Mass of the top quark and induced decay and mixing of neutral B mesons

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For a top quark of large mass certain processes, normally suppressed by the extended Glashow-Iliopoulos-Maiani mechanism, may proceed at a measurable rate. We examine the induced decays $b \to s+\gamma$, $B_s \to \tau^+\tau^-$, and $B_s \to \gamma\gamma$ and the mixings of B_d ($\bar{B}_d$) and B_s ($\bar{B}_s$) mesons; the $b \to s+\gamma$ decay and the neutral-meson mixings give sensitive tests of the top-quark mass.

In the Kobayashi-Maskawa[1] (KM) six-quark (and -lepton) model the tree-level couplings of the neutral gauge bosons (and Z) are naturally flavor diagonal in the basis of quark mass eigenstates. Furthermore, this extended Glashow-Iliopoulos-Maiani[2] (GIM) cancellation implies that flavor-changing neutral-current couplings induced at the one-loop level will vanish if the virtual quarks appearing in the loop are degenerate in mass [and similarly for induced mixings of neutral mesons such as $K^0 \Longleftrightarrow \bar{K}^0$ (Ref. 3)]. As such, flavor-changing neutral currents and induced neutral-meson mixings provide a measure of the mass splitting between the virtual quarks involved.

In this paper we will examine the constraints one may put on the mass of the as-yet-unobserved top (t) quark from limits on the mixing or neutral-current decay of B mesons. This follows the approach[3] used to infer limits on the charmed quark mass from the decays and mixing of neutral kaons. Examination of these kaon processes in the six-quark KM model has recently been undertaken[4,5]; these authors have extended previous treatments by calculating the induced vertices for arbitrary intermediate-quark mass, whereas previous work[6] considered the limit $M_Q/M_W \ll 1$.

We wish to extend this analysis to the decays and mixings of b quarks; when combined with the constraints from K decays, these processes present direct and experimentally feasible tests on the t-quark mass. There have been analyses of the mixing of neutral B mesons previously; however these utilized expressions for the mixing matrix element only valid for $M_t \ll M_W$.[8,9]

Consider first those decays of the b quark which involve the emission of a neutral gauge boson: $b \to s+\gamma$, $b\bar{s} \to \tau^+\tau^-$; $b\bar{s} \to \gamma\gamma$. We estimate the branching ratios by dividing the computed width for the decay in question by the total B-decay width, which we take[8] to be

$$\Gamma(B) \simeq \Gamma(b \to c+W^- \to c+X)$$

$$\simeq \frac{G_F{}^2 M_B{}^5}{192\pi^3} 2\,|V_{bc}|^2 \tag{1}$$

$$\simeq \frac{G_F{}^2 M_B{}^5}{192\pi^3} 2(s_2{}^2+s_3{}^2+2s_2 s_3\cos\delta)\ , \tag{2}$$

where V_{bc} is the (b,c) entry in the KM[1] mixing matrix,[10] with $s_i=\sin\theta_i$, $c_i=\cos\theta_i$, $0 \le \theta_i \le \pi/2$, $i=1,3$. In Eq. (2) we use $\theta_i \ll 1$ to retain only leading terms in $\sin\theta_i$, and use the explicit solutions of Ref. 10 in the numerical computations.

(i) $b(p_1) \to s(p_2)+\gamma(q,\epsilon)$. This will proceed via an induced magnetic dipole transition matrix element:

$$M=\frac{G_F}{2\sqrt{2}}\left[\frac{e}{2\pi^2}\right]\sum_i V_{ib}V_{is}^{*}F_2^i q^\mu \epsilon^\nu \bar{s}(p_2)\sigma_{\mu\nu}(M_b R+M_s L)b(p_1)\ , \tag{3}$$

where L (R) are the left- (right-) projection operators, the sum is over intermediate quark states, we have dropped terms suppressed by powers of the initial- or final-quark masses (assumed small), and[4,5]

$$F^i(x)=Q_i\left\{\left[-\frac{1}{4}\frac{1}{(x-1)}+\frac{3}{4}\frac{1}{(x-1)^2}+\frac{3}{2}\frac{1}{(x-1)^3}\right]x-\frac{3}{2}\frac{x^2}{(x-1)^4}\ln x\right\}\ .$$
$$-\left[\frac{1}{2}\frac{1}{(x-1)}+\frac{9}{4}\frac{1}{(x-1)^2}+\frac{3}{2}\frac{1}{(x-1)^3}\right]+\frac{3}{2}\frac{x^3}{(x-1)^4}\ln x\ , \qquad (4)$$

where $Q_i=\frac{2}{3}$ is the charge of the intermediate quark and $x=M_i^2/M_W^2$. The resulting decay width is then

$$\Gamma=\frac{\alpha G_F^2}{32\pi^4}M_b^5\left[1-\frac{M_s}{M_b}\right]^3\left[1+\frac{M_s}{M_b}\right]\left|\sum_k V_{is}V_{ib}^{*}F_2^i\right|^2\ . \qquad (5)$$

An inspection of the KM mixing matrix shows that the $b\to s+\gamma$ decay is favored over $b\to d+\gamma$; to leading order in s_i the KM mixing angles enter in the same way for this process (and the other rare decay processes) as in the leading-order term for the full b decay width and so do not appear in the branching ratios. The branching ratio for this process is shown in Fig. 1.

(ii) $B_s^0\to\tau^+\tau^-$. The width for the decay is[4,5]

$$\Gamma(B_s^0\to\tau^+\tau^-)=\left[\frac{G_F\alpha}{4\pi\sin^2\theta_W}\right]^2\frac{f_{B_s}^2}{8\pi}M_{B_s}M_\tau^2\left[1-\frac{4M_\tau^2}{M_{B_s}^2}\right]\left|\sum_i V_{is}V_{ib}^{*}C^i\right|^2\ , \qquad (6)$$

where

$$C^i(x)=-\frac{3}{4}\left[\frac{x}{x-1}\right]^2\ln x-\frac{x}{4}+\frac{3}{4}\left[\frac{x}{x-1}\right] \qquad (7)$$

and $x\equiv M_q^2/M_W^2$ for the ith quark in $C^i(x)$, and f_{B_s} is the pseudoscalar meson decay constant. The branching ratio is shown in Fig. 1 by the long dashed line and would seem to be too small for it to be seen in the foreseeable future. The decays $B_s^0\to\mu^+\mu^-$ or e^+e^- are further suppressed by the helicity mass factor.

(iii) $B_s^0\to\gamma\gamma$. The width for this decay is[5]

$$\Gamma(B_s^0\to\gamma\gamma)=\left[\frac{G_F\alpha}{4\pi}\right]^2\frac{f_{B_s}^2}{4\pi}M_{B_s}^3\left|\sum_i V_{is}V_{ib}^{*}I^i\right|^2\ , \qquad (8)$$

where

$$I^i(x)=\frac{4(Q_d+1)^2}{(1-x)}f(x/x_M)+Q_d^2\left[\frac{1-5x-2x^2}{(1-x)^3}-\frac{6x^2\ln x}{(1-x)^4}\right]+Q_d\left[\frac{3-9x}{(1-x)^2}-\frac{6x^2\ln x}{(1-x)^3}\right]\ , \qquad (9)$$

where $f(\beta)=2-2\pi^2\beta$ if $4\beta\leq 1$ and $f(\beta)=2-8\beta[\sin^{-1}(1/\sqrt{2\beta})]^2$ when $4\beta\geq 1$, and $x_M=M_B^2/M_W^2$. Again the branching ratio appears to be too

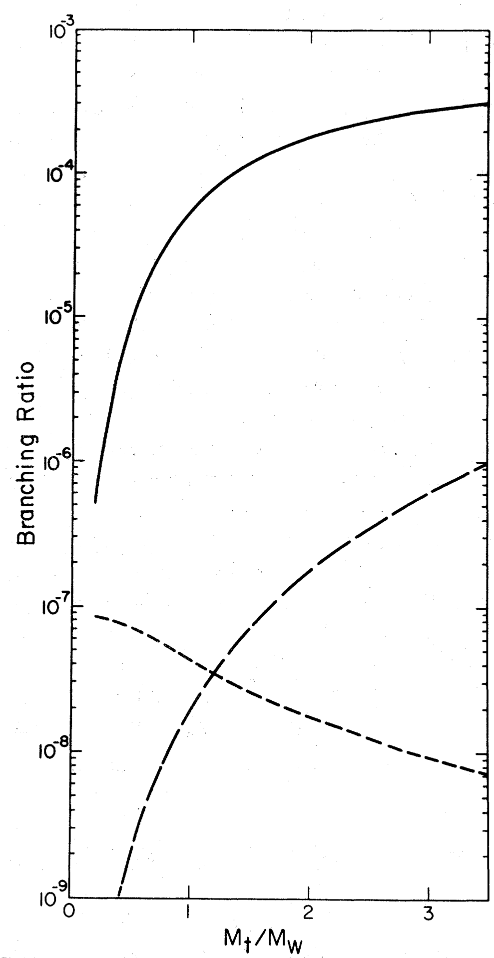

FIG. 1. Branching ratios as a function of quark mass for the decays: $b\to s+\gamma$ (solid line), $B_s\to\tau^+\tau^-$ (long-dashed line), $B_s\to\gamma\gamma$ (short-dashed line).

small for observation at present.

For the $b \to s + \gamma$ mode, in the rest frame of the B meson, the final state will involve a strange quark "jet" recoiling against a photon. As the jet is of low energy ($\sim M_B/2$), it should have dominant few-body modes such as $K^* \to K\pi$ or $Q \to K\pi\pi$. Hence the signature for $b \to s + \gamma$ will be $B \to K\pi\gamma$ or $B \to K\pi\pi\gamma$ with the γ *monochromatic* and the $K\pi$ (or $K\pi\pi$) system reconstructing to a $K^*(890)$ (or Q) meson.[11] (Note: the analogous decay $b \to s + G$ should proceed at an even larger rate, but unfortunately does not seem to have a distinct final state in which it could be isolated).

The analysis of mixings in the neutral-B_q^0-meson system proceeds[7–9] by direct analogy to that of the K-meson system. The CP-conserving piece of the mass mixing element for the B_q^0 mesons is given by

$$\Delta M = 2\mathrm{Re}\langle \bar{B}_q^0 | -\mathscr{L}_{\rm eff} | B_q^0 \rangle$$
$$= \frac{2}{3}\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi\sin^2\theta_W} f_{B_q^0}{}^2 M_{B_q^0} \mathrm{Re}(E_{B_q^0}) \,, \tag{10}$$

where

$$E = \sum_{j,k} V_{jq}^* V_{jb} V_{kq}^* V_{kb} \bar{E}(x_j, x_k)$$

with

$$\bar{E}(x_j,x_k) = -x_j x_k \left\{ \frac{1}{(x_j - x_k)}\left[\frac{1}{4} - \frac{3}{2}\frac{1}{(x_j-1)} - \frac{3}{4}\frac{1}{(x_j-1)^2}\right]\ln x_j \right. \tag{11}$$
$$\left. + \frac{1}{(x_k - x_j)}\left[\frac{1}{4} - \frac{3}{2}\frac{1}{(x_k-1)} - \frac{3}{4}\frac{1}{(x_k-1)^2}\right]\ln x_k - \frac{3}{4}\frac{1}{(x_j-1)(x_k-1)} \right\} ,$$

where $x_{j,k} = M_{j,k}{}^2/M_W{}^2$ with j,k the virtual quarks. In these expressions we have retained the full dependence[4–7] on the intermediate quark mass, including both gauge-boson and Higgs-boson-exchange contributions to the box diagrams. Renormalization effects from the strong chromodynamic interaction are expected[8,16] to be small and are ignored. To leading order in s_i the mixing functions are

$$\mathrm{Re}(E_{B_d^0}) = s_1{}^2(s_3{}^2 + 2s_2s_3\cos\delta + s_2{}^2\cos2\delta)\bar{E}(x_c,x_c)$$
$$-2s_1{}^2(s_2s_3\cos\delta + s_2{}^2\cos2\delta)\bar{E}(x_c,x_t) + s_1{}^2s_2{}^2\cos2\delta\bar{E}(x_t,x_t) \,, \tag{12}$$

$$\mathrm{Re}(E_{B_s^0}) = (s_3{}^2 + 2s_2s_3\cos\delta + s_2{}^2\cos2\delta)[\bar{E}(x_c,x_c) - 2\bar{E}(x_c,x_t) + \bar{E}(x_t,x_t)] \,. \tag{13}$$

This mixing will have an effect on the decays of B_q^0 meson pairs produced in e^+e^- annihilation[12,13] with the most experimentally distinctive characteristic of mixing occurring when both members of the B_q^0 ($\bar{B}_q^0$) pair undergo primary decay semileptonically. The fraction of same-sign dileptons produced is[12,13]

$$R \equiv \frac{N^{++} + N^{--}}{N^{++} + N^{--} + N^{+-} + N^{-+}}$$
$$= \frac{[4(\Delta M)^2 + (\Delta\Gamma)^2][8\Gamma^2 + 4(\Delta M)^2 - (\Delta\Gamma)^2]}{32[\Gamma^2 + (\Delta M)^2]^2} \,, \tag{14}$$

where Γ is the width of the B_q^0 meson and $\Delta\Gamma$, which is the difference in width of the CP-even (-odd) eigenstates of the B_q^0 system, is small compared to Γ.

There are two physically distinct sets of KM parameters[14–16] corresponding to $\delta \sim 0$ (quadrant I) or $\delta \sim \pi$ (quadrant II). This gives an ambiguity for R in the case of B_d^0 ($\bar{B}_d^0$) mixing but, *to leading orders in s_i*, does *not* affect R for B_s^0 ($\bar{B}_s^0$) mixing (nor, as noted above, the branching ratio for $b \to s + \gamma$).

In Fig. 2 we show the ratio R computed using the vacuum insertion approximation for ΔM and taking a range of pseudoscalar decay constants f_{B_q}, spanning the values given by the papers in Ref. 17. From Fig. 2 we see that for B_s^0 ($\bar{B}_s^0$) mixing, R rises to within 10% of its maximum value by a t mass of 0.4 M_W. For B_d^0 ($\bar{B}_d^0$) mixing on the other

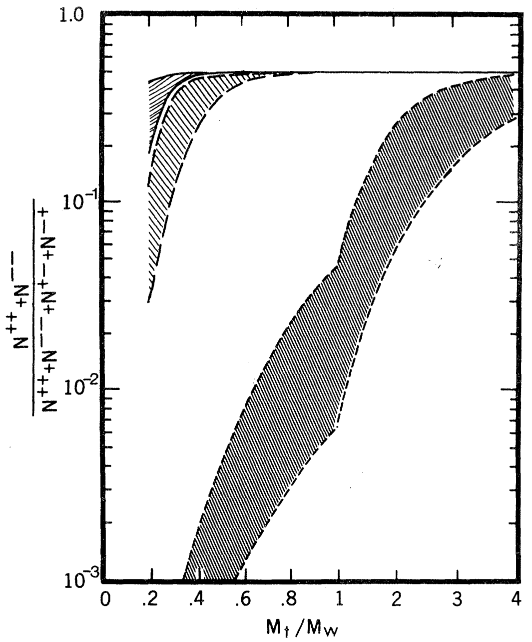

FIG. 2. Fractions of dileptons of same sign in primary decay of B^0-$\bar{B}^0$ pairs: B_s^0-$\bar{B}_s^0$ (solid line), B_d^0-$\bar{B}_d^0$ [short (long)] dash corresponds to KM parameters for solution with δ in quadrant I (II)]. The shaded areas between each pair of lines shows the effect of uncertainties in the value of the pseudoscalar coupling constants. Note the change of scale in the abcissa at $M_t/M_W=1$.

hand, R rises more slowly and is sensitive to the t mass over a larger range; however for this system the quadrant ambiguity must be resolved before one can infer the t mass (conversely, if one knew the t mass from direct measurement this would be a convincing way of resolving the quadrant ambiguity).

In summary, the decay $b \to s+\gamma$ is sensitive to the t mass over a wide range; furthermore, it has the distinctive signatures $B \to K^*\gamma \to K\pi\gamma$ and $B \to Q\gamma \to K\pi\pi\gamma$. The mixing of neutral B mesons is sensitive only to light t masses for B_s^0 ($\bar{B}_s^0$) and for B_d^0 ($\bar{B}_d^0$) suffers from the present quadrant ambiguity in the determination of the KM matrix.

Note added. After completion of the present work we received a paper by Buras,[18] in which he derives constraints on the t-quark mass from the K_L-K_S mass difference and the decay rate for $K_L \to \mu^+\mu^-$.

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