# Cosmological bounds on the masses of stable, right-handed neutrinos

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Previous authors have used the present mass density of the universe to constrain the masses of stable, left-handed neutrinos. We generalize these arguments to constrain the masses of stable, right-handed neutrinos which interact with strength $G \lesssim G_F$. Light neutrinos ($\lesssim 1$ MeV) are constrained to be less massive than $\sim 100$ eV–2 keV, depending upon $G$ and whether they are of the Dirac or Majorana type. Heavy neutrinos must be more massive than $\simeq (G_F/G)$ GeV, and less massive than about 10 TeV.

Within the context of the standard hot big-bang model[1] previous authors have used the present mass density of the universe to constrain the allowed masses of stable, left-handed neutrinos. If they are not to contribute too much mass density today ($\Omega \leqslant 2$), then for light neutrinos ($\leqslant 1$ MeV) the sum of their masses must be $\leqslant 200$ eV (Ref. 2), and for heavy neutrinos each must be more massive than ~1 GeV and less massive than ~4 TeV (Refs. 3 and 4). As we shall see, the upper limit (~4 TeV) depends upon the details of the interactions; a similar upper bound can be obtained for any stable particle, regardless of its interactions, that has a particle-antiparticle asymmetry of comparable magnitude to the baryon asymmetry, $n_B/n_\gamma \simeq 10^{-10\pm 1}$ (Refs. 5 and 6).

In this paper we generalize the previous results to stable ($\tau \gg \tau_{\rm universe} \sim 10^{18}$ sec), right-handed neutrinos which interact with effective strength $G \leqslant G_F \simeq 1.15 \times 10^{-5}$ GeV$^{-2}$. Such particles are predicted in many theories, particularly those with right-left symmetry.[7] The arguments we present can be generalized to make them applicable to any stable, weakly interacting neutral particle.

We assume that the temperature of the universe was once much greater than max $[m_R,\ 1\ {\rm MeV}\ (G_F/G)^{2/3}]$, so that these weakly interacting right-handed neutrinos and their antiparticles (denoted by $R$ and $\bar{R}$) were once present in equilibrium numbers, i.e., with a number density $n_0(T)$ for $R$ and $\bar{R}$ given by

$$n_0(T) = \frac{1}{(2\pi)^3}\int_0^\infty 4\pi p^2 dp \left\{ \exp\left[\frac{(m_R{}^2 + p^2)^{1/2}}{T}\right] + 1 \right\}^{-1} . \tag{1}$$

Throughout we shall use units such that $\hbar = k_B = c = 1$. We then compute their present contribution to the mass density $\rho_R$ and insist that it be less than the observed mass density of the universe:

$$\rho_R \leqslant (1.88 \times 10^{-29}\ {\rm g\ cm^{-3}})\Omega h^2$$
$$\leqslant (4 \times 10^{-29}\ {\rm g\ cm^{-3}})(\Omega h^2/2) , \tag{2}$$

where $\Omega$ is the ratio of the density of the universe to the critical density, and the Hubble parameter $H_0 = 100\ h\,{\rm km\,sec^{-1}Mpc^{-1}}$. We shall take $\Omega h^2$ to be less than 2; however, we explicitly exhibit how our results scale with $\Omega h^2$.

Light neutrinos (<1 MeV) decouple at a temperature $T_d \geqslant 1$ MeV, i.e., for $T \leqslant T_d$ their interaction rate $\Gamma$ is less than the expansion rate $H \equiv \dot{R}/R$, so that the total number of $R,\bar{R}$'s has not changed since their decoupling. Full-strength neutrinos (i.e., $G \simeq G_F$) decouple when $T \simeq 1$ MeV. Since $\Gamma \sim G^2 T^5$ and $H \sim G_N{}^{1/2} g_*{}^{1/2}(T) T^2$, $T_d \simeq 1$ MeV $(G_F/G)^{2/3}$ [ignoring the weak dependence on $g_*(T_d)$]. Here $g_* = \sum(g_{\rm boson} + \frac{7}{8} g_{\rm fermion})$ counts the total number of degrees of freedom of all the relativistic species and $G_N = 6.74 \times 10^{-39}$ GeV$^{-2}$ is Newton's constant. When they decoupled, $R,\bar{R}$'s were relativistic ($T_d \gg m_R$), and their combined (i.e., $R$ and $\bar{R}$) number density was a factor of $\frac{3}{4}$ less than that of the photons, due to Fermi statistics. However, after decoupling, the photons increased in number when various species annihilated (e.g., $e^\pm$ at $T \simeq \frac{1}{2}$ MeV). From entropy conservation it is simple to calculate the present number density of $R,\bar{R}$'s in terms of the temperature they would have today if they were massless, $T_R$, and $T_\gamma$, the present photon temperature ($2.7\ {\rm K} \leqslant T_\gamma \leqslant 3.0$ K)

$$(T_R/T_\gamma)^3 = 3.9/g_*(T_d) , \tag{3}$$

where $g_*(T_d)$ counts only those relativistic species still in thermal contact with the photons.[8] For full-strength neutrinos this yields the familiar result $(T_\nu/T_\gamma)^3 = \frac{4}{11}$. The contribution of $R,\bar{R}$'s to the present mass density is

 

$$\rho_R = m_R (T_R/T_\gamma)^3 \tfrac{3}{4} n_\gamma$$
$$\geq 5.3\times 10^{-31} [m_R\ (\mathrm{eV})] \left(\frac{T_R}{T_\gamma}\right)^3 \mathrm{g\,cm^{-3}}, \qquad (4)$$

where $n_\gamma \geq 400\ \mathrm{cm}^{-3}$ since $T_\gamma \geq 2.7$ K. The cosmological constraint on $m_R$ depends on whether $R$ is a "Dirac" (four-component neutrino with mass term $m\bar{\nu}_R \nu_L$), or a "Majorana" (two-component neutrino with mass term $m\nu_R\nu_R$) species.

If $R$ is a Majorana species, then from (2) we obtain

$$\sum_R m_R (T_R/T_\gamma)^3 \leq (71\ \mathrm{eV})(\Omega h^2/2), \qquad (5)$$

where $\sum_R$ is the sum over all such species. Note that for full-strength neutrinos (5) reduces to $\sum m_\nu \leq (200\ \mathrm{eV})(\Omega h^2/2)$—the usual bound. We have obtained $(T_R/T_\gamma)$ as a function of $(G_F/G)$ from Ref. 8, and in Fig. 1 we show the upper bound on $\sum_R m_R$ as a function of $(G_F/G)$. In the model of Mohapatra and Senjanovic[7] $m_{Ri} \simeq m_i^2/gM_{W_R}$ $(i=e,\mu,\tau)$, and they use the overly restrictive cosmological bound of $m_\nu < 10$ eV to obtain $M_{W_R} \gtrsim 3\times 10^8$ GeV. From Fig. 1 we obtain $M_{W_R} \gtrsim 3\times 10^6$ GeV.

It is interesting to note that for $G_F/G \geq 3\times 10^2$, $m_R$ can be as large as 1 keV or so. This is of some interest for galaxy formation in a neutrino-dominated universe. Because of neutrino free-streaming, perturbations in the neutrinos on scales $\leq 4\times 10^{18}\ M_\odot/[m_\nu\ (\mathrm{eV})]^2$ are damped.[9] For a full-strength neutrino of mass ~200 eV this scale is $\sim 10^{14}\ M_\odot \gg M_{\mathrm{galaxy}} \simeq 10^{12}\ M_\odot$. However, for a weakly interacting, right-handed neutrino the damping scale becomes $\sim 4\times 10^{18}\ M_\odot [m_R\ (\mathrm{eV})]^{-2} \times (T_R/T_\nu)^3$—the factor of $(T_R/T_\nu)^3$ arising because $R,\bar{R}$'s are less abundant than full-strength neutrinos by this factor. For $m_R \simeq 1$ keV and $G_F/G > 3\times 10^2$, $(T_R/T_\nu)^3 \sim 10^{-1}$, so that the damping mass is $\sim 4\times 10^{11}\ M_\odot \sim M_{\mathrm{galaxy}}$. Thus it is possible for galaxies to form from initial perturbations in the neutrino sea of such a species.

If $R$ is a Dirac species, then there exists a left-handed counterpart to $R$ with mass $m_L = m_R$, and if it interacts with strength $G_F$ as other left-handed species do, the limit which results from the contribution of both the right and left components to the present mass density is

$$\sum_R m_R [1 + \tfrac{11}{4}(T_R/T_\gamma)^3] \leq (200\ \mathrm{eV})(\Omega h^2/2). \qquad (6)$$

The corresponding bounds on $\sum_R m_R$ as a function of $(G_F/G)$ are also shown in Fig. 1. We mention in passing that big-bang nucleosynthesis restricts the effective number of light, two-component neutrino species to $\leq 4$ (Refs. 5, 10, and 11). Since three left-handed neutrino species are already

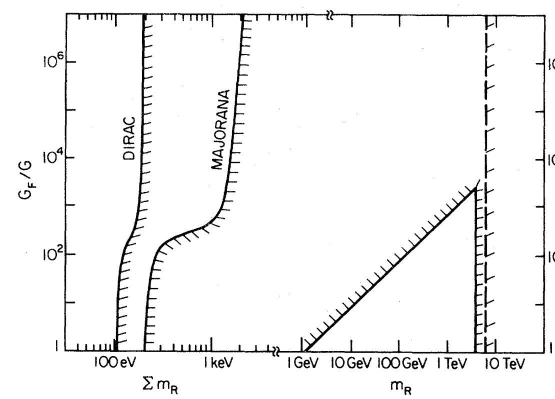


FIG. 1. Cosmological constraints from the present mass density of the universe on the possible masses of stable, right-handed neutrinos as a function of interaction strength $G$. Allowed masses are to the left of the curves marked "Dirac" (for Dirac neutrinos) and "Majorana" (for Majorana neutrinos), and inside the triangular region. No stable species with a particle-antiparticle asymmetry of the same magnitude as $n_B/n_\gamma$ more massive than ~5 TeV is permitted (indicated by the broken line). The abrupt change in the "Dirac" and "Majorana" curves for $G_F/G \sim 3\times 10^2$ is a result of the fact that for $G_F/G \geq 3\times 10^2$ these neutrinos decouple *before* $\mu^\pm$ and $\pi^\pm$, $\pi^0$ annihilations, and hence are ~3 to 4 times less abundant (relative to $\gamma$'s) today than a species which decouples after these species annihilate $(G_F/G < 3\times 10^2)$.

known to exist, if these species are Dirac species, then their right-handed components must interact very weakly ($G_F/G > 3\times 10^2$, Ref. 8) so that they do not contribute more than 1 to the total number of effective species. For an arbitrary, weakly interacting species which decouples at a temperature $T_d \gtrsim m$ (i.e., the rate of its thermalizing interactions $\simeq H$ for $T = T_d$), the bound on $m$ is obtained from (5), where a factor of $g$ (= number of degrees of freedom of the species) must be included in the sum.

The story for heavy $R,\bar{R}$'s (>1 MeV) is somewhat different because these neutrinos decouple at a temperature $< m_R$, and hence can annihilate, resulting in a lower final abundance than light (<1 MeV) neutrinos. As Lee and Weinberg[3] pointed out their final abundance is not just $\sim n_\gamma \exp(-m_R/T_d)$, because although they remain in thermal contact with the rest of the universe by their weak interactions (e.g., $Re^- \rightarrow Re^-$, etc.), their annihilations (via channels such as $R\bar{R}\rightarrow e^\pm$, $\mu^\pm, \pi^\pm$, etc.), which allow their number density "to track" its equilibrium value [given by (1)], "freeze out" at a higher temperature $(T_f \sim \frac{1}{20} m_R)$, and they drop out of chemical equilibrium at this temperature. For $T \leq T_f$ their total number remains constant.

To be quantitative, one must solve the rate (Boltzmann) equation for their number density $n(T)$,

$$\frac{dn}{dt} = -3\frac{\dot{R}}{R}n - \langle\sigma v\rangle(n^2 - n_0^2)\,, \tag{7}$$

where $\langle\sigma v\rangle$ is the thermal average of their annihilation cross×relative velocity, and $\dot{R}/R = -\dot{T}/T = (4\pi^3 G_N/45)^{1/2} g_*^{1/2}(T)T^2$. Following Lee and Weinberg,[3] for $m_R \leq M_{W_R}$ [$M_{W_R}$ is the mass of the gauge boson(s) which mediates the interactions of $R$] we take

$$\langle\sigma v\rangle = G^2 m_R^2 N_A/2\pi\,, \tag{8}$$

where $N_A$ is a dimensionless factor of $O(10)$ which depends upon the details of $R\bar{R}$ annihilations and counts the number of annihilation channels (e.g., if $m_R \sim 5$ GeV, the number of channels $\geq 18$, $R\bar{R} \to e^\pm, \mu^\pm, \tau^\pm, u\bar{u}, d\bar{d}, c\bar{c}, s\bar{s}, b\bar{b}$). For $m_R \geq M_{W_R}$, $m_R^2$ rather than $M_{W_R}^2$ dominates the propagator and we use

$$\langle\sigma v\rangle = G^2(M_{W_R}^4/m_R^2)N_A/2\pi\,. \tag{9}$$

Without a specific model one cannot relate $G$ and $M_{W_R}$; wherever necessary we shall assume $M_{W_R} \simeq (G_F/G)^{1/2} M_W$. Equation (7) can be recast in a more useful form:

$$df/dx = C\mu^3(f^2 - f_0^2)\,, \tag{10}$$

$$x = T/m_R\,,\quad C = G_F^2\left(\frac{45}{32\pi^5 G_N}\right)^{1/2}\,,$$
$$f_0(x) = n_0/T^3\,,\quad f(x;\mu) = n/T^3\,, \tag{11}$$

$$\mu^3 = (G/G_F)^2 m_R^3 N_A (g_f/2)^{-1/2} \times \begin{cases} 1, & m_R \leq M_{W_R} \\ (M_{W_R}/m_R)^4, & m_R \geq M_{W_R} \end{cases}\,, \tag{12}$$

where $g_f \simeq g_*(T_f \simeq m_R/20)$. The present number density of $R, \bar{R}$ is then just $2(T_R/T_\gamma)^3 f(0;\mu)T_\gamma^3$, the factor of $(T_R/T_\gamma)^3$ accounts for the increase in photons due to species which annihilate for $T < T_f$, and $(T_R/T_\gamma)^3 \simeq 3.9/g_f$. Using the numerical solution to (10) from Ref. 3 we obtain

$$\rho_R \gtrsim (1.6\times10^{-27}\ \mathrm{g\,cm^{-3}})(G_F/G)^{1.9}[m_R\ (\mathrm{GeV})]^{-1.85} N_A^{-0.95}(g_f/2)^{-0.52}\quad (m_R \leq M_{W_R})\,, \tag{13a}$$

$$\rho_R \gtrsim (9.6\times10^{-35}\ \mathrm{g\,cm^{-3}})[m_R\ (\mathrm{GeV})]^{1.95} N_A^{-0.95}(g_f/2)^{-0.52}\quad (m_R \geq M_{W_R})\,. \tag{13b}$$

For a variety of models for $N_A$ and $m_R = 0.1$ GeV–10 TeV we find that $N_A^{-0.95}(g_f/2)^{-0.52} \simeq 1.3$–$4.7\times10^{-2}$, and using $N_A^{-0.95}(g_f/2)^{-0.52} \simeq 3\times10^{-2}$ we obtain the following constraints on $m_R$:

$$m_R \geq (1.2\ \mathrm{GeV})(G_F/G)^{1.03}(\Omega h^2/2)^{-0.54}[N_A^{-0.51}(g_f/2)^{-0.28}/0.15]\quad (m_R \leq M_{W_R})\,, \tag{14a}$$

$$m_R \leq (3.6\ \mathrm{TeV})(\Omega h^2/2)^{0.51}[0.15 N_A^{0.49}(g_f/2)^{0.27}]\quad (m_R \geq M_{W_R})\,. \tag{14b}$$

These constraints are shown in Fig. 1. These results are only valid for Majorana neutrinos. If $R$ were a Dirac species, then during the epoch of annihilation, $m_R \geq T \geq T_F$, the $R, \bar{R}$'s are nonrelativistic, and the right and left components are mixed by the Dirac mass term, so that a Dirac species $R$ would, for all intents and purposes, interact with strength $G_F$, and the results of Lee and Weinberg[3] would be applicable. Note that both these constraints [(14a) and (14b)] are model dependent since they depend on $N_A$ and $g_f/2$; in addition, (14b) depends upon the precise relationship between $G$ and $M_{W_R}$ which we have assumed is $M_{W_R} \simeq M_W(G_F/G)^{1/2}$. These results also apply to an arbitrary, weakly interacting species whose annihilation rate is given by an expression similar to (8).

If the species $R$ possesses an $R$-$\bar{R}$ asymmetry of similar magnitude as the baryon asymmetry we can obtain another upper limit on $m_R$. We shall assume that $|(n_R - n_{\bar{R}})/n_\gamma| \simeq n_B/n_\gamma \simeq 10^{-10\pm1}$ (Ref. 5). The contribution to the mass density due to this asymmetry is given by

$$\rho_R \geq [m_R\ (\mathrm{GeV})] \times 7.1\times10^{-33}\ \mathrm{g\,cm^{-3}}\,, \tag{15}$$

and results in the constraint

$$\sum m_R \leq (5\ \mathrm{TeV})(\Omega h^2/2)\,, \tag{16}$$

which is shown as the broken line in Fig. 1. We note that this bound is comparable to that obtained in (14b); however, it is independent of the details of $R$-$\bar{R}$ annihilations, etc., and is valid for any stable species that has an asymmetry of the same magnitude as $n_B/n_\gamma$.

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