Estimate of the quark-gluon coupling strength from baryon masses

J. Franklin
Physics Department, Temple University, Philadelphia, Pennsylvania 19122

D. B. Lichtenberg
Physics Department, Indiana University, Bloomington, Indiana 47405
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The magnetic and color-magnetic spin-spin interactions are used to relate the fine-structure constant α to the corresponding strong-interaction constant α_s in terms of measured baryon masses and the nucleon-quark effective mass obtained from nucleon magnetic moments. An expression for the effective mass difference between the d and u quark is also obtained. The method gives $\alpha_s=0.65$ and $d-u=2.8$ MeV.

Sakharov[1] has used the magnetic and color-magnetic quark spin-spin interactions[2,3] to relate the quark-gluon coupling constant α_s to the electromagnetic constant α. The Sakharov approach exploits the similarity between the chromodynamic and electromagnetic spin-spin interactions and assumes approximate SU(6) symmetry of the hadron wave functions to derive sum rules that give the ratio α_s/α independently of other details of the hadron wave functions.[4] It is complementary to calculations that assume more specific wave functions to calculate baryon masses explicitly in terms of various parameters. With a lack of information about the complete Hamiltonian and the difficulty involved in making a full, relativistic calculation, it is of interest to pursue both approaches.

Sakharov used vector- and pseudoscalar-meson masses as well as baryon decuplet and octet masses to relate the two coupling constants. His result thus depends on connecting mesons and baryons via the assumed color interaction. Furthermore, because he used electromagnetic mass differences of resonances which are not precisely measured, his result is not very accurate. In this note, we use the magnetic and color-magnetic spin-spin interactions to relate the two constants using only baryon masses and only octet electromagnetic mass differences. To do this we have to assume that these spin-spin interactions are the only SU(3)-breaking interactions. The Fermi-Breit interaction[3] suggested by one-gluon exchange in quantum chromodynamics (QCD) contains other, spin-independent, SU(3)-breaking terms which are not included in our derivation. However, the exact form of SU(3) breaking is still a phenomenological question, and there is no indication that additional SU(3)-breaking interactions are required to correlate hadron masses. The assumption that the spin-spin interaction is the only one which breaks SU(3) has been used in calculating the strange-quark $-$nucleon-quark mass difference[5] and Σ^0-Λ mixing,[6] as well as hadron mass differences.[2,7] In each case, the result agrees well with calculations not making this assumption.

An approximate expression for the energy V_{ij} arising from the color-magnetic and electromagnetic spin-spin interactions of quarks of charge Q_i, Q_j in a relative s state is given by[3]

$$V_{ij}=(\lambda Q_iQ_j+\lambda_s)\vec{s}_i\cdot\vec{s}_j/m_im_j\ , \tag{1}$$

where m_i, m_j represent effective quark masses including possible relativistic contributions. The constants λ and λ_s are given by

$$\lambda=-(8\pi/3)\alpha\,|\,\psi_{ij}(0)\,|^2 \tag{2}$$

and

$$\lambda_s=(8\pi/3)(\tfrac{2}{3})\alpha_s\,|\,\psi_{ij}(0)\,|^2\ , \tag{3}$$

where $\alpha=\frac{1}{137}$ is the electromagnetic coupling, α_s is the corresponding quark-gluon coupling in QCD, and $\psi_{ij}(0)$ is the square of the quark-quark wave function at the origin ($\vec{r}_{ij}=0$). In what follows we shall neglect the dependence of α_s on momentum transfer in the various baryons of the octet and decuplet. This means that the value of α_s we obtain is an average appropriate to these baryons.

Using Eq. (1), and neglecting λ compared to λ_s, we obtain the result that a measure of λ_s is given by[8]

$$\Sigma^0-\Lambda=(\lambda_s/m^2)(1-m/s) \tag{4}$$

or by

$$\Xi^*-\Xi=\tfrac{3}{2}\lambda_s/ms\ , \tag{5}$$

with m being an average nucleon-quark (u or d) effective mass. We can obtain other measures of λ_s, which do not differ much from that of Eq. (5), in terms of the masses of the Σ^* or Δ decuplet baryons. We use the Ξ^* because its narrow width permits less ambiguity in relating the resonance peak to a quark-model mass.[9] Equations (4) and (5) can also be combined to give a measure of s/m[10]:

$$s/m=1+\tfrac{3}{2}[(\Sigma^0-\Lambda)/(\Xi^*-\Xi)] =1.535\pm0.002\ . \tag{6}$$

We can get a measure of λ if we make the assumption that the spin-spin interaction of Eq. (1) is the only SU(3)-breaking interaction energy. Then we find

$$n-p+\Sigma^+-\Sigma^0=\left[2+\frac{s}{m}\right]\left[\frac{\lambda}{12ms}-\frac{1}{4}\frac{\delta\lambda_s}{ms}\right]\ , \tag{7}$$

where

$$\delta=(d-u)/m\ . \tag{8}$$

The quantities λ and $\delta\lambda_s$ are comparable in size, and neither can be neglected compared to the other. Equations (2) through (8) can be combined to give the ratio

$$\frac{\alpha}{\alpha_s}=-\frac{2\lambda}{3\lambda_s}=\frac{8(\Sigma^0-\Sigma^++p-n)}{2(\Xi^*-\Xi)+(\Sigma^0-\Lambda)}-2\delta =0.0285\pm0.0016-2\delta\ . \tag{9}$$

As a check on our assumptions, we can also find linear combinations including electromagnetic mass differences of baryon resonances to measure α/α_s. Since these depend only on spin differences they isolate the spin interaction of Eq. (1) [as did Eqs. (4) and (5)], and no further assumption is necessary. We find[11]

$$\frac{\alpha}{\alpha_s}=\frac{2(\Xi^{*0}-\Xi^{*-}+\Xi^--\Xi^0)}{(\Xi^*-\Xi)}-2\delta =0.030\pm0.008-2\delta \tag{10}$$

and

$$\frac{\alpha}{\alpha_s}=\frac{2(\Sigma^{*+}-\Sigma^{*-}+\Sigma^--\Sigma^+)}{(\Sigma^*-\Sigma)}-2\delta =0.029\pm0.008-2\delta\ , \tag{11}$$

which are very similar to formulas given by Sakharov.[1] These three estimates of α/α_s agree well within the experimental errors and provide added confidence in the assumption used in deriving Eq. (9). Beyond this confirmation, we do not make further use of Eqs. (10) and (11) in obtaining our final result.

In order to get a measure of δ, we again assume that Eq. (1) is the only two-body SU(3)-breaking interaction and, further, that the one-body energy difference between the d and u quarks is equal to $m\delta$. This is true in the nonrelativistic limit and may be approximately true including some relativistic effects.[12] Then we can write

$$n-p+\tfrac{1}{3}(\Sigma^++\Sigma^--2\Sigma^0)=m\delta-\delta\lambda_s/(2m^2)\ . \tag{12}$$

Solving Eq. (12) for δ, we obtain

$$\delta=\frac{6(n-p)+2(\Sigma^++\Sigma^--2\Sigma^0)}{6m-2(\Xi^*-\Xi)-3(\Sigma^0-\Lambda)}\ , \tag{13}$$

where we have also used Eqs. (4) and (6).

We use the proton and neutron magnetic moments to estimate the average effective nucleon-quark mass to be 330 MeV.[13] Then, from Eqs. (8) and (13), we get

$$\delta=0.0086\pm0.0002\ , \quad d-u=2.8\pm0.1\text{ MeV}\ . \tag{14}$$

An alternate measure of m can be obtained from the relation[5]

$$s-m=\Lambda-N\ , \tag{15}$$

which follows from the assumption that the spin-spin term is the only SU(3)-breaking interaction. Combining Eqs. (15) and (6), we obtain

$$m=\frac{2(\Lambda-N)(\Xi^*-\Xi)}{3(\Sigma^0-\Lambda)}=330\text{ MeV}\ , \tag{16}$$

in agreement with the value from magnetic moments. This result provides more evidence for our assumptions.

With these values of m and δ, our final result for α/α_s from Eq. (9) is

$$\alpha/\alpha_s=0.0113\pm0.0016\ , \tag{17}$$

so that

$$\alpha_s = 0.65 \pm 0.09 \ . \qquad (18)$$

The value of α_s in Sakharov's approach, obtained from the $\Xi^{*0,-}$, $\Xi^{0,-}$, $K^{*0,+}$, and $K^{0,+}$ masses, is $\alpha_s = 0.25^{+0.09}_{-0.05}$. (This value differs somewhat from that in Sakharov's paper because we have used the more recent experimental masses of Ref. 10 in Sakharov's equations.) The reason for the small value of α_s obtained by Sakharov is that his equations give essentially zero for the $d-u$ mass difference. This may be connected with experimental uncertainties in the $K^{*0}-K^{*+}$ mass difference.

Le Yaouanc *et al.*[14] estimated α_s in an approach similar to ours, but neglecting the effect of quark mass differences on the spin-spin interaction. They obtained equations similar to Eqs. (7) and (9) but without the s/m factor in Eq. (7) or the term -2δ in Eq. (9), leading to a value $\alpha_s = 0.3$. They also estimated $|\psi_{ij}(0)|^2 = 1.5 \text{ fm}^{-3}$ for which we find, from Eqs. (3), (4), and (18), $|\psi_{ij}(0)|^2 = 0.86 \pm 0.12 \text{ fm}^{-3}$.

Our result for α_s agrees well with the value $\alpha_s = 0.60$ obtained by Itoh *et al.*[15] and the value $\alpha_s = 0.65$ obtained by Miura,[16] both using oscillator models with relativistic kinematics to fit hadron electromagnetic mass differences. However, Itoh *et al.* and Miura obtained a somewhat larger value of the $d-u$ mass difference (3.8 MeV) than our effective mass difference. We note that our value of α_s is considerably smaller than the values $\alpha_s = 1.6$ to 1.8 used in fits to strong baryon mass differences,[17,18] with the neglect of electromagnetic effects. Isgur[19] has extended the model of Ref. 18 to electromagnetic mass differences using a nucleon quark mass difference of 6 MeV. However, the quantity measured by our result $d-u = 2.8$ MeV is the difference in one-body quark energies (including kinetic energies) for which Isgur also finds 3 MeV.

The Isgur-Karl model of Refs. 18 and 19 includes some wave-function distortion (in perturbation theory) that is neglected in our approach, but the main reason we find a smaller value of α_s from electromagnetic mass differences is the difference in our treatment of the energy denominator of the spin-spin interaction of Eq. (1). We consider the m_i, m_j to be one-body quark energies taken approximately equal to the mass plus other one-body energies used in baryon mass calculations. The conditions for which this may be appropriate are discussed by Cohen and Lipkin[12] who also emphasize that this procedure may account for some of the surprising success of apparently nonrelativistic quark models in what is probably a relativistic regime. In Refs. 18 and 19 these energy denominators are taken as pure quark masses with no correction, while the full one-body energy is used elsewhere in the mass calculations. It can be seen from Eq. (9) that increasing δ, which would happen if pure quark masses were used in Eq. (8), would increase our result for α_s.

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