Evidence for SU(3) breaking in Cabibbo-type fits of semileptonic hyperon decay

John F. Donoghue and Barry R. Holstein
Department of Physics and Astronomy, University of Massachusetts, Amherst, Massachusetts

(Received 18 December 1981)

We analyze recent precision experiments involving semileptonic hyperon decay and demonstrate that there exists definite evidence for SU(3) breaking. The type of breaking observed is a difference in the $D/(F+D)$ ratios in the $\Delta S=0$ and $\Delta S=1$ sectors, which is at variance with the simplest quark-model expectations. We comment on the relevance of symmetry breaking to the determination of weak mixing angles.

Since 1963 when Cabibbo wrote his landmark paper,[1] the analysis of semileptonic hyperon decays has been based upon this very simple and elegant framework—a current-current weak interaction

$$H_w=\frac{G_\mu}{\sqrt{2}}J_\lambda l^\lambda \tag{1}$$

where $G_\mu=1.166\,32\times10^{-5}$ GeV^{-2} is the weak coupling constant observed in muon decay,[2]

$$l^\lambda=\bar\psi_e\gamma^\lambda(1+\gamma_5)\psi_{\nu_e}+\bar\psi_\mu\gamma^\lambda(1+\gamma_5)\psi_{\nu_\mu} \tag{2}$$

is the leptonic current, and

$$J_\lambda=\cos\theta_C\bar u\gamma_\lambda(1+\gamma_5)d+\sin\theta_C\bar u\gamma_\lambda(1+\gamma_5)s \tag{3}$$

is the weak hadronic current. Taking matrix elements between two hyperons, one finds

$$\langle B'_{p_2}|J_\lambda|B_{p_1}\rangle=\begin{Bmatrix}\cos\theta_C\\ \text{or}\\ \sin\theta_C\end{Bmatrix}\bar u(p_2)\left\{f_1\gamma_\lambda-i\sigma_{\lambda\nu}q^\nu\frac{f_2}{m_1+m_2}+\frac{f_3}{m_1+m_2}q_\lambda+g_1\gamma_\lambda\gamma_5-i\sigma_{\lambda\nu}q^\nu\gamma_5\frac{g_2}{m_1+m_2}+\frac{g_3}{m_1+m_2}q_\lambda\gamma_5\right\}u(p_1)\ , \tag{4}$$

where $q=p_1-p_2$ and the various form factors $f_1,\ldots,g_3$ are functions of q^2.

Analysis of hyperon decay has conventionally utilized the following.

(i) $f_3=g_2=0$ from G invariance plus SU(3) symmetry.[3]

(ii) f_1 and f_2 determined from CVC (conserved vector current) and SU(3). That is, characterizing the SU(3) transformation properties of B',J_λ,B by i,j,k, respectively, we write

$$\begin{aligned}f_1(0)&=if_{ijk}\ ,\\ f_2(0)&=if_{ijk}f+d_{ijk}d\ .\end{aligned} \tag{5}$$

The Ademollo-Gatto theorem[4] guarantees that deviations of $f_1(0)$ from this result will be second order in symmetry breaking. However, no such assurance exists for f_2. Nevertheless, using exact SU(3) we can write the couplings f,d in terms of neutron and proton anomalous moments

$$f=\kappa_p+\tfrac{1}{2}\kappa_n=0.84\ ,\quad d=-\tfrac{3}{2}\kappa_n=2.86\ . \tag{6}$$

(iii) g_1 parametrized by SU(3). Although again there is no Ademollo-Gatto theorem in this case so that effects can arise in principle which are first order in symmetry breaking, we utilize exact SU(3) to write

$$g_1(0)=if_{ijk}F+d_{ijk}D\ . \tag{7}$$

Fits to date, using these assumptions and including no symmetry breaking, have been uniformly excellent. Typical is a recent fit by Shrock and Wang,[5] which yields

$$\sin\theta_C=0.222,\quad F=0.429,\quad D=0.824\ , \tag{8}$$

and a $\chi^2=6.9$ for six degrees of freedom.

The surprising thing about these results is that although SU(3) breaking is generally of order 20%, the standard Cabibbo framework, employing no breaking at all, provides an excellent fit, as indicated above. However, modern quark models generally predict that definite SU(3) breaking should exist at some level thereby affecting both the determination of the Cabibbo angle and the quality of the semileptonic fits.[6]

In the six-quark version of the weak interaction the semileptonic interaction of Eq. (4) retains the same form but the coefficients "$\cos\theta_C$", "$\sin\theta_C$" of the $\Delta S=0$, $\Delta S=1$ currents are now written as

$$\begin{aligned}\cos\theta_C&=\cos\theta_1\ ,\\ \sin\theta_C&=\sin\theta_1\cos\theta_3\ ,\end{aligned}$$

where θ_1, θ_3 are the usual Kobayashi-Maskawa (KM) angles. Thus we now have

$$\cos^2\theta_1+\sin^2\theta_1\cos^2\theta_3<1 \ , \tag{9}$$

rather than by Cabibbo universality

$$\cos^2\theta_C+\sin^2\theta_C=1 \ . \tag{10}$$

This feature should *also* invalidate the conventional Cabibbo fit at some level. Hyperon semileptonic decays are then especially interesting in this context as they provide the strongest information on the KM angles, bounding θ_3 from above.[5]

Recently, high-statistics experiments on various hyperon decay modes have become available, involving thousands of events rather than the previous hundreds.[7] We have studied the new data within the Cabibbo framework searching for the SU(3) breaking predicted by the quark model and assessing its effects on the weak mixing angles. This Communication reports that a definite failure of the SU(3) Cabibbo fits is indicated, revealing, however, a different pattern of symmetry breaking than expected in the simplest quark models.

For our primary analysis, we shall fit only the experimental hyperon semileptonic rates. The g_1/f_1 ratios and asymmetry measurements have in general been analyzed using at least some of the assumptions of SU(3) symmetry (i.e., $g_2=0$). However, strong correlations can exist between the measured ratios and some symmetry breaking (i.e., a nonzero g_2).[8] Since we do find evidence for SU(3) breaking in the analysis of the branching ratios, the measured values of g_1/f_1 need to be treated with some caution. Thus we shall indicate the effects of including g_1/f_1 values, but our principal analysis will utilize the decay rates.

After completing most of our work, we learned that Garcia and Kielanowski have independently uncovered similar evidence for SU(3) breaking.[9] Although clearly related, their analysis is somewhat different and complementary to ours.

An important feature of our approach is the treatment of the radiative corrections, which must be carefully handled at the present levels of experimental precision. We separate the radiative correction into three components: (i) a model-independent and lepton-energy-dependent term whose explicit form has been given by Sirlin[10]; (ii) for the case of neutral-hyperon decay a Coulombic $\pi^2\alpha/v$ correction term which accounts for the interaction between the final charged baryon and the charged lepton; and (iii) a model-dependent and lepton-energy-independent piece whose form has been given by Abers, Dicus, Norton, and Quinn[11] and improved by Sirlin, who has shown recently that the same form should obtain for Fermi and for Gamow-Teller theory.[12]

We now begin by fixing $\cos\theta_1$ from analysis of 0^+-0^+ Fermi transitions in nuclear β decay[13]

$$\cos\theta_1=0.9737\pm0.0025 \ . \tag{11}$$

Secondly, from measurement of the electron and neutrino asymmetries in neutron β decay we fix the axial-vector coupling in neutron β decay[7]

$$g_1^{np}(0)=1.253\pm0.007 \ . \tag{12}$$

Assuming now *no* SU(3) breaking—i.e., the usual Cabibbo fit—we attempt to fit the ten branching ratios in terms of two free parameters—the ratio

$$\alpha_D=\frac{D}{F+D} \ ,$$

and the $\Delta S=1$ current coupling

$$\sin\theta_1\cos\theta_3 \ .$$

The results are shown in Table I, with the best-fit

TABLE I. The results of an SU(3) fit to semileptonic data. Experimental data are from Ref. 7. The fitted parameters are quoted in the text.

Quantity	Experiment	Rates alone: Fitted value	Rates alone: Contribution to χ^2	Rates plus g_1/f_1: Fitted value	Rates plus g_1/f_1: Contribution to χ^2
$\Gamma(n\to pe\nu)$	$(1.091\pm0.017)\times10^{-3}$	1.10×10^{-3}	0.58	1.10×10^{-3}	0.58
$\Gamma(\Sigma^+\to\Lambda e\nu)$	$(0.253\pm0.059)\times10^{6}$	0.276×10^{6}	0.15	0.276×10^{6}	0.16
$\Gamma(\Sigma^-\to\Lambda e\nu)$	$(0.370\pm0.020)\times10^{6}$	0.456×10^{6}	18.6	0.458×10^{6}	19.2
$\Gamma(\Lambda\to pe\nu)$	$(3.19\pm0.05)\times10^{6}$	3.23×10^{6}	0.57	3.25×10^{6}	1.54
$\Gamma(\Lambda\to p\mu\nu)$	$(0.597\pm0.133)\times10^{6}$	0.552×10^{6}	0.11	5.57×10^{6}	0.09
$\Gamma(\Sigma^-\to ne\nu)$	$(7.20\pm0.20)\times10^{6}$	6.62×10^{6}	4.09	6.70×10^{6}	2.55
$\Gamma(\Sigma^-\to n\mu\nu)$	$(3.04\pm0.27)\times10^{6}$	3.11×10^{6}	0.08	3.15×10^{6}	0.18
$\Gamma(\Xi^-\to\Lambda e\nu)$	$(3.27\pm0.22)\times10^{6}$	2.88×10^{6}	3.12	2.90×10^{6}	2.80
$\Gamma(\Xi^-\to\Sigma^0 e\nu)$	$(0.52\pm0.12)\times10^{6}$	0.508×10^{6}	0.09	0.513×10^{6}	0.02
$(g_1/f_1)(\Lambda\to pe\nu)$	0.717 ± 0.03	...	...	0.723	0.04
$(g_1/f_1)(\Sigma^-\to ne\nu)$	-0.387 ± 0.034	...	...	-0.339	2.03
$(g_1/f_1)(\Xi^-\to\Lambda e\nu)$	0.248 ± 0.050	...	...	0.192	1.24
$(f_1/g_1)(\Sigma\to\Lambda e\nu)$	0.034 ± 0.090	...	...	0	0.14

value of

$$\alpha_D = 0.634 \pm 0.008 \tag{13}$$

very close to the SU(6) value

$$\alpha_D = \frac{3}{5} \ , \tag{14}$$

while the best-fit value of

$$\sin\theta_1 \cos\theta_3 = 0.224 \pm 0.002 \tag{15}$$

is consistent with the value

$$\sin\theta_1 = 0.228 \pm 0.010 \tag{16}$$

expected if $\theta_3 = 0$. However, the overall $\chi^2 \cong 27$ for the fit is obviously much too large, given the seven degrees of freedom, to be acceptable. The quoted errors correspond to an increase in χ^2 by one unit.

The origin of this large χ^2 is in the $\Delta S = 0$ mode $\Sigma^- \to \Lambda e^- \bar{\nu}_e$, which alone contributes about 19 to the χ^2. In fact, if we omit the $\Sigma^\pm \to \Lambda e^- \bar{\nu}_e$ rates from consideration a very good fit is found, as shown in Table II, with a $\chi^2 \cong 5$ for four degrees of freedom. The fitted values

$$\alpha_D = 0.644 \pm 0.009, \quad \sin\theta_1 \cos\theta_3 = 0.225 \pm 0.002 \tag{17}$$

are very close to the values found in the overall fit.

On the other hand, a fit of the $\Delta S = 0$ decays alone, $n \to p e^- \bar{\nu}_e$, $\Sigma^\pm \to \Lambda e^\pm \nu_e$, as shown in Table II yields

$$\alpha_D = 0.572 \pm 0.015 \ , \tag{18}$$

which is now *less than* the SU(6) value. This clash between $\Delta S = 0$ decays requiring $\alpha_D < 0.6$ and $\Delta S = 1$ decays demanding $\alpha_D > 0.6$ is the origin of the poor fit obtained when both are taken together.

These conclusions are unchanged if g_1/f_1 values are included. Fitting all available data yields $\chi^2 = 31$ for eleven degrees of freedom with

$$\alpha_D = 0.635 \pm 0.005, \quad \sin\theta_C = 0.225 \pm 0.002 \ . \tag{19}$$

Use of $\Delta S = 1$ data alone ($\chi^2 = 7.3$ for seven degrees of freedom) gives

$$\alpha_D = 0.645 \pm 0.006, \quad \sin\theta_C = 0.225 \pm 0.002 \ , \tag{20}$$

while fitting $\Delta S = 0$ data ($\chi^2 = 1.0$ for three degrees of freedom) gives

$$\alpha_D = 0.57 \pm 0.015 \ . \tag{21}$$

This pattern of symmetry breaking is not, however, the one expected from simple quark models,[6] which suggest that g_1 (and to a lesser extent f_1) will undergo an overall shift in scale when comparing $\Delta S = 0$ to $\Delta S = 1$ (due to the difference between $d \to u$ and $s \to u$ transitions) but that the $\alpha_D = D/(D+F)$ ratio should be the same for both the $\Delta S = 0$ and $\Delta S = 1$ sectors. This pattern would have a strong effect on the mixing-angle determination, as a relative shift in $\Delta S = 1$ compared to $\Delta S = 0$ would be compensated in the fit by a corresponding shift in the apparent mixing angle, $\sin\theta_1 \cos\theta_3$.

We have attempted to see if such a quark-model pattern might be acceptable, by including a renormalization of all $\Delta S = 1$ axial-vector couplings by a factor η_A [$\eta_A = 1$ corresponds to the SU(3) limit], and have refit the decay rates using η_A as a free parameter. Again the fit is poor, with

$$\begin{aligned} &\eta_A = 0.94 \pm 0.01, \quad \alpha_D = 0.625 \pm 0.008 \ , \\ &\sin\theta_1 \cos\theta_3 = 0.231 \pm 0.002 \end{aligned} \tag{22}$$

and $\chi^2 = 20.6$ for six degrees of freedom. Once more it is the decay $\Sigma^- \to \Lambda e \nu$ which is responsible for the poor fit.

Removing $\Sigma \to \Lambda e \nu$ from consideration, we can look at the $\Delta S = 1$ decays by themselves. Including only the rates, one can obtain good fits to the rate data for a wide range of values of η_A, $0.8 \leq \eta_A \leq 1.05$, due to the compensation of a shift in η_A by a shift in $\sin\theta_1 \cos\theta_3$. The correlation is roughly

$$\begin{aligned} &\sin\theta_1 \cos\theta_3 = 0.225 + 0.01(1 - \eta_A) \ , \\ &\frac{D}{D+F} = 0.63 - 0.1(1 - \eta_A) \ . \end{aligned} \tag{23}$$

TABLE II. Separate fits to $\Delta S = 0$ and $\Delta S = 1$ rates. The fitted parameters are quoted in the text.

		$\Delta S = 0$		$\Delta S = 1$ fit	
Rate	Experiment	Fitted value	Contribution to χ^2	Fitted value	Contribution to χ^2
$n \to p e \nu$	$(1.091 \pm 0.014) \times 10^{-3}$	1.10×10^{-3}	0.58	...	...
$\Sigma^+ \to \Lambda e \nu$	$(0.253 \pm 0.059) \times 10^6$	0.22×10^6	0.42	...	...
$\Sigma^- \to \Lambda e \nu$	$(0.370 \pm 0.020) \times 10^6$	0.36×10^6	0.49	...	...
$\Lambda \to p e \nu$	$(3.19 \pm 0.05) \times 10^6$	...	...	3.21×10^6	0.17
$\Lambda \to p \mu \nu$	$(0.597 \pm 0.133) \times 10^6$	...	...	0.55×10^6	0.13
$\Sigma^- \to n e \nu$	$(7.02 \pm 0.20) \times 10^6$	...	...	6.94×10^6	0.15
$\Sigma^- \to n \mu \nu$	$(3.04 \pm 0.27) \times 10^6$	...	...	3.27×10^6	0.72
$\Xi^- \to n e \nu$	$(3.27 \pm 0.22) \times 10^6$	...	...	2.86×10^6	3.53
$\Xi^- \to \Sigma^0 e \nu$	$(0.52 \pm 0.12) \times 10^6$	...	...	0.51×10^6	0.03

If in addition the vector charge had some SU(3) breaking (parametrized by η_V) the mixing angles could change more dramatically

$$\sin\theta_1\cos\theta_3=\frac{0.225}{\eta_V}+0.01\frac{(1-\eta_A)}{\eta_V} \quad . \tag{24}$$

The range of allowed axial-vector couplings can be reduced by inclusion of the g_1/f_1 values. With them, a good fit can be obtained for η_A (or η_A/η_V if $\eta_V \neq 1$) ranging from 0.95 to 1.05, with the best fit being

$$\eta_A=0.99\pm0.06,\quad \alpha_D=0.644\pm0.006\ ,$$
$$\sin\theta_1\cos\theta_3=0.226\pm0.002 \tag{25}$$

with $\chi^2=7.2$ for seven degrees of freedom. There is no evidence for $\eta_A \neq 1$; however, small (5%) SU(3) breaking of this sort can be accommodated.

In summary, present data yield a clear indication of symmetry breaking, with the rate for $\Sigma^- \rightarrow \Lambda e \nu$ being four standard deviations from the value required by the Cabibbo fit to $\Delta S=1$ decays. Confirmation of this measurement would be very helpful. The pattern of symmetry breaking appears to be that the axial-vector current transforms with a slightly different SU(3) ratio α_D in the $\Delta S=0$ and $\Delta S=1$ sectors. In addition 5% SU(3)-violating shifts in the overall scale of the $\Delta S=1$ axial-vector-current couplings may be present without significantly changing the quality of the $\Delta S=1$ fit. The corresponding range in $\sin\theta_1\cos\theta_3$ is

$$0.220 \leq \sin\theta_1\cos\theta_3 \leq 0.230 \quad . \tag{26}$$

Further work on the mechanisms of symmetry breaking is needed in order to fully assess the effect on the determination of the weak mixing angles.

It is a pleasure to thank D. Jensen, A. Garcia, and M. Kreisler for many useful discussions. This work is supported in part by the National Science Foundation.

---

[1]N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963).

[2]We use the muon lifetime quoted by the Particle Data Group [Rev. Mod. Phys. 52, S1 (1980)], together with

$$G_\mu{}^2=\frac{192\pi^3}{\tau_\mu}\frac{m_\mu{}^2}{m_e{}^2}\frac{1}{f(1-(\alpha/2\pi)\frac{1}{4}(\pi^2-25))}\ ,$$

where

$$f(x)=1-8x+8x^3-x^4-12x^2\ln x\ .$$

[3]See, e.g., R. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley Interscience, New York, 1969).

[4]M. Ademollo and R. Gatto, Phys. Lett. 13, 264 (1964).

[5]R. E. Shrock and L. L. Wang, Phys. Rev. Lett. 41, 1692 (1978).

[6]See, e.g., J. F. Donoghue and R. R. Holstein, Phys. Rev. D 25, 206 (1982); B. H. Kellett, Phys. Rev. D 10, 2269 (1974)

[7]M. Bourquin *et al.* (WA2 collaboration), Université Paris-Sud Report No. LAL-81/18 and contribution to International Conference on High Energy Physics, Lisbon, Portugal, 1981 (unpublished); J. Wise *et al.*, Phys. Lett. 91B, 165 (1980); 98B, 123 (1981); Particle Data Group, Rev. Mod. Phys. 52, S1 (1980).

[8]D. A. Jensen (private communication).

[9]A. Garcia and P. Kielanowski, Phys. Lett. (to be published); A. Garcia (private communication).

[10]A. Sirlin, Phys. Rev. 164, 1767 (1967).

[11]E. Abers, D. Dicus, R. Norton, and H. Quinn, Phys. Rev. 167, 1461 (1968).

[12]A. Sirlin, NYU report, 1981 (unpublished); see also A. Garcia, Phys. Rev. D 25, 1348 (1982); A. Sirlin, Rev. Mod. Phys. 50, 576 (1978).

[13]D. H. Wilkinson and D. Alburger, Phys. Rev. C 13, 2517 (1976).