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Aspects of the locality of bilocal fields

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We show that a Lorentz-invariant "bilocal" field whose mass-squared operator has a complete set of bound-state eigenfunctions can be local in its center-of-mass coordinate only if it is a sum of products of local fields and c-number functions.

When Yukawa[1] introduced the concept of bilocal fields he defined them simply as fields depending on two space-time variables. It seems, however, that a more restrictive definition with more emphasis on the word "local" in "bilocal" is necessary. Just what the actual definition of "bilocal" should be has not yet been settled.

A possible definition of bilocality modeled on the conventional notion of locality was given by Capri and Chiang.[2] Somewhat later[3] explicit fields displaying this kind of locality were constructed. The definition in this case was based on the notion that the space-time coordinates of a bilocal field somehow mimic the presence of constituents such as quarks. Therefore fields $\phi(x_1,x_2)$, $\phi(y_1,y_2)$ were defined as relatively bilocal if they commuted whenever both coordinates of one were spacelike with respect to both coordinates of the other:

$$[\phi(x_1,x_2),\phi(y_1,y_2)]=0 \text{ if } (x_i-x_j)^2<0 \tag{1}$$

for all possible i,j.

Another notion of bilocality that is implied in several papers[4] on multilocal fields is to have two fields relatively bilocal if they commute whenever their centers of mass are spacelike or

$$[\phi(X,x),\phi(Y,y)]=0 \text{ if } (X-Y)^2<0 \,. \tag{2}$$

The purpose of this note is to show that this second kind of bilocality can occur for a "free bilocal" field only if the field $\phi(X,x)$ is simply a sum of products of local fields $\phi_{n,l}(x)$ and c-number functions $f_{n,l}(x)$:

$$\phi(X,x)=\sum f_{n,l}(x)\phi_{n,l}(X) \,. \tag{3}$$

A further assumption required is that the mass-squared operator has a complete set of bound-state eigenfunctions as is, for example, the case in the model described by Capri and Chiang.[2] Thus it follows that the bilocality of such fields is essentially trivial.

The reason for wanting such locality is that it facilitates the construction of advanced and retarded Green's functions Δ_A, Δ_R satisfying the condition

$$\Delta(X-Y;x,y)=\Delta_A(X-Y;x,y)-\Delta_R(X-Y;x,y) \,, \tag{4}$$

where

$$i\Delta(X-Y;x,y)=[\phi(X,x),\phi(Y,y)] \,. \tag{5}$$

In fact one can simply define

$$\Delta_A(X-Y;x,y)=\theta(-X^0+Y^0)\Delta(X-Y;x,y) \,, \tag{6}$$

$$\Delta_R(X-Y;x,y)=-\theta(X^0-Y^0)\Delta(X-Y;x,y) \,.$$

To prove the assertion made above, we assume that $\phi(X,x)$ satisfies the Heisenberg equation

$$(P^2-M^2)\phi(X,x)=0 \,, \tag{7}$$

where

$$P_\mu=-i\frac{\partial}{\partial X^\mu} \tag{8}$$

and M^2 is the mass-squared operator and hence a Lorentz scalar. We further assume that M^2 is self-adjoint and has a complete set of bound-state eigenfunctions which may, for convenience, be chosen real:

$$M^2 f_{n,l}(p,x)=m_n{}^2 f_{n,l}(p,x) \,. \tag{9}$$

Here p is the eigenvalue of P, and l is a label distinguishing mass-degenerate states. The labels n and l are Lorentz scalars and the eigenfunctions are also assumed Lorentz invariant.

The field $\phi(X,x)$ is further assumed to have a Fock-space representation built up from the "one-particle" states:

$$f_{n,l}(p_n,x)e^{-ip_n\cdot X} ,$$

where

$$p_n{}^2=m_n{}^2 . \tag{10}$$

Thus we introduce the following annihilation and creation operators:

$$[a_{n,l}(\vec{p}),a_{n',l'}(\vec{p}\,')]=[a^\dagger_{n,l}(\vec{p}),a^\dagger_{n',l'}(\vec{p}\,')]=0 , \tag{11}$$

$$[a_{n,l}(\vec{p}),a^\dagger_{n',l'}(\vec{p}\,')]=\omega_n(\vec{p})\delta(\vec{p}-\vec{p}\,')\delta_{n,n'}\delta_{l,l'} , \tag{12}$$

where

$$\omega_n(\vec{p})=(\vec{p}^2+m_n{}^2)^{1/2}. \tag{13}$$

The field operator now has the expansion

$$\phi(X,x)=\frac{1}{[2(2\pi)^3]^{1/2}}\sum_{n,l}\int\frac{d^3p}{\omega_n(\vec{p})}[e^{-ip_n\cdot X}f_{n,l}(p_n,x)a_{n,l}(\vec{p})+e^{ip_n\cdot X}f_{n,l}(p_n,x)a^\dagger_{n,l}(\vec{p})] . \tag{14}$$

It is now straightforward to evaluate the commutator:

$$[\phi(X,x),\phi(Y,y)]=i\Delta(X-Y;x,y)$$
$$=\frac{1}{2(2\pi)^3}\sum_{n}\int\frac{d^3p}{\omega_n(p)}[e^{-ip_n\cdot(X-Y)}f_{n,l}(p_n,x)f_{n,l}(p_n,y)-e^{ip_n\cdot(X-Y)}f_{n,l}(p_n,x)f_{n,l}(p_n,y)] . \tag{15}$$

Now, using Lorentz invariance, it follows that a necessary and sufficient condition for $\Delta(X-Y;x-y)$ to vanish for spacelike separation of X and Y is that

$$\Delta(X-Y;x-y)\Big|_{X^0=Y^0}=0 \text{ for } \vec{X}\neq\vec{Y} . \tag{16}$$

Written out, this produces the condition

$$\frac{1}{2(2\pi)^3}\sum_{n,l}\int\frac{d^3p}{\omega_n(\vec{p})}e^{i\vec{p}\cdot(\vec{X}-\vec{Y})}[f_{n,l}(\omega_n,\vec{p};x)f_{n,l}(\omega_n,\vec{p};y)-f_{n,l}(\omega_n,-\vec{p};x)f_{n,l}(\omega_n,-\vec{p};y)]=0 . \tag{17}$$

Hence we require

$$\sum_l f_{n,l}(\omega_n,\vec{p};x)f_{n,l}(\omega_n,\vec{p};y)=\sum_l f_{n,l}(\omega_n,-\vec{p};x)f_{n,l}(\omega_n,-\vec{p};y) . \tag{18}$$

Furthermore these sums are Lorentz invariant since the $f_{n,l}$ are. Thus we can define

$$\rho_n(x^2,y^2,x\cdot y,p_n\cdot x,p_n\cdot y)=\sum_l f_{n,l}(\omega_n,\vec{p};x)f_{n,l}(\omega_n,\vec{p};y) , \tag{19}$$

where

$$p_n\cdot x=\omega_n x^0-\vec{p}\cdot\vec{x}$$

and the Lorentz invariance has been made explicit.

The center-of-mass locality condition [Eq. (18)] now reduces to

$$\rho_n(x^2,y^2,x\cdot y,\omega_n x^0-\vec{p}\cdot\vec{x},\omega_n y^0-\vec{p}\cdot\vec{y})$$
$$=\rho_n(y^2,x^2,x\cdot y,\omega_n y^0+\vec{p}\cdot\vec{y},\omega_n x^0+\vec{p}\cdot\vec{x}). \tag{20}$$

This immediately implies that

$$\rho_n(x^2,y^2,x\cdot y,p_n\cdot x,p_n\cdot y)$$

is independent of $p_n\cdot x$ and $p_n\cdot y$ and is a function of only x^2, y^2, $x\cdot y$, and $p_n{}^2=m_n{}^2$.

It now follows that the functions $f_{n,l}(p_n,x)$ are also independent of $p_n\cdot x$ and depend on p_n only trivially, i.e., via $p_n{}^2$. To see this call $p_n\cdot x=\xi$ and $p_n\cdot y=\eta$ and suppress temporarily all further x and y dependence so that

$$f_{n,l}(p_n,x)=g_{n,l}(\xi) ,$$
$$f_{n,l}(p_n,y)=g_{n,l}(\eta) . \tag{21}$$

The fact that ρ_n is independent of $p_n\cdot x$ and $p_n\cdot y$ now reads

$$\sum_l g_{n,l}(\xi)g_{n,l}(\eta)=\text{constant} \quad (22)$$

as far as the variables ξ and η are concerned. Differentiating first with respect to η and then with respect to ξ and setting $\eta=\xi$, we get

$$\sum_l \left|\frac{dg_{n,l}(\xi)}{d\xi}\right|^2=0 .$$

Therefore

$$\frac{dg_{n,l}(\xi)}{d\xi}=0$$

or

$$g_{n,l}(\xi)=\text{constant} .$$

Thus the functions $f_{n,l}(p_n,x)$ are in fact functions of x^2 and $p_n{}^2$ only, so the field $\phi(X,x)$ has the form

$$\phi(X,x)=\frac{1}{[2(2\pi)^3]^{1/2}}\sum_{n,l} f_{n,l}(m_n{}^2,x^2)\int\frac{d^3p}{\omega_n(\vec{p})}[e^{-ip_n\cdot X}a_{n,l}(\vec{p})+e^{ip_n\cdot X}a_{n,l}^\dagger(\vec{p})]$$
$$=\sum_{n,l} f_{n,l}(m_n{}^2,x^2)\phi_{n,l}(X) \quad (23)$$

as stated.

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