2\rangle$. The state $|20, 1_2^+, 8_2\rangle$ gets mixed in the ground state thanks to the spin-orbit interaction. Since in our Hamiltonian the spin-orbit part contains also the Thomas term we have a small numerical value for this mixing and therefore it gives also a small contribution in the baryon spectroscopy.[13] This implies also that the state $|20, 1_2^+, 8_2\rangle$ could become visible in πN scattering at the level of 4% (that is below the present experimental uncertainties).

To evaluate the magnetic moments of the low-lying baryons one substitutes in Eq. (56) the matrix elements of $\overline{H}_S$, $\overline{H}_{SO}$, and $\overline{H}_T$ given in Eqs. (36b), (36c), and (36d), as well as the corresponding magnetic moments of the excited states listed in Tables II and III. The final results, shown in Table IV, depend on five parameters m_u, m_d, m_s, α_s, and ω. We have assumed Dirac magnetic moments and have made three fits using the naive quark model and our work with and without spin-orbit forces. We took in the last two cases $\omega = m_p/2$, where m_p is the proton mass, as suggested by spectroscopy and then minimized

$$\chi^2 \equiv \sum_i (\mu_i^{\text{th}} - \mu_i^{\text{exp}})^2/(\Delta\mu_i^{\text{exp}})^2\,,$$

where $i = p, n, \ldots, \Sigma^0\Lambda$ under the restriction $m_u = m_d$ (we have also put by hand $\Delta\mu_p^{\text{exp}} = 0.08$, $\Delta\mu_n^{\text{exp}}$

TABLE IV. Comparison of our numerical results with experimental data (Ref. 16), with the symmetric-quark-model results, and with those obtained without spin-orbit interaction.

	Expt.	No spin forces	With spin forces	Without spin-orbit	Ref. 17
$\mu(p)$	2.793	2.74	2.83	2.83	2.85
$\mu(n)$	−1.913	−1.83	−1.78	−1.78	−1.85
$\mu(\Lambda)$	−0.6138 ± 0.0047	−0.60	−0.58	−0.58	−0.61
$\mu(\Sigma^+)$	2.33 ± 0.13	2.63	2.72	2.72	−2.54
$\mu(\Sigma^-)$	−1.4 ± 0.25	−1.02	−1.15	−1.15	−1.00
$\mu(\Xi^0)$	−1.236 ± 0.014	−1.41	−1.37	−1.37	−1.20
$\mu(\Xi^-)$	−0.75 ± 0.07	−0.49	−0.63	−0.63	−0.43
$\mu(\Sigma^0\Lambda)$	$-1.82^{+0.25}_{-0.18}$	−1.58	−1.54	−1.54	−1.51
m_u(MeV)		343	266	288	330
m_s(MeV)		524	391	440	550
ω(MeV)			470	470	310
α_s			0.69	1.06	1.62

$=0.05$, $\Delta\mu_{\Lambda}^{\rm exp}=0.025$, and $\Delta\mu_{\Xi^0}^{\rm exp}=0.05$ in order not to overestimate the weight of the best-measured magnetic moments). The results of the fits for the magnetic moments, the quark masses, and α_s are displayed in Table IV.

In the last column of the same Table we show for comparison the results of Isgur and Karl.[17] One can see that the order of magnitude of our results agrees with that of Ref. 17 although according to their approach the excited states $[(70,0_2^+,8)$, $(70,0_2^+,10)$, $(70,0_2^+,1)]$ taken into account are not the same as those of ours [in addition $(56,0_R^+,8)$, $(20,1_2^+,8)$, $(70,2_2^+,8)$, $(70,0_2^+,1)]$. As we have indicated the contribution from the states $(70,0_2^+,10)$ and $(70,0_2^+,1)$ [as well as a part of $(70,0_2^+,8)$] cancels because the contribution from H_Δ is already contained in the zeroth-order results (thus in our formulas there is no further first-order breaking by quark mass differences than that already included in the lowest order).

Our model with spin forces has a better χ^2 than that of the simple quark model (30.5 against 38) but it is still not good enough to "explain" the present experimental data on the magnetic moments of the baryons. The resulting quark masses, $m_u=m_d=266$ MeV, $m_s=391$ MeV are slightly smaller than those obtained by fitting the naive quark model and also smaller than those arising from a fit to the baryon spectroscopy.[18] A better fit to the magnetic moments was obtained by dropping the constraint $m_u=m_d$ but then we got, as in the naive quark model, $m_u>m_d$. That is, spin forces are not sufficient to turn this relation around.

Since we have not found a good fit to the experimental data we conclude that other effects, such as relativistic corrections, exchange currents, and three-body forces, are important. In particular, our expressions for the magnetic moments do not contain the nonanalytic terms (in the current quark masses) predicted by chiral perturbation theory.[19] (This is of no surprise since the quark model does not contain the Goldstone phenomenon). Since this kind of term arises from the electromagnetic properties of the virtual pions it is obvious that to obtain them within the quark model one has to incorporate exchange currents. In principle one should also include electromagnetic corrections but they are expected to be quite small.

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[1]Under the assumption that the hadronic electromagnetic current transforms according to the adjoint representation of SU(6), M. A. B. Bég, B. W. Lee, and A. Pais, Phys. Rev. Lett. 13, 514 (1964).

[2]Under the assumption that the hadronic electromagnetic current transforms according to the adjoint representation of SU(3), S. Coleman and S. L. Glashow, Phys. Rev. Lett. 6, 423 (1961).

[3]J. Franklin, Phys. Rev. 172, 1807 (1968).

[4]S. Okubo, Phys. Lett. 4, 14 (1963); A. Sirlin, Nucl. Phys. B161, 301 (1979).

[5]H. Fritzsch and M. Gell-Mann, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1972), Vol. 2, p. 135; D. J. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973); S. Weinberg, *ibid.* 31, 494 (1973); H. Fritzsch, M. Gell-Mann, and H. Leutwyler, Phys. Lett. 47B, 365 (1973).

[6]A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975).

[7]The complete list of references on this subject is too extensive to be quoted here but it can be traced back from M. Böhm, Z. Phys. C 3, 321 (1980); N. Isgur, G. Karl, and R. Koniuk, Phys. Rev. Lett. 41, 1269 (1978); N. Isgur, Phys. Rev. D 21, 779 (1980).

[8]H. Fritzsch, CERN Report No. TH. 2647, 1979, (unpublished).

[9]J. Franklin, Phys. Rev. D 20, 1742 (1979).

[10]R. B. Teese and R. Settles, Phys. Lett. 87B, 111 (1979).

[11]M. Böhm, R. Huerta, and A. Zepeda, in *Lecture Notes in Physics*, Vol. 135, *Proceedings of the IV International Colloquium on Group Theoretical Methods in Physics*, edited by K. B. Wolf (Springer, Berlin, 1980), p. 219.

[12]D. B. Lichtenberg, *Unitary Symmetry and Elementary Particles*, 2nd ed. (Academic, New York, 1978).

[13]M. Böhm, Ref. 7.

[14]N. Isgur and G. Karl, Phys. Rev. D 18, 4187 (1978).

[15]In Table I and Eq. (37) we have omitted the isospin-breaking mixing since its contribution to the magnetic moments is already contained in the lowest-order results [see remark after Eq. (26)].

[16]L. Schachinger *et al.*, Phys. Rev. Lett. 41, 1348 (1978); G. Bunce *et al.*, Phys. Lett. 86B, 386 (1979); R. Settles *et al.*, Phys. Rev. D 20, 2154 (1979); O. Overseth *et*

al., in *Baryon 80*, proceedings of the IVth International Conference on Baryonic Resonances, Toronto, edited by N. Isgur (Univ. of Toronto, Toronto, 1981); Particle Data Group, Rev. Mod. Phys. 52, S1 (1980).

[17] N. Isgur and G. Karl, Phys. Rev. D 21, 3175 (1980).
[18] N. Isgur and G. Karl, Phys. Rev. D 20, 1191 (1979).
[19] D. G. Caldi and H. Pagels, Phys. Rev. D 10, 3739 (1974).