On the experimental determination of the leptonic decay constant of pseudoscalar mesons

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The decay constant in M_{l2} decays cannot be determined precisely because of the presence of model dependence in the virtual radiative corrections and in the accompanying bremsstrahlung process. In a previous paper we showed that this problem can be solved, at least as far as experimental analysis is concerned, by introducing a modified decay constant and provided hard bremsstrahlung photons are discriminated. Nevertheless, the present experimental detection of energetic photons does not yet allow for the discrimination ability required by our results. In order to give our results a more immediate usefulness, we explore the possibility of relaxing the requirement on the elimination of all such photons. We find that in $M_{\mu 2}$ decays this requirement can be substantially relaxed, allowing for less restrictive experimental setups. Our result can be traced back to a theorem due to Burnett and Kroll and Bell and Van Royen.

I. INTRODUCTION

The experimental determination of the decay constant of the leptonic decays[1] of pseudoscalar mesons M_{l2},

$$M\to l\bar{\nu}_l ,$$

is rendered ambiguous by the model dependence of the radiative corrections. The decay vertex can be shown to be unambiguously defined by introducing a modified decay constant.[2,3] Under the most general assumptions of quantum electrodynamics and Lorentz covariance, all the model dependence due to strong interactions, the intermediate vector boson, and the ultraviolet cutoff, afflicting the calculation of radiative corrections to first order in α, can be absorbed into the decay contant provided that hard bremsstrahlung photons are discriminated. The modified decay constant can be experimentally determined without theoretical biases.

The advantage of this approach lies in that the experimental value of the modified decay constant contains information complementary to that of the structure-dependent form factors of radiative M_{l2} decays.[4] The decay constants of M_{l2} decays can thus play a similar role as the modified form factors[5] of neutron decay, by putting rather stringent restrictions on the models for the structure of hadrons and the nature of the intermediate boson.[6]

In practice it may turn out to be difficult to perform an experiment where all the hard photons are discriminated and, at the same time, still have high statistics. In Ref. 3, we asked that Δk, the maximum energy of unobserved photons, be around 1 or 2 MeV. In this work we shall explore to what extent this restriction on Δk can be relaxed.

We shall see that in electron-mode decays Δk can be allowed to be somewhat larger, and that in muon-mode decays, Δk can be allowed to be substantially larger. In Sec. II, we shall study the structure-dependent contributions to the bremsstrahlung transition probability as Δk is allowed to become larger. In Sec. III, we shall derive an expression for the M_{l2} transition rate valid for more relaxed values of Δk and we shall discuss our results.

II. STRUCTURE-DEPENDENT CONTRIBUTIONS

The bremsstrahlung transition amplitude is given by

$$M_B=\frac{Ge}{\sqrt{2}}\bar{u}_l O_\mu v_\nu a p_\mu \left[\frac{2l\cdot\epsilon}{\lambda^2+2l\cdot k}+\frac{2p\cdot\epsilon-k\cdot\epsilon}{\lambda^2-2p\cdot\epsilon}\right]+ap_\mu\bar{u}_l\frac{\not\epsilon\not k O_\mu}{2l\cdot k}v_\nu+\bar{u}_l O_\mu v_\nu T_{\mu\lambda}\epsilon_\lambda$$
$$\equiv[1]+[2]+[3] . \quad (1)$$

We use the same notation as in Ref. 3. The structure dependence is contained in the tensor $T_{\mu\lambda}$. Its most general form[4] is given by gauge invariance and Lorentz covariance as

$$T_{\mu\lambda}=\frac{A}{M^2}(k_\mu p_\lambda - g_{\mu\lambda} p\cdot k)+i\frac{V}{M^2}\epsilon_{\mu\lambda\alpha\beta}p_\alpha k_\beta \ . \tag{2}$$

The form factors A and V have the same dimensions as the decay constant a. We shall assume that strong interactions are well enough behaved so that the three are of comparable magnitude. This seems to be supported by experiment.[4]

In this section, we shall concentrate on the structure-dependent contributions to the transition rate; the structure-independent ones will be considered in the next section. We must then consider the contributions arising from the interference of [1] and [3], [2] and [3], and the square of [3] of Eq. (1).

For the time being, let us take A and V as constants. Later on we shall discuss their dependence on $q^2=(p-k)^2$. Equation (1) must be squared, summed over the spins of the leptons and polarization of the photon, multiplied by the phase-space factor, and integrated over the photon momentum direction and energy, up to a maximum energy Δk. The contribution to the transition rate of the structure-dependent (SD) part, Eq. (2) up to terms of order $(\Delta k/M)^3$, is

$$\begin{aligned}\Gamma_{\rm SD}=\frac{G^2}{2}\frac{1}{4\pi}\frac{\alpha}{\pi}\frac{m^2}{M^3}(M^2-m^2)^2\left[\frac{\Delta k}{M}\right]^2\Bigg\{&\Bigg[\frac{M^2+m^2}{M^2-m^2}-\frac{m^2}{M^2}\frac{4M^4}{(M^2-m^2)^2}\ln\frac{M}{m}\\&+\frac{2\Delta k}{3M}\left[\frac{2M^2}{M^2-m^2}-\frac{4M^4}{(M^2-m^2)^2}\ln\frac{M}{m}\right]\Bigg]\mathrm{Re}(aA^*)\\&+\frac{2\Delta k}{3M}\left[-\frac{2M^2}{M^2-m^2}+\frac{4M^4}{(M^2-m^2)^2}\ln\frac{M}{m}\right]\mathrm{Re}(aV^*)\Bigg\}\ .\end{aligned} \tag{3}$$

For purposes of comparison, we recall the uncorrected decay rate

$$\Gamma_0=\frac{G^2}{2}\frac{1}{4\pi}\frac{m^2}{M^2}(M^2-m^2)^2|a|^2\ . \tag{4}$$

It is important to notice that the power $\Delta k/M$ does not appear and that the factor m^2/M^2 appears as a common factor in Eq. (3). That is, the structure-dependent contribution to order $(\Delta k/M)^3$ carries the same suppression factor due to the charged-lepton mass as the uncorrected rate, Eq. (4). This should be contrasted with contributions of higher powers of $\Delta k/M$. Computing some contributions of fourth order in $\Delta k/M$, we get, for example, terms such as

$$\frac{G^2}{2}\frac{1}{4\pi}\frac{\alpha}{\pi}\frac{(M^2-m^2)^2}{M}\left[\frac{\Delta k}{M}\right]^4(|A|^2+|V|^2)\left[\frac{1}{3}\frac{M^2}{M^2-m^2}-\frac{1}{6}\frac{m^2(M^2+m^2)}{M^2(M^2-m^2)}\right]\ . \tag{5}$$

It is clear then that as Δk is made larger compared to M, the contributions with higher powers of $\Delta k/M$ will take over those of Eq. (3), and therefore Δk cannot be allowed to grow too much. In order to estimate the maximum value for Δk, we shall require that the contributions of Eq. (3) and of terms such as (5) be of the order of $\frac{1}{100}$ of a percent of the uncorrected transition rate Eq. (4), since it is at this level of precision that one expects second-order radiative corrections to show up. Following this criterion we are led to two equations for Δk. They determine two values for it, which we distinguish by the subscripts a and b:

$$\frac{\alpha}{\pi}\frac{m^2}{M^2}\left[\frac{\Delta k_a}{M}\right]^2=10^{-4}\frac{m^2}{M^2} \tag{6}$$

and

$$\frac{\alpha}{\pi}\left[\frac{\Delta k_b}{M}\right]^4=10^{-4}\frac{m^2}{M^2}\ . \tag{7}$$

All other factors in Eqs. (3) and (4), and the expression (5) are of the same size, if, as we assumed before, A and V are of the same order of magnitude as the decay constant a. The term $(\Delta k/M)^3$ in Eq. (3) gives a bound for Δk that falls in between the two above. Replacing the numerical values of the different masses involved, we obtain the results of Table I for pion, kaon, and D-meson leptonic decays. All the electron modes have Δk restricted by Eq. (7). The reason is that the uncorrected transition rate is very much suppressed by the smallness of electron mass. This repeats again in $D_{\mu 2}$ decay, while in $\pi_{\mu 2}$ and $K_{\mu 2}$ decays it is Eq. (6) that restricts Δk.

We shall conclude this section by discussing the q^2 dependence of A and V. It is reasonable to expect that they are well enough behaved to be expanded in powers of q^2/M^2,

$$V(q^2)=V(0)+\lambda_V \frac{q^2}{M^2}+\dots \quad (8)$$

and

$$A(q^2)=A(0)+\lambda_A \frac{q^2}{M^2}+\dots \quad . \quad (9)$$

Using $q^2=M^2-2Mk_0$, it is clear that the contributions of λ_V and λ_A, and of other terms in Eqs. (8) and (9) either behave as the contributions of $V(0)$ and $A(0)$, as discussed above, or are further suppressed by higher powers of $\Delta k/M$. Therefore, the q^2 dependence does not provide new bounds on Δk.

TABLE I. Numerical values of the allowed values of Δk in MeV corresponding to Eqs. (6) and (7).

	Δk_a	Δk_b
π_{e2}	29	4
K_{e2}	103	7
D_{e2}	387	14
$\pi_{\mu 2}$	29	56
$K_{\mu 2}$	102	104
$D_{\mu 2}$	387	202

III. RESULTS AND DISCUSSION

Although the structure-dependent part of the bremsstrahlung does not contribute to order $\Delta k/M$, there will be structure-independent contributions to this order. Incorporating them, the result for the total transition rate, including the virtual radiative corrections, is

$$\Gamma_{\text{tot}}=\frac{G^2}{2}\frac{|a'|^2}{4\pi}\frac{m^2}{M^3}(M^2-m^2)^2\left[1+\frac{\alpha}{\pi}\Phi+\frac{\alpha}{\pi}\Phi'\right], \quad (10)$$

where

$$\Phi=2\ln\left[\frac{2\Delta k}{\sqrt{mM}}\right]\left[-1+\frac{M^2+m^2}{M^2-m^2}\ln\frac{M}{m}\right]+\frac{5}{8}+\left[-\frac{1}{2}+\frac{M^2+m^2}{M^2-m^2}\right]\ln\frac{M}{m}$$
$$+\frac{M^2+m^2}{M^2-m^2}\left[L\left[\frac{M^2-m^2}{M^2}\right]-\ln^2\frac{M}{m}\right] \quad (11)$$

and

$$\Phi'=\frac{4\Delta k}{M}\left[\frac{m^2}{M^2-m^2}\left[-1+\frac{M^2+m^2}{M^2-m^2}\ln\frac{M}{m}\right]-\ln\frac{M}{m}\right]. \quad (12)$$

a' is the modified decay constant and L is the Spence function.

In Table II, we list the numerical values of $(\alpha/\pi)\Phi$ and $(\alpha/\pi)\Phi'$ for different decays. The value of Δk used is the smaller of the two values given in each row of Table I. The above expression for the decay rate is accurate up to a few hundredths of a percent. This is as far as one can go before considering second-order radiative corrections, as mentioned before.

For the electron-mode decays, incorporating the $\Delta k/M$ contribution allows for some relaxation of the experimental requirements for the discrimination of hard photons. It is for the muon-mode de-

TABLE II. Numerical values of the model-independent radiative corrections in Eqs. (10), (11), and (12), for π, K, and D leptonic decays. The value for Δk used is the smaller of the two in the corresponding row in Table I.

	$\frac{\alpha}{\pi}(\Phi+\Phi')$
π_{e2}	-0.0716
K_{e2}	-0.1085
D_{e2}	-0.1531
$\pi_{\mu 2}$	-0.0019
$K_{\mu 2}$	-0.0094
$D_{\mu 2}$	-0.0220

cays that a substantial improvement is obtained. For instance, in $\pi_{\mu 2}$ decays Δk can be as large as 29 MeV which practically covers all the allowed range for the photon energy. The muon-mode decays, in addition to being overwhelmingly favored by the $V-A$ theory, have such a relaxed upper bound on Δk that they allow for a highly accurate determination of the modified decay constant a'.

It is then clear from our analysis that, just as the radiative M_{e2} decays are appropriate to determine the structure dependence[3] in the bremsstrahlung vertex, the $M_{\mu 2}$ decays are appropriate to determine the model dependence in the proper decay vertex. The reason for this complementary role of $M_{\mu 2}$ decays and radiative M_{e2} decays lies in that there is neat separation between the structure-independent and structure-dependent bremsstrahlung contributions, as the energy of the emitted photon is allowed to grow. This can be visualized in the figure of Ref. 7. The neat separation can be traced back to a theorem due to Bell and Van Royen, and Burnett and Kroll.[8]

Let us conclude by emphasizing once more the relevance that the accurate determination of the modified decay constant a' has. The model-dependent contributions to a' are rich on information about strong and weak interactions.[9] Therefore, an accurate value of a' can be very useful in discriminating different theoretical approaches, thus guiding towards the ultimate formulation of strong and weak interactions.

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[1] *M* represents a pion, a kaon, a *D* meson, or any other pseudoscalar meson; *l* represents an electron or a muon.

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