Origin of the Universe as a quantum tunneling event

David Atkatz*† and Heinz Pagels
The Rockefeller University, New York, New York 10021
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We present a nonsingular model of cosmogenesis in which the Universe arises as a result of quantum-mechanical barrier penetration. The Universe is described throughout its evolution by a Friedmann-Robertson-Walker (FRW) metric, and the matter distribution by a perfect fluid, whose equation of state is chosen so as to allow the tunneling to occur. Cosmic evolution proceeds in three stages; an initial static spacetime configuration tunnels into a "fireball" state in which particle creation occurs. As the fireball expands, particle creation ends, and the Universe enters the "post-big-bang" epoch of adiabatic expansion. We find that within the context of the FRW ansatz, only a spatially closed universe may originate in this manner. Implications of this creation scheme and possible generalizations are discussed. As a by-product of this investigation we find that the evolution of the Universe is described by a Gell-Mann—Low equation with the β function specified by the equation of state.

I. INTRODUCTION

How did the Universe begin? Present data are consistent with, and in fact form a strong body of evidence in support of, the "standard" big-bang model. The Universe was once at enormous temperature, an expanding fireball of quarks, leptons, and gluons. But where did *that* come from? Extrapolating the standard model backwards in time leads to a singular solution of the Einstein equations; indeed these equations predict that singularities are inevitable under a very general and physically reasonable choice of initial conditions.[1] In that case, answering such a question is not possible within the current framework of physics. But can the Universe begin with a nonsingular (although obviously extremely violent) event?

One possibility is that the Universe "bounces"; terms in the effective gravitational action induced by quantum effects reverse the collapse.[2] If the matter density is sufficient to close such a universe, the question of origin need not arise; in certain models these universes have always existed, eternally expanding and contracting. In this paper we will explore another possibility, one in which the Universe originated as a tunneling event from a classically stable, static spacetime configuration. The big bang is analogous to a single radioactive decay, on a huge scale.

Speculations on the quantum origin of the Universe began with a paper by Tryon,[3] who suggested that the Universe might be a vacuum fluctuation; it began as nothing at all. If this is so then the net quantum numbers of the Universe must be zero. The total electric charge of the Universe is consistent with zero. The fact that total baryon number is not zero is not so troubling today, as the "grand unified" field theories of strong, weak, and electromagnetic interactions imply proton instability. In addition, Tryon adopts the view that total energy must be strictly conserved in the creation process. Therefore a universe which originated as a vacuum fluctuation *must* have zero total energy. He presented a simple "semi-Newtonian" argument, in which the positive mass-energy of the galaxies is balanced by their negative gravitational potential, to within a factor of order unity. Thus the quantum numbers of the Universe could be the same as those of the vacuum (at very early times), and the vacuum-fluctuation picture appears plausible. In a fully relativistic model, however, Tryon's argument fails. Total energy may only be rigorously defined in asymptotically flat spacetimes. In such spacetimes the energy of the gravitational field itself is positive semidefinite, and the only spacetime with zero total energy is flat and empty everywhere.[4] Tryon went on to present a generally relativistic argument due to Bergmann, indicating that any closed universe has zero energy; hence his main prediction, based purely on energy conservation, is that we live in a closed universe. But for arbitrary non-asymptotically flat geometries, open or closed, total energy is not well defined. In addition, any homogeneous, isotropic, nonempty universe cannot be asymptotically flat. Total-energy considerations are therefore irrelevant for creating a universe of this type from the vacuum; only absolutely conserved charges must globally add to zero, a condition which is automatic in spatially closed geometries.

Brout, Englert, and Gunzig[5] have further developed the idea of the vacuum origin of the Universe. According to their view, the Universe is initiated by some *local* quantum fluctuation of the spacetime metric. Varying metrics are well known to result in particle creation.[6] This creation of matter causes a further change in the metric, and a cooperative process is set up. During this fireball stage of particle creation, characterized phenomenologically by negative pressure, the Universe is an open de Sitter spacetime which will develop a singularity, a future event horizon, within a finite proper time. Before this horizon is reached, however, the authors postulate that the cooperative process stops, and particle creation ends. Then begins the second stage of the Universe's evolution, adiabatic free expansion with positive pressure, the usual post-big-bang expansion. The authors examine in detail the particle creation mechanism and the joining of the fireball and big-bang stages, but the origin of the first quantum fluctuation is not examined.

We will address the question of vacuum cosmogony. Clearly this was a very violent event. If we examine well-understood physical processes and ask what are some of the most violent, then radioactive decay comes immediately to mind. Imagine an oppositely charged pair of particles, bound by a short-range attractive nuclear potential. This state is classically stable. Increase the strength of the nuclear force; the binding energy increases, and the total energy can become zero. The bound state becomes degenerate with the vacuum. There is a finite probability for the particles to quantum mechanically tunnel through the barrier and separate. Massive, expanding matter has been "created" from the vacuum.

In analogy to this decay process we assume that the Universe began as a classically stable, static configuration of spacetime. We examine the conditions for the existence of a finite tunneling amplitude between this initial state and a fireball state with subsequent adiabatic expansion. Our principal assumptions are that during its entire evolution the Universe may be described by a homogeneous, isotropic, Robertson-Walker metric, and matter treated phenomenologically as a perfect fluid. In addition, we assume the Hilbert-Einstein action, although this assumption can be altered without changing our conclusions dramatically. Finally, we allow ourselves the freedom of choosing an equation of state for matter (analogous to picking the potential in the nuclear decay problem) so that the tunneling process is possible.

The tunneling amplitude between initial and final states may be expressed as a Feynman path integral over metrics. In order that the path integral have well-behaved convergence properties, we analytically continue the initial and final metrics to the Euclidean section; i.e., we Wick rotate to imaginary times, and integrate over all metrics with Euclidean signature $[+\ +\ +\ +]$. The path integral may then be evaluated semiclassically. We discuss later the problem of the lack of positivity of the Euclidean Hilbert-Einstein action.

The Robertson-Walker ansatz describes a spacetime containing a three-dimensional spacelike hypersurface of homogeneity; for any fixed value of the global Gaussian time coordinate t, the three-spaces are locally isotropic and homogeneous. They are either open or closed, and for a particular choice of coordinates the three-space topology may be parametrized by the values -1, 0, $+1$ of an integer k. We find a finite tunneling amplitude exists *only* for those spaces with finite three-volume on the Euclidean section. There are two such spaces, the closed spherical universe characterized by $k=+1$, $\Lambda \geq 0$ (Λ is the cosmological constant), and the de Sitter universe, $k=0$, $\Lambda > 0$. Although $k=0$ in the de Sitter case, corresponding to an infinite three-space, the Euclidean section is an S^4, of radius $(3/\Lambda)^{1/2}$, and is compact.[7] The noncompact spaces have infinite volume, and this implies an infinite action and vanishing tunneling amplitude. Loosely speaking, infinite energy is required to transform the metric everywhere on an open space.

II. THE FRIEDMANN-ROBERTSON-WALKER ANSATZ AND THE FIELD EQUATIONS

The most general line element describing a homogeneous, isotropic spacetime is of Friedmann-Robertson-Walker (FRW) form, and may be written

$$ds^2 = -c^2(t)dt^2 + a^2(t)d\sigma^2 \,, \qquad (1)$$
$$d\sigma^2 \equiv h_{ij}dx^i dx^j = \frac{\delta_{ij}dx^i dx^j}{(1+\frac{1}{4}K\delta_{ij}x^i x^j)^2} \,,$$

with Lorentzian signature $[g_{\mu\nu}]=[-+++]$. Here $a(t)$ is the cosmic expansion factor, $c(t)$, inserted for convenience, represents the scale of time translations, and can always be transformed to 1 by a rescaling of the timelike coordinate, and h_{ij} is the metric on the three-dimensional spacelike surface of homogeneity. K is a constant curvature parameter. The Riemann tensor on the spacelike surface is given by

$$^{(3)}R^i{}_{jkl}=-K(\delta^i_k h_{jl}-\delta^i_l h_{jk}) ; \tag{2}$$

thus K parametrizes the intrinsic curvature of the three-space, which is hyperbolic, flat, or spherical as $K<0$, $K=0$, or $K>0$. With a suitable choice of spacelike coordinates, the metric (1) may be cast in the form

$$d\sigma^2=\begin{cases} kK^{-1}(d\chi^2+\sinh^2\chi\, d\Omega^2), & k=-1 \\ d\chi^2+\chi^2 d\Omega^2, & k=0 \\ kK^{-1}(d\chi^2+\sin^2\chi\, d\Omega^2), & k=1 , \end{cases} \tag{3}$$

where $k\equiv K/|K|$, and $d\Omega^2=d\theta^2+\sin^2\theta\, d\phi^2$. The three-valued integer k now classifies the three-space topology. For the cases where the intrinsic three-curvature is nonvanishing we follow the convention of rescaling the expansion factor through $a(t)\rightarrow |K|^{1/2}a(t)$. Only the spherical three-space ($k=1$) is of finite extent, with volume $V_3=2\pi^2K^{-3/2}$. We note at this point that the Weyl tensor $C_{\mu\nu\lambda\gamma}$ evaluated on an FRW spacetime vanishes; these spacetimes are conformally flat. Suitable coodinate transformations can always be found such that the FRW metrics with $k=-1,0$ and Lorentzian signature $[-+++]$, or the $k=0,1$ metrics with Euclidean signature $[++++]$ can be put in the form

$$ds^2=\Omega^2(\pm dt'^2+dx'^2) , \tag{4}$$

where Ω is a function of $(t'^2+kx'^2)$, and the signature of the flat line element in (4) is that of the metric being transformed.

The total action is given by

$$S=S_G+S_M ,$$
$$S_G=-\frac{1}{2\kappa}\int d^4x\sqrt{-g}\,(R+2\Lambda) , \tag{5}$$
$$S_M=\int d^4x\sqrt{-g}\,\mathscr{L}_M ;$$

S_G is the gravitational contribution, S_M that of matter, and $\kappa^2=8\pi G$ with G the Newton-Cavendish constant. A cosmological constant Λ has been introduced for generality. The vanishing of the metric variation of the action yields the Einstein field equations,

$$\begin{aligned} &G_{\mu\nu}-g_{\mu\nu}\Lambda=-\kappa\theta_{\mu\nu} , \\ &G_{\mu\nu}\equiv R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R , \end{aligned} \tag{6}$$

where the matter stress tensor $\theta_{\mu\nu}$ is formally defined by

$$\delta S_M\equiv-\tfrac{1}{2}\int d^4x\sqrt{-g}\,\theta_{\mu\nu}\delta g^{\mu\nu} . \tag{7}$$

The Einstein equations (6) are satisfied by the FRW ansatz, provided the most general, homogeneous, isotropic form is used for the matter stress tensor, that of a perfect fluid,

$$\theta_{\mu\nu}=[\epsilon(t)+p(t)]U_\mu U_\nu+g_{\mu\nu}p(t) , \tag{8}$$

where $\epsilon(t)$ is the energy density of the fluid, $p(t)$ the pressure, and U_μ the four-velocity. In addition, the "energy" equation

$$\dot a^2-\frac{\kappa}{3}a^2\epsilon=-k+\frac{a^2\Lambda}{3} \tag{9}$$

and the continuity equation

$$(a^3\epsilon)^{\cdot}=-p(a^3)^{\cdot} , \tag{10}$$

must be satisfied; the dot denotes d/dt. The latter equation may be rewritten as

$$a\frac{d\epsilon}{da}=-3(\epsilon+p) , \tag{11}$$

and thus implies that ϵ depends on t only through the expansion factor $a(t)$. The matter Lagrangian density appearing in (5) is chosen by the requirement that the variation of the total action with respect to c (which may then be evaluated at $c=1$), yield the Einstein energy equation (9). Using the FRW ansatz, we find

$$\begin{aligned} &S_G=\frac{3V_3}{\kappa}\int dt\, c^{-1}\left[a\dot a^2+a^2\ddot a+c^2ak-\frac{\dot c}{c}a^2\dot a-\frac{c^2a^3\Lambda}{3}\right] , \\ &S_M=V_3\int dt\, ca^3\epsilon(a) , \end{aligned} \tag{12}$$

where $\int d^4x(-g)^{1/2} \equiv V_3 c a^3$, and V_3 is formally infinite for the open spaces.

A second field equation follows from the variation of the action (12) with respect to $a(t)$ if we assume the continuity equation (11) for the variation of $\epsilon(a)$ with respect to a:

$$2\frac{\ddot{a}}{a} + \left[\frac{\dot{a}}{a}\right]^2 + \frac{k}{a^2} - \frac{2}{3}\frac{\Lambda}{a} = -\kappa p \ . \quad (13)$$

This equation has only the pressure on the matter side; the continuity equation can be thought of as a definition of the pressure in terms of $d\epsilon/da$. But (13) may also be obtained by taking the time derivative of (9), and employing (10), and thus is not independent. Alternately, of course, the continuity equation follows from the two field equations.

There are, however, three unknown functions appearing in these equations, $a(t)$, $\epsilon(a)$, and p, and a third, independent, equation is required for their solution. This is the matter equation of state, which we write in the form

$$p = p(\epsilon) \ . \quad (14)$$

Equations (9), (10), and (the yet unspecified) (14) completely describe an FRW cosmological model.

III. THE GELL-MANN–LOW EQUATION, THE EQUATION OF STATE, AND TUNNELING

Determining the state of the FRW universe at any time now proceeds in two steps. The equation of state $p = p(\epsilon)$ is specified, and the continuity equation (11) solved for $\epsilon(a)$; this result is then inserted into the field equation (9), yielding the expansion factor $a(t)$.

It is interesting to note that the simple scaling and conformal properties of FRW spacetimes become manifest through a renormalization-group–type equation. Defining a Callan-Symanzik β function,[8]

$$\beta(\epsilon) \equiv -3[\epsilon + p(\epsilon)] \ , \quad (15)$$

and integrating the continuity equation (11) yields the Gell-Mann–Low equation[9]

$$\ln\left[\frac{a}{a_0}\right] = \int_{\epsilon_0}^{\epsilon(a)} \frac{dz}{\beta(z)} \ , \quad (16)$$

where $\epsilon_0 = \epsilon(a_0)$, and a_0 is the scale factor at some reference time t_0, which may be conveniently chosen to be the present. Thus $\epsilon(a)$ plays the role of the running coupling constant, and the expansion factor $a(t)$ that of the momentum scale at which the coupling constant is evaluated. With this correspondence, the evolution of the Universe may be described in the language of the renormalization group. Suppose that $\epsilon_0 < \epsilon_c$, the critical density as determined from the field equations. [This is the density required to ensure that the rate of expansion vanishes asymptotically, and is found by setting the "total energy" k equal to zero in Eq. (9); in the absence of a cosmological constant, $\epsilon_c = 3H^2/8\pi G$, where $H = \dot{a}/a$ is the Hubble constant.] Then the scale factor increases without limit, and in the asymptotic region $a \to \infty$. Specifying the equation of state directly determines the β function. For the late universe, the energy density is small (we are in the "weak-coupling" regime); particle creation has long since ceased, and the Universe is undergoing adiabatic expansion. Particle creation implies that $(\epsilon a^3)^{\cdot} > 0$, or, through the continuity equation, negative pressure. Pressure in the late universe is therefore positive semidefinite, and in the region of small ϵ, β is negative. The usual equations of state for the late universe satisfy $p(0)=0$; thus $\beta(0)=0$. The point $\epsilon=0$ is an "ultraviolet-stable fixed point", we reside at present within its domain of attraction, and the Universe is asymptotically free. This is just an unfamiliar way of stating that for an open universe the expansion is unbounded, and the energy density and pressure attenuate to zero. As an example, consider the equation of state

$$p(\epsilon) = \frac{\gamma}{3}\epsilon \ ; \quad (17)$$

$\gamma = 0, 1$ correspond to a matter- or radiation-dominated late universe, respectively. The asymptotic energy density obtained from the Gell-Mann–Low equation (16) is

$$\epsilon(a) = \epsilon_0 \left[\frac{a_0}{a}\right]^{3+\gamma} \ , \quad (18)$$

and vanishes as $a \to \infty$.

If $\epsilon_0 > \epsilon_c$, the Einstein equation (9) ensures that the scale factor satisfies $a(t) < a_{max}$; it is bounded from above. Although the Gell-Mann–Low equation still applies, its solution in the region $a > a_{max}$ has no physical significance.

The energy density of the early universe is of course quite large, corresponding to the strong-coupling regime of the β function, and little is known about the equation of state. We will simply assume here an equation which allows the universe to originate as a tunneling event from a static geometry. Ultimately one would like to derive this equation from a fundamental theory; we do not address this problem here.

In the usual manner we interpret the equation of motion (9), with $\Lambda=0$, as that of a classical particle of twice unit mass moving in a potential

$$V(a)=\frac{-\kappa}{3}a^2\epsilon(a) \tag{19}$$

with total energy $-k$; i.e.,

$$\dot{a}^2+V(a)=-k\ . \tag{20}$$

The classically allowed region satisfies $k+V(a)<0$, while in the unphysical tunneling region $k+V(a)>0$. In the asymptotic region (late times), for a matter- or radiation-dominated universe,

$$V(a) \underset{a\to\infty}{\sim} \frac{\kappa}{3}\epsilon_0 a^2\left[\frac{a_0}{a}\right]^{3+\gamma}; \tag{21}$$

for $a(t)$ small (early times) we assume the potential shown in Fig. 1. This potential has been chosen such that

$$\dot{a}_{|a_I}=0\ . \tag{22}$$

In order to ensure this static pretunneling configuration, the presence of a constant positive (negative) energy density is required in the case $k=1$ $(k=-1)$.

The continuity equation, together with Eq. (19), imply

$$\frac{dV}{da}=a(3p+\epsilon)\ ; \tag{23}$$

we use this relation and its derivative to study the qualitative features of the equation of state which leads to our potential. For simplicity we analyze the case $k=0$; there are no qualitative differences for nonvanishing k. The potential is shown in Fig. 2(a), the equation of state in Fig. 2(b). The initial static state occurs at A; this state tunnels quantum mechanically through region I. The Universe is created at B; here $V=\epsilon=0$, while $dV/da<0$. The pressure is therefore negative, implying particle creation, which continues throughout region II. Region III represents the post-big-bang stage; here the pressure is again positive, particle creation has ended, and the Universe is expanding.

This equation of state defines a β function which, when continued to complex arguments, possesses a series of square-root branch points

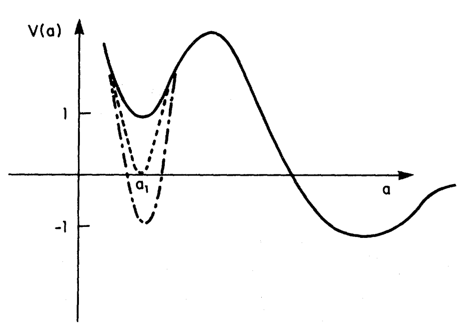

FIG. 1. The potential at early times. The solid, dotted, and broken lines denote the potential for the $k=-1$, 0, 1 universes, respectively.

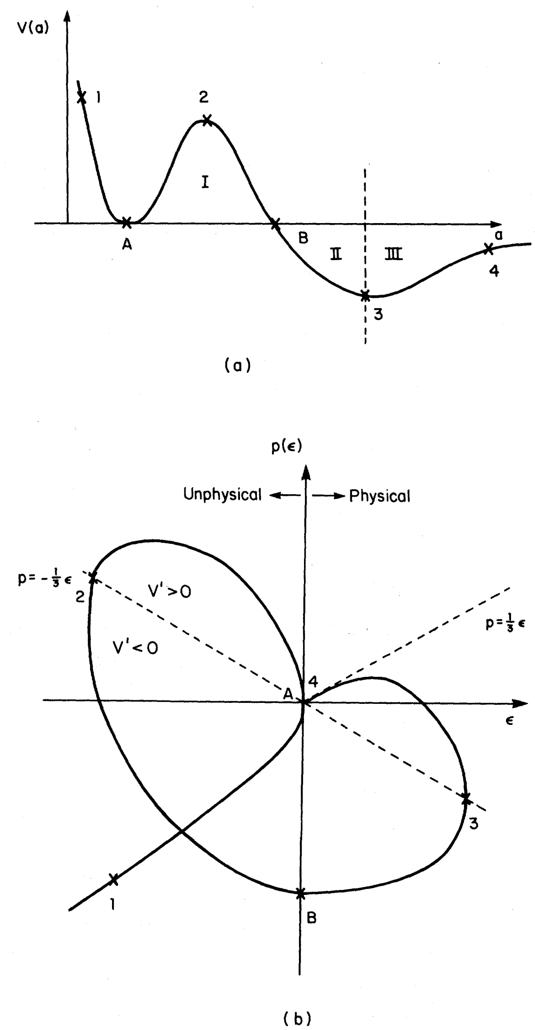

FIG. 2. (a) The potential for $k=0$. (b) The equation of state.

corresponding to the extrema of the potential $V(a)$. The analytic structure of the inverse β function is depicted in Fig. 3; it is a multisheeted function with branch points and a pole at the origin.

We have nothing to say as to whether such equations of state are physically reasonable, or indeed if the phenomenology of a perfect fluid can be continued to a state of such high density and pressure. No doubt quantum processes play an important role in this regime, and it is not clear if the phenomenological approach we use can include them even approximately.

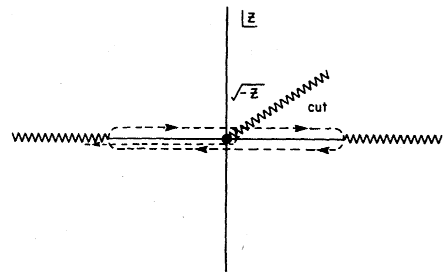

FIG. 3. Singularity structure of the inverse β function in the complex z plane.

IV. CALCULATION OF THE TUNNELING AMPLITUDE

The tunneling amplitude is obtained by a semiclassical evaluation of the Euclidean path integral

$$\Gamma = N \int [dg_{\mu\nu}] \exp\{-[S_G^E(g_{\mu\nu}) + S_M^E(\epsilon, g_{\mu\nu})]\} \ . \tag{24}$$

S_G^E and S_M^E are the gravitational and matter contributions to the total Euclidean action, respectively; N is a normalization factor. The metric is expanded around the FRW form and terms up to second order in the variation are retained in the action. Then, to leading order in $\hbar$, the tunneling amplitude is given by

$$\Gamma = A \exp\{-[S_E(a,k,\epsilon) - S_E(a,k,0)]\} \ , \tag{25}$$

where A is a determinantal factor, $S_E(a,k,\epsilon)$ is the total Euclidean action evaluated in the FRW ansatz, and $S_E(a,k,0)$ is the Euclidean FRW vacuum action. For the FRW ansatz the continuation to the Euclidean section is accomplished by the replacement $t \to -ix_0$. The Euclidean action S_E is then given by

$$\begin{aligned} S_E &= S_G^E + S_M^E \ , \\ S_G^E &= \frac{3}{\kappa} V_3 \int dx_0 \left[aa'^2 + a^2 a'' - ak + \frac{a^3 \Lambda}{3} \right] , \\ S_M^E &= V_3 \int dx_0 a^3 \epsilon(a) \ , \end{aligned} \tag{26}$$

where a prime denotes d/dx_0. The second derivative a'' appearing in S_G^E is removed by a partial integration, and the Euclidean equation of motion

$$(a')^2 + \frac{a^2 \Lambda}{3} = k + V(a) \tag{27}$$

is used to eliminate a'. By performing the change of variables $dx_0 = da/a'$ the simple formula

$$S_E = \frac{-3V_3}{\kappa} \int_{a_1^2}^{a_2^2} da^2 \left[V(a) + k - \frac{a^2 \Lambda}{3} \right]^{1/2} \tag{28}$$

obtains. The integration is over the unphysical region, where the integrand is real.

The Euclidean action is proportional to V_3, the volume of the spacelike sections. Thus a finite action configuration exists only for finite V_3; the three-space must be closed. For the compact space $k=1$, $V_3 = 2\pi^2$, and we have (for the case $\Lambda = 0$)

$$S_E - S_E^0 = \frac{-6\pi^2}{\kappa} \int_{a_1^2}^{a_2^2} da^2 \{ [V(a)+1]^{1/2} - 1 \} \ . \tag{29}$$

If we approximate the potential in the tunneling region by a square-well potential of height V_0 between a_1 and a_2 [A and B in Fig. 2(a)], then

$$S_E - S_E^0 = \frac{-6\pi^2}{\kappa}\Delta^2[(V_0+1)^{1/2}-1] , \tag{30}$$

where $\Delta^2 = {a_2}^2 - {a_1}^2$ is a measure of the width of the potential barrier.

For the FRW universe, we find the Euclidean action is negative definite, even after vacuum subtraction. This is a well-known problem in gravitational theory; in general, the Euclidean action is not positive-definite, even for real positive-definite metrics. To remedy this situation, Hawking suggests[10] that in evaluating the path integral the integration be split into two parts. The space of all positive-definite metrics on the manifold in question is divided into conformal equivalence classes. In each class one picks the metric that satisfies the Einstein equations, integrates over each of these solutions, and then over the conformal factor Ω. In order to render the path integral convergent the allowed conformal factors must be restricted; the integration is carried out only over Ω of the form $1+i\xi$, ξ real. *This prescription is a redefinition of the gravitational path integral,* chosen to provide proper convergence properties. When applied in the case of the pure gravitational action evaluated on an FRW spacetime, Hawking's prescription results in a positive-definite quantity. That this is so is easily shown. Recall that the Euclidean FRW metric can be written as $g_{\mu\nu} = \Omega^2(\tau)\delta_{\mu\nu}$, where $\delta_{\mu\nu}$ is diagonal [1111], and $\tau^2 = x'_\mu x^{\mu'}$. This yields for the Hilbert-Einstein action

$$S_G^E = \frac{-6\pi^2}{\kappa}\int d\tau\, \tau^3 \dot{\Omega}^2 ; \tag{31}$$

here the overdot denotes differentiation with respect to τ. For conformal factors of the form $1+i\xi$, we find

$$S_G^E = \frac{6\pi^2}{\kappa}\int d\tau\, \tau^3 \dot{\xi}^2 \geq 0 . \tag{32}$$

However, this simple behavior does *not* obtain in the presence of a matter distribution characterized by a stress tensor whose trace is nonvanishing, i.e., when the matter action is not conformally invariant. Unfortunately, this situation is the one we are dealing with in our simple model. The perfect-fluid matter action (in an FRW spacetime) can be written

$$S_M^E = 2\pi^2 \int d\tau\, \tau^3 \Omega^4 \epsilon , \tag{33}$$

and depends upon the conformal factor algebraically; letting $\Omega = 1+i\xi$ does not result in a simple change of sign.

The lack of positivity of the gravitational action reflects the fact that there is no consistent generally relativistic definition of energy for spaces satisfying arbitrary boundary conditions. This is a fundamental problem in the path-integral approach to quantum gravity, and must be resolved before a complete and unambiguous calculation of the tunneling amplitude (indeed of any quantum gravitational transition probability) is possible. We believe, however, that when a complete path-integral quantization scheme for gravity-matter systems is developed, our principle conclusion, that only a compact universe may originate via tunneling, will stand.

The only other finite-action FRW solution is a $\Lambda > 0$ de Sitter spacetime; its Euclidean section is an S^4, with finite three-volume. To see this, note that the Euclidean de Sitter line element can be written

$$ds^2 = \left[1 - \frac{\Lambda}{3}r^2\right]dt^2 + \left[1 - \frac{\Lambda}{3}r^2\right]^{-1} dr^2 + r^2 d\Omega^2 ; \tag{34}$$

a rescaling of the form $r \rightarrow \bar{r} = (\Lambda/3)^{1/2} r$ casts the three-space section in the form

$$d\sigma^2 = \frac{d\bar{r}^2}{1-\bar{r}^2} + \bar{r}^2 d\Omega^2 . \tag{35}$$

In spherical polar coordinates, the FRW three-space metric becomes

$$d\sigma^2 = \frac{dr^2}{1-kr^2} + r^2 d\Omega^2 ; \tag{36}$$

the correspondence is clear, the de Sitter three-space can be cast in a $k=1$ form. This is due to the fact that only the four-space curvature is a coordinate invariant.

V. DISCUSSION AND CONCLUSIONS

A principal conclusion of our investigation is that only a spatially compact universe can originate as a quantum tunneling event. We find this conclusion somewhat disappointing, as we had hoped that the Universe could have originated from flat empty space, a configuration that truly corresponds to nothing at all. Unfortunately this possibility seems to be ruled out by the positive-energy theorem,[4] which requires flat, empty spacetime to be absolutely stable; it is the lowest-energy state. Any deviation from such a geometry must be paid for by a corresponding increase in the action, and changing the action over an infinite volume costs infinite energy.

The initial state of our Universe has a compact $k=1$ geometry, and the question arises: Where did *that* come from? This is a question we cannot answer. However, a desirable feature of such a compact space is that all total charges associated with conserved currents must vanish for topological reasons, a feature we would ordinarily associate with a "vacuum."

An interesting interpretation of the bounce solutions for the early Universe emerges from our work. The bounce solutions are possible providing we add R^2 terms to the Hilbert-Einstein action. In fact in the conformally flat FRW geometries the most general modification is just the squared scalar curvature; the squares of the Riemann and Ricci tensors may be expressed in terms of R^2, up to a total divergence. We may view the bouncing universe as a classical particle hitting a potential barrier and bouncing off, remaining in the classically allowed energy region. The barrier, however, may be of finite height and width, with a classically stable, static state existing on the other side. If a finite tunneling amplitude exists between these classical states, the static state may be the true initial state of the Universe. We have examined this interpretation of the bounce solutions, and have found that a constant energy density is required in order that this initial state be static.

What is the Euclidean vacuum action for gravity? Gibbons and Hawking[9] suggest

$$S=\frac{1}{2\kappa}\int_M d^4x\sqrt{g}\,(R+2\Lambda)+\frac{1}{\kappa}\int_{\partial M} d^3x\sqrt{h}\,K+C\ , \qquad (37)$$

where the first integral is over the manifold M. The second is over its boundary ∂M, h is the Euclidean metric induced on this boundary, and K is the trace of the second fundamental form of the boundary. C is a constant term which depends only on h, and not on the values of g on the interior of M. (As we find the first term in the action finite only for closed spaces, we have not discussed the surface term, since it vanishes identically in such spaces.) Hawking argues that for flat space $R=\Lambda=0$, and the first term vanishes. The constant C is chosen by the requirement that the flat-space vacuum action vanish; thus

$$C\equiv-\frac{1}{\kappa}\int d^3x\sqrt{h}\,K^0\ , \qquad (38)$$

where K^0 is the trace of the second fundamental form embedded in flat space. However, it is not completely clear that the first term does indeed vanish. Consider approaching a flat-space configuration through a sequence of spaces of constant curvature, where $R=-4\Lambda$, and let $\Lambda\to 0$. For these spaces $\int d^4x(g)^{1/2}\sim 1/\Lambda^2$, and

$$S=\frac{-A}{\kappa\Lambda}\ , \qquad (39)$$

with A a numerical constant. As $\Lambda\to 0$, this expression is unbounded from below by a multiplicative (rather than additive) infinity. The fact that the vacuum action is poorly defined in general relativity creates a problem for our computation of the tunneling amplitude. It is not clear that a completely satisfactory estimate of the tunneling amplitude can be given until the problem of defining the action is resolved.

A possible extension of this work is to consider departures from the FRW geometries. One could then entertain the idea that the tunneling event began locally rather than over the whole spacetime. Local creation seems desirable from the standpoint of resolving the horizon problem.

There have been suggestions,[11] along the line of the Kaluza-Klein five-dimensional theory and its generalization to non-Abelian gauge theories, that internal symmetries are manifestations of compact higher-dimensional manifolds with a radius of curvature on the order of the Planck length. Conceivably these intriguing ideas can be integrated into our picture of cosmogenesis. What we envision is that the Universe began as a compact manifold of dimension $N>4$. A four-dimensional subspace of this manifold then tunnels into the fireball configuration, leaving the remainder as the observed internal symmetries. While highly speculative, we believe this idea is worth pursuing.

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*On leave from Queen Mary College, University of London, London, E1 4NS, England.

†Present address: Bell Laboratories, Holmdel, New Jersey 07733.

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