Disappearing dyons

Curtis G. Callan, Jr.
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08544
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I show that in a world with massless fermions the interaction between the charge rotator degree of freedom of the magnetic monopole and the massless-fermion vacuum replaces the dyon states with a set of excitations having zero expectation value of the electric field.

I. THE PROBLEM

One of the surprising properties of the magnetic monopole of weak-interaction gauge theory[1] is that it has an internal degree of freedom which allows it to exist in states of nonzero electric charge[2] and to readily exchange charge with ordinary particles. More surprising yet, the charges of these "dyon" states are not integer multiples of e if the vacuum angle of the world is nonzero.[3] The analysis which leads to these conclusions applies, strictly speaking, only to the pure gauge theory. In this paper, I will show that if the gauge fields couple to fermions of mass much less than the mass m_W, of the vector boson, the above picture is substantially modified. In particular, if *zero*-mass fermions are present, there are *no* electrically charged monopole states. Instead the monopole is an island of chiral-symmetry breaking in the Fermi sea, characterized by large *fluctuations,* but zero expectation value, of the electric field. The properties of the zero-fermion-mass monopole turn out to be rather easy to derive and will be presented in detail in the rest of the paper. Since light quarks *do* exist, the zero-mass picture is presumably more phenomenologically relevant than the conventional picture of the monopole.

The following simple argument shows that something must happen to the dyon in the presence of massless fermions: The anomaly equation for the chiral charge is

$$\frac{dQ_5}{dt}=c\int d^3x\vec{\mathrm{E}}\cdot\vec{\mathrm{B}}\ .$$

Consider the expectation of this equation in any dyonic energy eigenstate. The left-hand side is zero in any energy eigenstate, while the right-hand side is nonzero in a dyon state, since E and B are both radial fields of definite sign. To see the way out of this paradox, consider how the dyon states arise in the first place.[4] The minimum-energy gauge field configuration in the monopole sector is not unique but is parametrized by all possible gauge rotations of a basic configuration $A_{ia}^0(\vec{\mathrm{x}})$. To get the low-lying states it suffices (in the $A_0=0$ gauge) to quantize motions of the system in the configuration space of minimum-energy solutions (the rationale is that there is a gap of order m_W between these and all other configurations). These motions are parametrized by a time-dependent, gauge-rotation angle $\alpha(t)$. The action is proportional to $\dot{\alpha}^2$, and the energy eigenstates are rotator-like states with nonzero expectation of $\dot{\alpha}$ (and therefore of $\vec{\mathrm{E}}$) and energies of order e^2m_W. If light fermions are present, the space of almost-zero-energy configurations is clearly enlarged. Now the fermions can couple strongly to the gauge-rotation angle α, and there is no longer any reason to expect α to have a simple rotator action. To escape the above-described paradox, we have only to show that the new effective action for α does not have solutions with nonzero expectation of $\dot{\alpha}$ (and therefore of E). All the relevant physics is contained in the problem of massless fermions interacting with gauge fields which are time-dependent gauge rotations of the basic monopole field.

The problem can be further simplified by ignoring all fermion variables except those belonging to a special partial wave. The rationale for this simplification is given in the next section. This strategy is borrowed from the work of Blaer *et al.*,[5] who have studied the way in which a dyon configuration with definite electric field begins to decay towards the dyon ground state. The following sections show how a similar strategy can be used to deal directly with the ground state of the dyon.

II. THE APPROXIMATION

For simplicity we work with the monopole of the Georgi-Glashow model interacting with one flavor of $I=\frac{1}{2}$ massless fermion. The monopole configuration is

$$A^0_{ia}(x)=\epsilon_{aij}\hat{x}_j A(r) ,$$

$$A(o)=0,\ \ A(r)=\frac{1}{r}\ \text{for}\ r\gtrsim\frac{1}{m_W} ,$$

$$\phi_a(x)=\hat{x}_a H(r) ,$$

where ϕ is the Higgs field. The relevant gauge rotations, $U_\lambda=\exp(i\lambda\hat{x}\cdot\vec{\tau}/2)$, are those which leave the Higgs field invariant. Consequently the gauge degrees of freedom in an $A_0=0$ gauge treatment reduce to the single function $\lambda(\vec{x},t)$. The fermions satisfy a Dirac equation which has been thoroughly analyzed by Jackiw and Rebbi.[6] A key point is that the angular momentum $\vec{J}=\vec{L}+\vec{S}+\vec{T}$ is conserved and can be used to do a partial-wave analysis. The $J=0$ partial wave is the only one for which the fermions are not kept away from the monopole core (of radius $r_0=m_W{}^{-1}$) by a centrifugal barrier. Since we are interested in states of energy much less than m_W, the centrifugal barrier will decouple all but the $J=0$ partial waves. Once we restrict our attention to s-wave fermions we may also limit ourselves to spherically symmetric gauge functions $\lambda(r,t)$.

The Dirac equation for ψ is

$$\left[\gamma_0\frac{\partial}{\partial t}+\gamma_i\left[\frac{\partial}{\partial x_i}-iA_i^\lambda\right]\right]\psi=0 ,$$

where

$$A_i^\lambda=U_\lambda A_i^0 U_\lambda{}^{-1}+iU_\lambda\nabla_i U_\lambda{}^{-1}$$

(in the $A_0=0$ gauge we have fermions propagating in a time-varying vector potential). The $J=0$ Fermi field may be decomposed as

$$\psi^{(\pm)}_{J=0}=\begin{bmatrix}X_\pm\\ \pm X_\pm\end{bmatrix} ,$$

$$X_\pm=\frac{1}{\sqrt{8\pi r}}(g_\pm+p_\pm\hat{x}\cdot\vec{\tau})\tau_2 ,$$

where $\pm$ refer to helicity and X is a 2×2 matrix, one index describing spin, the other describing isospin. The normalization of g and p is chosen so that they have anticommutation relations with standard normalization. When restricted to $J=0$, the Dirac equation is a set of equations for the functions g and p which may be written in the form

$$\left[\tau_3\left[\partial_t\mp i\frac{\lambda'}{2}\right]+i\tau_1\partial_r + \left[A-\frac{1}{r}\right]\tau_2 e^{\pm i\lambda\tau_2}\right]\chi_\pm=0 ,$$

$$\chi_\pm=\begin{bmatrix}g_\pm\\ \pm ip_\pm\end{bmatrix} .$$

The equations for the two helicities of course decouple since we are dealing with massless fermions. If we make the identification

$$\bar{\gamma}_0=\tau_3,\ \ \bar{\gamma}_1=i\tau_1,\ \ \bar{\gamma}_5=\bar{\gamma}_0\bar{\gamma}_1=\tau_2$$

(note that $\{\gamma_\mu,\gamma_\nu\}=2\bar{g}_{\mu\nu}$, where $\bar{g}_{\mu\nu}$ is the one-dimensional Minkowski metric), this equation clearly has the structure of the Dirac equation for a one-dimensional fermion interacting with an Abelian vector potential $A_\mu=\pm(\lambda'/2)\delta_{\mu0}$ and having a position-dependent "mass term"

$$M(r)=\left[A-\frac{1}{r}\right]\bar{\gamma}_5 e^{\mp i\lambda\bar{\gamma}_5} .$$

[$M(r)$ has nothing to do with the physical mass of the three-dimensional fermion.] The function $A(r)$ is such that $M(r)$ vanishes exponentially rapidly outside the monopole core ($r>r_0$). Since we are interested in states of energy small compared to m_W, the details of fermion wave functions for $r<r_0$ are irrelevant and it should be sufficient to set the mass term equal to zero and exclude the fermion from the interior of the monopole by imposing an appropriate boundary condition at $r=r_0$ (we could just as well have set the boundary at $r=0$ except that the finite core size provides a necessary cutoff on certain electromagnetic self-energies). The proper boundary condition[7] follows from the fact that $M(r)$ diverges as $-1/r$ at $r=0$: unless the *lower* component of χ vanishes just outside the monopole core, χ will have a nonintegrable singularity at $r=0$. In other words we want to set $\tau_+\chi(r_0)=0$. This is a standard bag boundary condition for one-dimensional massless fermions. In order to tame infrared singularities it will be convenient to impose the same boundary condition at a large radius R, as well. Eventually we will let $R\to\infty$.

To summarize, we have reduced the fermion system to two massless one-dimensional fermions (one for each physical helicity) confined to a box of size $R-r_0$ and interacting with an Abelian gauge field:

$$\bar{\gamma}_\mu(\partial_\mu-iA^\pm_\mu)\chi_\pm=0 ,$$

$$A_\mu^{\pm}=\pm\frac{\lambda'}{2}\delta_{\mu 0}\ ,$$

$$r_0<x_1\le R\ ,$$

$$\tau_+\chi=0 \text{ at } x_1=r_0, R\ .$$

For future reference, we will want to know what certain physical current densities are in terms of the pseudo one-dimensional Fermi fields $\chi_\pm$. Outside the monopole core, the electric charge direction in isospin space is parallel to the Higgs field $[\phi_a(x)=\hat{x}_a H(r)]$. Consequently, the electric charge and the *radial* electric current densities are

$$J_0=\bar{\psi}\hat{x}\cdot\frac{\vec{\tau}}{2}\gamma_0\psi\ ,$$

$$J_r=\bar{\psi}\hat{x}\cdot\frac{\vec{\tau}}{2}\hat{x}\cdot\vec{\gamma}\psi\ ,$$

where the γ matrices are the *four*- dimensional ones. They may be written in terms of the $J=0$ fields as follows:

$$J_0=-\frac{1}{8\pi r^2}(\bar{\chi}_+\bar{\gamma}_1\chi_+-\bar{\chi}_-\bar{\gamma}_1\chi_-)\ ,$$

$$J_r=-\frac{1}{8\pi r^2}(\bar{\chi}_+\bar{\gamma}_0\chi_+-\bar{\chi}_-\bar{\gamma}_0\chi_-)\ .$$

III. AN EFFECTIVE ACTION

Having reduced the fermion sector to a Schwinger model, we should be able to solve it using Schwinger's trick. Write

$$\chi_\pm=e^{i(a_\pm\gamma_5+b_\pm)}\chi_\pm^{(0)}\ ,$$

where $\chi^{(0)}$ satisfies the free Dirac equation with the boundary condition $\tau_+\chi^{(0)}=0$ at $r=r_0$, R. Then it is easy to show that $\chi_\pm$ satisfies an interacting one-dimensional Dirac equation if

$$\dot{a}+b'=0,\quad a'+\dot{b}=\pm\frac{\lambda'}{2}$$

or

$$\Box b=\pm\frac{\dot{\lambda}'}{2},\quad \Box a=\mp\frac{\lambda''}{2}\ ,$$

$$\Box=\partial_t{}^2-\partial_r{}^2\ .$$

In order that $\chi_\pm$ satisfy the chosen boundary condition, we need only impose the condition

$$a(r_0)=a(R)=0\ .$$

The boundary condition on b implied by the equation $\dot{a}+b'=0$ is then

$$b'(r_0)=b'(R)=0\ .$$

The functions a and b are completely determined by the gauge function λ and we may take any one of a, b, or λ as our independent variable describing the gauge field configuration.

Our strategy will be to integrate out the fermions and derive an effective action for the gauge field. In order to do this we need the currents induced in the vacuum by a given gauge field $\lambda(r,t)$. We make use of the usual point-separation definition

$$\langle\bar{\chi}\bar{\gamma}_\mu\chi(x)\rangle=\lim_{y\to x}[F(y,x;A)-F(y,x;0)]\ ,$$

$$F(x_1,x_2)=\exp(i\eta\int_l^2 dx\cdot A)$$
$$\times\operatorname{tr}[\bar{\gamma}_\mu\langle T(\chi(x_1)\bar{\chi}(x_2))\rangle]\ ,$$

where χ satisfies the one-dimensional Dirac equation $\bar{\gamma}_\mu(\partial^\mu-iA^\mu)\chi=0$; in taking the limit $y\to x$ one averages over the direction of $\Delta=y-x$; the coefficient η is chosen to ensure conservation of the physical charge current. In view of the relation between J_0, J_r and the one-dimensional current $K_\mu=\bar{\chi}\bar{\gamma}_\mu\chi$, we want to impose $\epsilon^{\mu\nu}\partial_\mu K_\nu=0$ (i.e., that the one-dimensional axial-vector current is conserved). This is achieved by setting $\eta=1$. The result of carrying out the point-separation procedure is

$$\langle\bar{\chi}\bar{\gamma}_0\chi\rangle=-\frac{1}{\pi}\dot{b}\ ,$$

$$\langle\bar{\chi}\bar{\gamma}_1\chi\rangle=\frac{1}{\pi}b'\ .$$

As in the usual Schwinger model, the fact that these currents are *linear* in the gauge field is the key to being able to compute the fermion determinant.

Now we must calculate the action functional, or fermion determinant, for a given gauge field time history. Recall that in ordinary electrodynamics, the variation of the fermion determinant under small variations of the gauge field is given by

$$\delta\ln\det[\gamma_\mu(\partial_\mu-iA_\mu)]=i\int d^4x\,\delta A^\mu\langle J_\mu\rangle_A\ ,$$

where $\langle J_\mu\rangle_A$ is the vacuum expectation of the charge current for the given vector potential. If $\langle J_\mu\rangle_A$ is linear in A, this is instantly soluble for the determinant,

$$\ln\det[\gamma_\mu(\partial_\mu-iA_\mu)]=\frac{i}{2}\int d^4x\,A^\mu\langle J_\mu\rangle_A\ .$$

In our problem the time-varying gauge function $\lambda(r,t)$ can be interpreted as producing a radial Abelian vector potential $A_r=-\lambda'$ (and a radial Abelian electric field $E_r=-\dot{\lambda}'$) outside the monopole core. We have already calculated the electric current induced by λ and found it to be linear in λ. We can therefore use the determinant formula derived above to evaluate the fermionic effective action

$$S_\psi=-\tfrac{1}{2}\int dt\int 4\pi r^2 dr\lambda'\langle J_r\rangle_\lambda$$
$$=\tfrac{1}{2}\int dt\int 4\pi r^2 dr\lambda'\frac{1}{8\pi r^2}(\langle\bar{\chi}_+\gamma_0\chi_+\rangle_\lambda$$
$$-\langle\bar{\chi}_-\gamma_0\chi_-\rangle_{-\lambda})$$
$$=-\frac{1}{2\pi}\int_{r_0}^{R} dt\,dr\lambda'\dot{b}\ .$$

Note that because of various sign changes previously noted, the two helicity states make equal contributions. Making use of $\Box b=\dot{\lambda}'/2$ and the fact that $b'=0$ on the spatial boundaries, we may integrate by parts to cast this into the very suggestive form

$$S_\psi=-\frac{1}{\pi}\int dt\int_{r_0}^{R} dr(\partial_\mu b)^2$$

To get the full action we must add the Coulomb energy of the radial electric field produced by λ,

$$S_{\text{Coul}}=\frac{1}{2e^2}\int dt\int 4\pi r^2 dr(\dot{\lambda}')^2$$
$$=\frac{8\pi}{e^2}\int dt\int_{r_0}^{R} dr r^2(\Box b)^2\ .$$

The full effective action is the sum of S_ψ and S_{Coul}:

$$S_{\text{eff}}(b)=\frac{8\pi}{e^2}\int dt\int dr r^2(\Box b)^2$$
$$-\frac{1}{\pi}\int dt\int dr\,\partial_\mu b\,\partial^\mu b\ .$$

Since it is quadratic, we should be able to solve it to find the low-lying excitations of the monopole/massless fermion system.

We have chosen to express S_{eff} as a functional of the derived variable b. When varying S_{eff} with respect to b it is important to impose the surface boundary condition $b'(r_0)=b'(R)=0$. This means that δb is a free variation on the surface, while $\delta b'$ must be zero on the surface. The differential equation and boundary conditions which follow from the variational principle are readily found to be

$$\frac{8\pi^2}{e^2}\Box(r^2\Box b)+\Box b=0\ ,$$
$$b'=0,\quad (r^2\Box b)'=0 \text{ at } r=r_0,R\ ,$$

where

$$\Box=\partial_t{}^2-\partial_r{}^2\ .$$

Before solving these equations, let us see how the conventional dyon solution arises. It should emerge if we neglect S_ψ, the response of the fermion vacuum to the gauge field. This looks superficially reasonable if e^2 is small. The equation to solve (with the same boundary conditions as before) is then

$$\Box(r^2\Box b)=0\ .$$

The solution for b and the corresponding solution for λ are

$$b=\frac{\alpha t^2}{4}+\frac{\alpha(r^2-2rr_0)}{4r}-\frac{\alpha r}{2}+\frac{\alpha r_0}{2}\ln r\ ,$$
$$\lambda=\alpha t\left[\frac{1}{r_0}-\frac{1}{r}\right],$$

where α is arbitrary. The corresponding electric field is $E_r=\dot{\lambda}'=\alpha/r^2$—a Coulomb field proportional to the growth rate of λ. This is the conventional dyon solution. For what comes next we want to note that the dyon solution is a *zero-frequency* normal mode—had it been of nonzero frequency, E_r would have been oscillatory in time and have had *zero* expectation value.

IV. SOLVING THE EFFECTIVE ACTION

Now to solve the full equation. We note that a solution of

$$\left[\Box+\frac{e^2}{8\pi^2 r^2}\right]b=0\ ,$$
$$b'(r_0)=b'(R)=0\ ,$$

automatically solves the full equation with its larger set of boundary conditions and must be the solution we want. We look for solutions of the form $b=e^{i\omega t}f(\omega r)$ and find that $f(x)$ is a Bessel function,

$$\left[\frac{d^2}{dx^2}+1-\frac{e^2}{8\pi^2 x^2}\right]f(x)=0,\quad x=\omega r$$
$$f(x)=\left[\frac{\pi x}{2}\right]^{1/2}J_\nu(x),\quad \nu^2=\frac{1}{2}+\frac{e^2}{8\pi^2}\ .$$

The boundary conditions determine the spectrum of allowed values of ω through a transcendental equation of a type familiar from quantum-mechanical scattering theory. If we write $\omega=\eta/R$ and $\alpha=e^2/4\pi$, the eigenvalue equation takes the form

$$\left[\frac{\alpha}{2\pi}\frac{R}{r_0}\right]\frac{1}{\eta}=\frac{\sin(\eta+\alpha)}{\cos(\eta-\alpha)} .$$

The essential results can be easily inferred from a graph of this equation. In the limit $R\rightarrow\infty$ (α fixed) the spectrum is

$$\omega_n=[(n+\tfrac{1}{2})\pi+\delta(\omega)]\frac{1}{R},\quad n=0,1,\ldots$$

$$\delta(\omega)\rightarrow\alpha \text{ for } \omega<<\alpha r_0{}^{-1} ,$$

$$\delta(\omega)\rightarrow\frac{\pi}{2}-\alpha \text{ for } \omega>>\alpha r_0{}^{-1} .$$

In the limit $\alpha\rightarrow 0$ (R fixed) the spectrum is

$$\omega_n=n\pi\frac{1}{R},\quad n=0,1,\ldots$$

(the continuum loses one state which becomes a zero-frequency mode). Only in this unphysical limit is there a zero-frequency mode which can be interpreted as a dyon state. In the physically relevant limit, this mode is driven into the continuum and is not to be distinguished from the other continuum states. The essential point is that if $\alpha>0$, there is *no* zero-frequency mode.

The standard dyon is recognizable by its long-range Coulomb field and we should ask what the electric field carried by our normal modes looks like at large distances. We have in general that $E_r=\dot{\lambda}'=2\Box b$. Because the normal modes satisfy $(\Box+e^2/8\pi^2r^2)b=0$, we have that $E_r=-(e^2/4\pi^2r^2)b$. It is easy to establish from the equation for b that the asymptotic behavior of b is $b\sim e^{i\omega t}\cos[\omega r+\Delta(\omega)]$. Although the electric field energy density $E_r{}^2$ is very similar to that of the dyon, the electric field itself has zero average value so long as $\omega>0$. Since ω is always positive, we say that the dyon disappears when the monopole couples to massless fermions.

To understand what is going on, it is helpful to have a physical picture of the normal modes we have been discussing. S-wave fermions are spherical shells of charge converging on, or receding from, the monopole at the speed of light. The b field evidently describes *pairs* of such particles. In the region between the two shells of such a pair there is a Coulomb field whose sign depends on the sign of the charge of the outermost particle. When a converging pair of shells scatters from the monopole and emerges as an outgoing pair the role of leading and following particle would normally be interchanged and the sign of the Coulomb field reversed. However, in the $J=0$ partial wave of a monopole the fermions *necessarily* change the sign of their charge when scattering from the monopole.[8] Thus a given pair has the same-sign Coulomb field between the two particles, whether it is converging on the monopole or receding from it. Furthermore, the sign of the field turns out to depend on the chirality of the particles making up the pair. Therefore the Coulomb field perceived far from the monopole depends crucially on the state of the fermion vacuum (i.e., the extent to which such pairs are present and their chirality properties). The method we have developed gives us a convenient way of describing this vacuum and its excitations.

V. CHIRAL SYMMETRY BREAKING AND FERMION MASS

Eventually we want to study the effect of a fermion mass term on all of this. As a first step, let us look at the two-point function of the density $D=\overline{\psi}_+\psi_-$. Treating D as we have treated the charge currents gives

$$D=\frac{1}{8\pi r^2}e^{2ib}D^{(0)} ,$$

$$D^{(0)}=\overline{\chi}_+^{(0)}\chi_-^{(0)} .$$

The two-point function is then

$$\langle D^\dagger(x_1)D(x_2)\rangle=\frac{1}{64\pi^2r_1{}^2r_2{}^2}\langle\exp\{2i[b(x_1)-b(x_2)]\}\rangle\langle D^{(0)\dagger}(x_1)D^{(0)}(x_2)\rangle .$$

The expectation of the scalar fields has to be taken using $S_{\text{eff}}(b)$ as the measure, while the $D^{(0)}$ expectation value is constructed using free Fermi propagators. The result is

$$\langle D^\dagger(x_1)D(x_2)\rangle=\frac{1}{64\pi^2r_1{}^2r_2{}^2}\frac{\exp[-2\langle b(x_1)b(x_1)\rangle]\exp[-2\langle b(x_2)b(x_2)\rangle]\exp[4\langle b(x_1)b(x_2)\rangle]}{2\pi^2(x_1-x_2)^2} ,$$

where $\langle b(x_1)b(x_2)\rangle$ is the propagator following from S_{eff}. That propagator can easily be seen to satisfy

$$\langle b(x_1)b(x_2)\rangle = G_0(x_1,x_2) - G_{e^2}(x_1,x_2) ,$$

where

$$\Box G_0 = \frac{\pi}{2}\delta(x_1-x_2) ,$$

$$\left[\Box + \frac{e^2}{8\pi^2 r^2}\right] G_{e^2} = \frac{\pi}{2}\delta(x_1-x_2) ,$$

and

$$\frac{\partial}{\partial r}G_0 = \frac{\partial}{\partial r}G_{e^2} = 0 \text{ at } r=R,\ r_0 .$$

Note that because S_{eff} is a higher-than-second-derivative action, $\langle b(x_1)b(x_2)\rangle$ is not singular at $x_1 = x_2$. With a little bit of work we can show that

$$\langle b(r,t)b(r,t)\rangle = \tfrac{1}{2}\ln\frac{r}{R} + C_1 .$$

We will also need the result

$$\langle b(r,t)b(r',0)\rangle \underset{t\to\infty}{\sim} \tfrac{1}{2}\ln\frac{t}{R} + C_2$$

(the quantities C_1 and C_2 are definite numerical constants). Putting all this together, we can show that

$$\langle D^+(r,t)D(r',0)\rangle \underset{t\to\infty}{\rightarrow} \frac{C}{r^3 r'^3}$$

(i.e., the dependences on t *and* R both cancel). This argument is not quite complete as we have not computed the determinant of the b field and it could depend on R. There are related examples where this happens.[9] By the cluster theorem, the above behavior of the two-point function means that

$$\langle D(r,t)\rangle = \frac{\bar{C}}{r^3} .$$

In other words, the monopole is the center of a region of chiral-symmetry breaking (driven by the anomaly) which is suppressed only by a power of the distance from the center of the monopole. External fermions can presumably scatter from this condensate with symmetry-breaking effects which would be very interesting to examine, especially for SU(5) monopoles where one might hope to see mixing of leptons and quarks.

Another important question is the effect of finite fermion masses on all of this. The above arguments and the bosonization treatment of the Schwinger model suggest that the fermion mass should be accounted for by adding a term proportional to $m \cos 2b$ to the effective action density. The normal modes now interact with each other and the system is no longer soluble. Furthermore, the action is no longer invariant to the addition of a constant to b—i.e., to global chiral rotations. Since the effect of a θ (vacuum angle) term can be implemented by a global chiral rotation, this means that the energies will now depend on θ. Once energies depend on θ, there can be an expectation value of $\vec{E}\cdot\vec{B}$ and the Coulomb field of the dyon can reappear. Understanding how it turns on as we increase the fermion mass from zero and how the standard dyon picture is recovered as the mass becomes large is a question to which we will return.

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