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Quark-model calculation of the weak electric coupling in semileptonic baryon decay

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The weak electric form factor g_2 is calculated in the nonrelativistic quark model (NRQM) and the MIT bag model for weak $\Delta S=1$ semileptonic transitions. The bag-model result is $g_2/g_1=0.30$ with the NRQM result roughly twice this. Other weak transition form factors are tabulated, including expected SU(3)-breaking effects.

Analysis of semileptonic hyperon decays remains of interest in studying the weak interaction[1] and determining Kobayashi-Maskawa (KM) mixing angles.[2] The Hamiltonian for semileptonic decay is given by

$$H_w=\frac{G}{\sqrt{2}}J_\lambda l^\lambda+\text{H.c.}\ , \tag{1}$$

where $G\cong 10^{-5}/m_p{}^2$ is the usual weak coupling constant,

$$l_\lambda=\bar{\psi}_e\gamma_\lambda(1+\gamma_5)\psi_{\nu_e}+\bar{\psi}_\mu\gamma_\lambda(1+\gamma_5)\psi_{\nu_\mu} \tag{2}$$

is the lepton current, and

$$J_\lambda=K_{11}\bar{u}\gamma_\lambda(1+\gamma_5)d+K_{12}\bar{u}\gamma_\lambda(1+\gamma_5)s \tag{3}$$

is the hadronic current. Here K_{ij} are elements of the KM matrix,[3] which in the usual notation are given by

$$K_{11}=\cos\theta_1\ ,\qquad K_{12}=\sin\theta_1\cos\theta_3\ . \tag{4}$$

The matrix element of the hadronic current between spin-$\frac{1}{2}$ states is conventionally written as[4]

$$\begin{aligned}\langle B'_{p_2}\,|\,J_\lambda\,|\,B_{p_1}\rangle=K_{ij}\bar{u}(p_2)\Bigg[&f_1(q^2)\gamma_\lambda-i\frac{1}{m_1+m_2}\sigma_{\lambda\eta}q^\eta f_2(q^2)+if_3(q^2)q_\lambda\frac{1}{m_1+m_2}\\&+g_1(q^2)\gamma_\lambda\gamma_5-i\frac{g_2(q^2)}{m_1+m_2}\sigma_{\lambda\eta}q^\eta\gamma_5+iq_\lambda\frac{1}{m_1+m_2}g_3(q^2)\gamma_5\Bigg]u(p_1)\ ,\end{aligned} \tag{5}$$

where $q=p_1-p_2$. Here f_1 and g_1 are the conventional vector and axial-vector form factors, f_3 and g_3 are the induced scalar and pseudoscalar form factors, and f_2 and g_2 are the weak magnetism and electric form factors, respectively. Via T invariance all form factors are relatively real. Fits to hyperon decay generally assume exact SU(3) symmetry for f_1, g_1, and f_2 while g_3 is determined via PCAC (partial conservation of axial-vector current)[5] and g_2, and f_3 are set equal to zero. The vanishing of f_3 and g_2 is a consequence of the invariance of $\bar{u}\gamma_\lambda(1+\gamma_5)d$ under the G-parity[6] operation

$$G=Ce^{-i\pi I_2}$$

for transitions between common members of an isotropic multiplet, for example,

$$n\to pe^-\bar{\nu}_e,\ \ \Xi^-\to\Xi^0e^-\bar{\nu}_e\ .$$

Similarly for transitions between common members of a V-spin multiplet, for example,

$$\Sigma^-\to ne^-\bar{\nu}_e,\ \ \Xi^0\to\Sigma^+e^-\bar{\nu}_e\ ,$$

invariance of $\bar{u}\gamma_\lambda(1+\gamma_5)s$ under G',

$$G'=Ce^{-i\pi V_2}\ ,$$

imposes $g_2=f_3=0$. Finally, for transitions involving more than one I-spin or V-spin multiplet, for example,

$$\Lambda\to pe^-\bar{\nu}_e,\ \ \Xi^-\to\Sigma^0e^-\bar{\nu}_e\ ,$$

SU(3) symmetry plus G,G' invariance yields $f_3 = g_2 = 0$.

Of course, SU(3) and G' invariance are rather badly broken symmetries in the real world. In the case of f_3, however, this assumption is not problematic, since even if $f_3 \neq 0$ the spacetime dependence on q_λ requires any dependence on f_3 in decay spectra to be of order[7]

$$\frac{m_l^2}{E_l(m_1+m_2)} <<< 1 , \tag{6}$$

where m_l and E_l are the final-charged-lepton mass and energy, respectively. However, spectral dependence on g_2 is of order[7]

$$\frac{E_l}{m_1+m_2} \tag{7}$$

and can make measurable changes in the analysis of semileptonic decays. In fact, several years ago Garcia[8] made an extensive analysis of the then existing data for $\Lambda \to p e^- \bar{\nu}_e$ and concluded that the best fit could be obtained for

$$\frac{g_2}{f_1} \approx -5 , \tag{8}$$

a surprisingly large value. Also, early experimental work involving measurements of the correlation between nuclear spin and electron momentum determined a large value of g_2 in *nuclear* β decay[9]:

$$\frac{g_2}{f_1} \approx -8 . \tag{9}$$

More recent work has cast considerable doubt on these preliminary results.[10] Nevertheless, it is of interest to know just how large g_2 is, especially in view of the high-statistics experiment on $\Lambda \to p e^- \bar{\nu}_e$ currently being analyzed.[11]

Although all g_2's would be zero in an SU(3)-invariant world, the real world involves broken SU(3)/SU(2) symmetry and thus we do not expect strict zero values for weak electricity. Since G invariance involves SU(2) symmetry while G' invariance involves SU(3) symmetry, we expect

$$\left|\frac{g_2}{f_2}\right| \lesssim \frac{m_n - m_p}{m_n + m_p} \sim 10^{-2} - 10^{-3}, \quad n \to p e^- \nu ,$$
$$\left|\frac{g_2}{f_2}\right| \lesssim \frac{m_\Lambda - m_p}{m_\Lambda + m_p} \sim 10^{-1}, \quad \Lambda \to p e^- \nu , \tag{10}$$

where we have scaled the weak electricity form factor in terms of the weak magnetism form factor since they have similar forms. Since[12]

$$f_2 \approx 3.7, \quad n \to p e^- \bar{\nu}_e ,$$
$$f_2 \approx 1.6, \quad \Lambda \to p e^- \bar{\nu}_e , \tag{11}$$

our crude estimate yields

$$|g_2| \lesssim 0.03, \quad n \to p e^- \bar{\nu}_e ,$$
$$|g_2| \lesssim 0.3, \quad \Lambda \to p e^- \bar{\nu}_e . \tag{12}$$

Thus g_2 can be neglected in analysis of neutron β decay but may be relevant in $\Delta S = 1$ semileptonic hyperon decays.

Obviously, one can do better than these simple estimates. There has been previous work on this question by Pritchett and Deshpande[13] in terms of a dispersive analysis. However, their work involved sums over many differing intermediate states and their result, while consistent with Eq. (12), involves considerable uncertainty.

We believe that a reasonable and reliable estimate of this effect can be provided by the quark model. Both of the coupling constants f_2 and g_2 are related to pieces of the quark wave function which determine magnetic moments. In order to display the relevant physics most simply, we will first calculate the results in the nonrelativistic quark model,[14] and then provide a more reliable calculation in the MIT bag model.[15]

In the multipole expansion of a polar vector current, the electric dipole term vanishes by parity and the first nonzero term is the magnetic moment[16]

$$\vec{\mu} = \tfrac{1}{2} \int d^3x \, \vec{r} \times \vec{V}(x) , \tag{13}$$

where $\vec{V}$ is the spatial component of the vector current. Upon taking the matrix element for a normalized nonrelativistic quark wave function (M_1^* is the quark mass)[17]

$$\psi_i(p) = \begin{bmatrix} 1 \\ \dfrac{1}{2M_i^*} \vec{\sigma} \cdot \vec{p} \end{bmatrix} \phi(p) \tag{14}$$

and using the vector current

$$V_{ij}^\mu(x) = \bar{\psi}_i(x) \gamma^\mu \psi_j(x) \tag{15}$$

one obtains the result

$$\vec{\mu}_{ij} = \frac{1}{2} \left[\frac{1}{2M_i^*} + \frac{1}{2M_j^*} \right] \vec{\sigma}_{ij} . \tag{16}$$

In the case $M_i = M_j$ this is just the usual magnetic moment

$$\vec{\mu}_{ii} = \frac{1}{2M_i^*} \vec{\sigma}_{ii} , \tag{17}$$

where the quark charge must be appended if one is considering the electromagnetic current. The matrix element of this quark operator is to be identifed with the *total* magnetic moment, which can be found from the nonrelativistic reduction of Eq. (5) to be

$$\begin{aligned} \langle \vec{\mu}_{ij} \rangle &= \left[\frac{1}{2} \left[\frac{1}{2m_1} + \frac{1}{2m_2} \right] f_1 + \frac{f_2}{m_1+m_2} \right] \chi_2^\dagger \sigma \chi_1 \\ &= \frac{1}{2} \left[\frac{1}{2M_1^*} + \frac{1}{2M_j^*} \right] \langle \vec{\sigma}_{ij} \rangle , \end{aligned} \tag{18}$$

where $\chi_{1,2}$ are the two-component Pauli spinors for the external states and $\langle \vec{\sigma}_{ij} \rangle$ is the matrix element of the spin operator connecting quarks i and j, between appropriate baryon states $| B \rangle$:

$$\langle \vec{\sigma}_{ij} \rangle = \langle B_2 | \sigma_{ij} | B_1 \rangle . \tag{19}$$

For normalization, one can calculate the magnetic moment of the proton

$$\mu_p = \left\langle p, \uparrow \left| \sum_{i=u,d} \frac{1}{2M_i^*} Q_i \sigma_{ii}^z \right| p, \uparrow \right\rangle . \tag{20}$$

Using SU(6) wave functions for the proton and $M_u^* = M_d^* = M^*$, one finds $\langle \sum_i Q_i \sigma_{ii}^z \rangle = 1$ and therefore

$$\mu_p = \frac{f_1 + f_2}{2M_p} = \frac{1}{2M^*} = \frac{2.79}{2M_p}$$

or

$$M^* = \frac{M_p}{2.79} = 330 \text{ MeV} , \tag{21}$$

which is the usual nonrelativistic result.

In the multipole expansion of an axial-vector current, it is the "magnetic" moment which vanishes by parity and the "electric dipole" which remains,

$$\vec{\mathrm{d}} = -i \int d^3x \, \vec{r} A^0(x) , \tag{22}$$

where A^0 is the time component of the axial-vector current. Again using the nonrelativisitic quark wave function, one obtains from the axial-vector current

$$A_{ij}^\mu = \psi_i \gamma^\mu \gamma_5 \psi_j \tag{23}$$

the result

$$\vec{\mathrm{d}}_{ij} = \frac{1}{2} \left[\frac{1}{2M_j^*} - \frac{1}{2M_i^*} \right] \vec{\sigma}_{ij} . \tag{24}$$

The electric dipole moment measures the difference of the initial and final quark moments, whereas the magnetic moment measures the sum. Again this must be identified with the total dipole moment, which can be found from the nonrelativistic reduction of the axial-vector term in Eq. (5), with the result

$$\begin{aligned} \langle \vec{\mathrm{d}}_{ij} \rangle &= \left[\frac{1}{2} \left[\frac{1}{2m_2} - \frac{1}{2m_1} \right] g_1 + \frac{g_2}{m_1+m_2} \right] \chi_2^\dagger \vec{\sigma} \chi_1 \\ &= \frac{1}{2} \left[\frac{1}{2M_j^*} - \frac{1}{2M_i^*} \right] \langle \vec{\sigma}_{ij} \rangle . \end{aligned} \tag{25}$$

As expected, g_2 vanishes in the SU(3) limit.

In the above, the nonrelativistic quark model was used only for illustration. The results in general are

$$\left[\frac{1}{2} \left[\frac{1}{2m_1} + \frac{1}{2m_2} \right] f_1 + \frac{f_2}{m_1+m_2} \right] = \left\langle B_2, \uparrow \left| \tfrac{1}{2} \int d^3x (\vec{r} \times \vec{J})_z \right| B_1, \uparrow \right\rangle \equiv \mu , \tag{26}$$

$$\left[\frac{1}{2} \left[\frac{1}{2m_2} - \frac{1}{2m_1} \right] g_1 + \frac{g_2}{m_1+m_2} \right] = \left\langle B_2, \uparrow \left| -i \int d^3x \, z A_0 \right| B_1, \uparrow \right\rangle \equiv d , \tag{27}$$

where $| B,s \rangle$ represent quark-model states for the parent and daughter baryons. These can also be derived simply from the wave-packet formalism of Donoghue and Johnson,[18] as shown in the Appendix.

We now proceed to calculate the required matrix elements in a more realistic model. The MIT bag model has relativistic quarks with the wave function

$$\psi(x) = \begin{bmatrix} iu(r) \chi_m \\ l(r) \vec{\sigma} \cdot \hat{r} \chi_m \end{bmatrix} . \tag{28}$$

Here

$$u = N j_0(pr) ,$$
$$l = -N \left[\frac{E-m}{E+m} \right]^{1/2} j_1(pr) , \tag{29}$$

p is an eigenvalue of the bag boundary-condition equation (if $m=0$, $p=2.0428/R$), and N is chosen such that

$$1 = \int d^3x\, \psi^\dagger(x)\psi(x) = \int d^3x\,(u^2+l^2) . \tag{30}$$

For the strangeness-changing current the magnetic-moment operator has matrix elements

$$\begin{aligned}\mu &= \tfrac{1}{3} \int d^3x\, r(u_u l_s + u_s l_u)\langle \sigma^3_{us} \rangle \\ &= g_1^{\mathrm{SU}(6)} \tfrac{1}{3} \int d^3x\, r(u_u l_s + u_s l_u) ,\end{aligned} \tag{31}$$

where the matrix element has been evaluated and the result is the SU(6) Clebsch Gordan coefficient for g_1, called $g_1^{\mathrm{SU}(6)}$ above. We remind the reader of the bag-model calculation of g_1 (Ref. 19),

$$\begin{aligned}g_1 &= \left\langle B_2, \uparrow \left| \int d^3x\, A_3 \right| B_1, \uparrow \right\rangle \\ &= \int d^3x\,(u_u u_s - \tfrac{1}{3} l_u l_s)\langle \sigma^3_{us} \rangle \\ &= g_1^{\mathrm{SU}(6)} \int d^3x\,(u_u u_s - \tfrac{1}{3} l_u l_s) ,\end{aligned} \tag{32}$$

where the integral is less than unity and renormalizes the SU(6) result down to reasonable values (i.e, for $\Delta S=0$, $g_1^{\mathrm{SU}(6)} = \frac{5}{3}$, $g_1 = \frac{5}{3} \times 0.7 = 1.2$, see Ref. 18). The "electric dipole" moment can be calculated in a similar fashion:

$$\begin{aligned}d &= \tfrac{1}{3} \int d^3x\, r(u_u l_s - u_s l_u)\langle \sigma^3_{us} \rangle \\ &= g_1^{\mathrm{SU}(6)} \tfrac{1}{3} \int d^3r\, r(u_u l_s - u_s l_u) .\end{aligned} \tag{33}$$

In order to reduce uncertainties we may normalize these results to the proton's total magnetic moment:

$$\begin{aligned}\mu &= \mu_p g_1^{\mathrm{SU}(6)} \left[\frac{\int d^3x\, r(u_u l_s + u_s l_u)}{\int d^3x\, r(2u_u l_u)} \right] \\ &\equiv g_1^{\mathrm{SU}(6)} \mu_p \rho_m ,\\ d &= \mu_p g_1^{\mathrm{SU}(6)} \left[\frac{\int d^3x\, r(u_u l_s - u_s l_u)}{\int d^3x\, r(2u_u l_u)} \right] \\ &\equiv g_1^{\mathrm{SU}(6)} \mu_p \rho_E .\end{aligned} \tag{34}$$

There is a very interesting universality feature to the above results, which is also present in the nonrelativistic quark model. If the wave-function overlaps do not depend much on the external states, which is generally true, then the total magnetic moment, the total $\Delta S=1$ electric dipole moment and g_1 (hence also g_2) all transform under SU(3) in an identical fashion. If one parametrizes the transformation properties by the usual d and f coefficients, this means that the calculated ratio using SU(6) wave functions is $d/f = \frac{2}{3}$ for all three. In fact, since in practice the d/f ratio for g_1 deviates slightly from $\frac{2}{3}$,[20] this universality can be generalized to state that g_1 and g_2 and the total transition magnetic moment all transform in the same way (i.e., with the same d/f ratio) when studying $\Delta S=1$ processes. The restriction to $\Delta S=1$ processes is important, as g_2 vanishes for $\Delta S=0$, while g_1 and the total moment undergo a shift in overall scale due to SU(3) breaking when transforming from $\Delta S=0$ to $\Delta S=1$.[20] Nevertheless the use of SU(3) parametrizations among the $\Delta S=1$ processes should be reliable.

In practice ρ_M and ρ_E do not vary much from state to state as long as one is dealing with $\Delta S=1$ transitions. A bag-model calculation, using the parameters of Ref. 8, yields

$$\rho_M = 0.86 ,\qquad \rho_E = 0.094 . \tag{35}$$

For comparison, the nonrelativistic quark model yields[21]

$$\begin{aligned}\rho_M &= \left[\frac{1}{M_u^*} + \frac{1}{M_s^*} \right] \frac{M_u^*}{2} \cong 0.8 ,\\ \rho_E &= \left[\frac{1}{M_u^*} - \frac{1}{M_s^*} \right] \frac{M_u^*}{2} \cong 0.2\end{aligned} \tag{36}$$

if we use $M_u^*/M_s^* \sim \frac{3}{5}$. Solving for g_2/g_1, we find

$$\begin{aligned}\frac{g_2}{g_1} &= \left[\frac{g_1^{\mathrm{SU}(6)}}{g_1} \right] (m_1+m_2)\mu_p \rho_E - \frac{1}{4} \left[\frac{M_1}{M_2} - \frac{M_2}{M_1} \right] \\ &= 3.7 \left[\frac{m_1+m_2}{2m_p} \right] \rho_E - \frac{1}{4} \left[\frac{M_1}{M_2} - \frac{M_2}{M_1} \right] ,\end{aligned} \tag{37}$$

where we have utilized the experimental g_1 in neutron β decay to determine $g_1^{\mathrm{SU}(6)}/g_1 = 5/3 \times 1.25$.

As the masses do not vary greatly, g_2/g_1 will be nearly a universal constant. We shall quote its value in Λ β decay, where the bag model predicts

$$\frac{g_2}{g_1} \cong 0.30 \tag{38}$$

while the nonrelativistic quark model yields

$$\frac{g_2}{g_1} \cong 0.73 \ . \tag{39}$$

The same can be done for f_2:

$$\frac{f_2}{f_1} = \frac{g_1}{f_1} \frac{g_1^{\mathrm{SU}(6)}}{g_1}(m_1+m_2)\mu_p\rho_M - \frac{1}{2}\left[\frac{m_2}{m_1}+\frac{m_1}{m_2}\right] . \tag{40}$$

In this case f_2/f_1 (or even f_2/g_1) is not universal as g_1/f_1 varies considerably from transition to transition.

In order to provide a benchmark for future experimental comparisons, we tabulate the bag-model values of f_1,f_2,g_1,g_2 for various transitions in Table I. In doing so, we have included SU(3) breaking in f_1 and g_1 due to wave-function mismatches between strange and nonstrange quarks (a renormalization factor which is calculated to be 0.97 for f_1 and 1.08 for g_1 when $\Delta S=1$), as first noted in Ref. 20.

The results of SU(3) breaking in the quark-model calculations of f_1, f_2, and g_1 are not new, although we do not know of any place where they are treated in a unified manner, and the universality of the SU(3) behavior of μ, g_1, and g_2 appears not to have been noted. The calculation of g_2 in the quark model is new, and we expect a nonzero g_2, contrary to common usage. This can be important, as experimental values of coupling constants extracted from the energy distributions often show strong correlations with g_2.[22] The sign of the quark-model result is quite clear, $g_2>0$. The magnitude depends on a cancellation of matrix elements and is somewhat more sensitive to model-dependent features. However, the bag-model value $g_2/g_1=0.30$ appears quite reasonable.

Note added in proof. A calculation of g_2 using broken SU(6) [B. H. Kellet, Phys. Rev. D 10, 2269 (1974)] has been pointed out to us. Kellet's results are similar to ours, except for his surprisingly large g_2 in $\Delta S=0$ neutron β decay.

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TABLE I. Form factors as predicted by the quark model for semileptonic baryon decay. Here $\alpha_D=D/(D+F)$ [exact SU(6) predicts $\alpha_D=\frac{3}{5}$], $g_A\cong 1.25$ is the axial-vector coupling in neutron β decay, and $\eta_V\cong 0.97$, $\eta_A\cong 1.08$, $\rho_E\cong 0.094$, and $\rho_M\cong 0.86$ are various SU(3)-breaking factors discussed in the text.

Reaction	f_1	f_2	g_1	g_2
$n\to pe^-\bar\nu_e$	1	$3.7g_1-f_1$	g_A	0
$\Sigma^+\to\Lambda e^+\nu_e$	0	$4.55g_1-1.00f_1$	$g_A(\frac{2}{3})^{1/2}\alpha_D$	$-0.03g_1$
$\Sigma^-\to\Lambda e^-\bar\nu_e$	0	$4.55g_1-1.00f_1$	$g_A(\frac{2}{3})^{1/2}\alpha_D$	$-0.03g_1$
$\Sigma^-\to\Sigma^0 e^-\bar\nu_e$	$\sqrt{2}$	$4.69g_1-f_1$	$g_A\sqrt{2}(1-\alpha_D)$	0
$\Sigma^0\to\Sigma^+ e^-\bar\nu_e$	$-\sqrt{2}$	$4.69g_1-f_1$	$-g_A\sqrt{2}(1-\alpha_D)$	0
$\Xi^-\to\Xi^0 e^-\bar\nu_e$	-1	$5.21g_1-f_1$	$-g_A(1-2\alpha_D)$	0
$\Lambda\to pe^-\bar\nu_e$	$-(\frac{3}{2})^{1/2}\eta_V$	$4.05\rho_M g_1-1.01f_1$	$-g_A(\frac{3}{2})^{1/2}\eta_A(1-\frac{2}{3}\alpha_D)$	$(4.05\rho_E-0.09)g_1$
$\Sigma^0\to pe^-\bar\nu_e$	$-\frac{1}{\sqrt{2}}\eta_V$	$4.20\rho_M g_1-1.03f_1$	$-g_A\frac{1}{\sqrt{2}}\eta_A(1-2\alpha_D)$	$(4.20\rho_E-0.12)g_1$
$\Sigma^-\to ne^-\bar\nu_e$	$-\eta_V$	$4.20\rho_M g_1-1.03f_1$	$-g_A\eta_A(1-2\alpha_D)$	$(4.20\rho_E-0.12)g_1$
$\Xi^-\to\Lambda e^-\bar\nu_e$	$(\frac{3}{2})^{1/2}\eta_V$	$4.80\rho_M g_1-1.01f_1$	$g_A(\frac{3}{2})^{1/2}\eta_A(1-\frac{4}{3}\alpha_D)$	$(4.80\rho_E-0.08)g_1$
$\Xi^-\to\Sigma^0 e^-\bar\nu_e$	$\frac{1}{\sqrt{2}}\eta_V$	$4.95\rho_M g_1-1.00f_1$	$g_A\frac{1}{\sqrt{2}}\eta_A$	$(4.95\rho_E-0.05)g_1$
$\Xi^0\to\Sigma^+ e^-\bar\nu_e$	η_V	$4.95\rho_M g_1-1.00f_1$	$g_A\eta_A$	$(4.95\rho_E-0.05)g_1$

APPENDIX

Equations (26) and (27) can be derived using the wave-packet formalism of Donoghue and Johnson.[18] A quark-model state is in general not a momentum eigenstate but can be described by a superposition of such states:

$$|B,s\rangle=\int d^3p\frac{m}{E}\chi_\lambda^s(p)\,|B(p),\lambda\rangle\ , \tag{A1}$$

where $\chi_\lambda^s(p)$ is the wave packet for constructing the localized quark state. For simplicity at low p, χ_λ^s may be taken to be $\chi_\lambda^s(p)=\delta_{s\lambda}\chi(p)$. The normalizations are

$$\langle B,s'\,|\,B,s\rangle=\delta_{ss'}\ ,$$

$$\langle B(p'),\lambda'\,|\,B(p),\lambda\rangle=\delta_{\lambda\lambda'}\frac{E}{m}(2\pi)^3\delta^3(p-p')\ , \tag{A2}$$

and

$$\int d^3p\frac{m}{E}\,|\chi(p)|^2(2\pi)^3=1\ . \tag{A3}$$

For most quark states $\langle p^2\rangle/M^2$ is small, and standard results are obtained by neglecting it. To calculate magnetic moments one studies

$$\begin{aligned}\vec{\mu}_i&=\left\langle B',s'\left|\int d^3r\,\tfrac{1}{2}(\vec{r}\times\vec{V})_i\right|B,s\right\rangle\\&=\int d^3r\,d^3p\frac{m}{E}d^3p'\frac{m'}{E'}\chi^*(p')\chi(p)\frac{\epsilon^{ijk}}{2}r_j\langle B'(p'),s'\,|\,\vec{V}_k(\vec{x})\,|\,B(p),s\rangle\\&=\int d^3r\,d^3p\frac{m}{E}d^3p'\frac{m'}{E'}\chi^*(p')\chi(p)\frac{\epsilon_{ijk}}{2}r_j\bar{u}(p')\left[f_1\gamma_k-i\frac{f_2\sigma_{k\nu}q^\nu}{m_1+m_2}\right]u(p)e^{i\vec{q}\cdot\vec{r}}\ .\end{aligned} \tag{A4}$$

Noting that (for $p/M<<1$)

$$\bar{u}(p')\left[f_1\gamma_k-i\frac{f_2\sigma_{k\nu}q^\nu}{m_1+m_2}\right]u(p)e^{i\vec{q}\cdot\vec{r}}\cong(\chi_{s'}^\dagger\vec{\sigma}\chi_s\times\vec{\nabla})_k\left[f_1\frac{1}{2}\left[\frac{1}{2m_1}+\frac{1}{2m_2}\right]+f_2\frac{1}{m_1+m_2}\right]e^{i\vec{q}\cdot\vec{r}}\ , \tag{A5}$$

we have, upon integration by parts,

$$\begin{aligned}\mu_i&=\chi_{s'}^\dagger\sigma_i\chi_s\left[f_1\frac{1}{2}\left[\frac{1}{2m_2}+\frac{1}{2m_1}\right]+f_2\frac{1}{m_1+m_2}\right]\int d^3r\int d^3p'd^3p\,\chi^*(p')\chi(p)e^{i(\vec{p}-\vec{p}\,')\cdot\vec{r}}\\&=\chi_{s'}^\dagger\sigma_i\chi_s\left[f_1\frac{1}{2}\left[\frac{1}{2m_1}+\frac{1}{2m_2}\right]+f_2\frac{1}{m_1+m_2}\right]\end{aligned} \tag{A6}$$

as promised.

In a corresponding way we can obtain Eq. (27). We note that

$$\begin{aligned}d_i&=\left\langle B',s'\left|-i\int d^3x\,r_iA_0(x)\right|B,s\right\rangle\\&=-i\int d^3r\int d^3p'\frac{m'}{E'}d^3p\frac{m}{E}\chi^*(p')\chi(p)r_i\bar{u}(p')\left[g_1\gamma_0\gamma_5-i\frac{g_2}{m_1+m_2}\sigma_{0\nu}q^\nu\gamma_5\right]u(p)e^{i\vec{q}\cdot\vec{r}}\ .\end{aligned} \tag{A7}$$

Since

$$\bar{u}(p')\left[g_1\gamma_0\gamma_5-i\frac{1}{m_1+m_2}g_2\sigma_{0\nu}q^\nu\gamma_5\right]u(p)e^{iq\cdot r}\approx-i\chi_{s'}^\dagger\vec{\sigma}\chi_s\cdot\vec{\nabla}\left[\tfrac{1}{2}g_1\left[\frac{1}{2m_2}-\frac{1}{2m_1}\right]+\frac{g_2}{m_1+m_2}\right]e^{i\vec{q}\cdot\vec{r}} \tag{A8}$$

an integration by parts yields

$$\begin{aligned}d_i&=\chi_{s'}^\dagger\sigma_i\chi_s\left[\frac{1}{2}g_1\left[\frac{1}{2m_2}-\frac{1}{2m_1}\right]+\frac{g_2}{m_1+m_2}\right]\int d^3r\int d^3p\,d^3p'\chi^*(p')\chi(p)e^{i(\vec{p}-\vec{p}\,')\cdot\vec{r}}\\&=\chi_{s'}^\dagger\sigma_i\chi_s\left[\frac{1}{2}g_1\left[\frac{1}{2m_2}-\frac{1}{2m_1}\right]+\frac{g_2}{m_1+m_2}\right]\end{aligned} \tag{A9}$$

which is the desired result.

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[12]We use

$$f_2=\mu_p-\mu_n-1=3.70, \quad n\to p \ ,$$

$$f_2=\frac{g_1}{f_1}\frac{4}{3}(m_\Lambda+m_p)\mu_p\rho_n - \frac{1}{2}\left[\frac{m_\Lambda}{m_p}+\frac{m_p}{m_\Lambda}\right]=1.61, \quad \Lambda\to p \ .$$

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[17]The normalization condition on the four-component momentum-space wave function $\psi(p)$ is

$$\int d^3x\, \psi^\dagger(x)\psi(x)=1 \ ,$$

where

$$\psi(x)=\int \frac{d^3p}{(2\pi)^{3/2}}e^{i\vec{p}\cdot\vec{x}}\psi(p) \ .$$

This is equivalent to

$$\int d^3p\, |\phi(p)|^2=1$$

if one drops terms of order $p^2/4M^2$.

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$$\int d^3p\, \phi_u^*(p)\phi_s(p)\le 1 \ .$$

In this sense, there does not exist a clear prediction in the case of the nonrelativistic quark model unless one knows $\phi(p)$ explicitly.

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