PHYSICAL REVIEW D VOLUME 25, NUMBER 7 1 APRIL 1982

Brief Reports

Brief Reports are short papers which report on completed research which, while meeting the usual **Physical Review** *standards of scientific quality, does not warrant a regular article. (Addenda to papers previously published in the* **Physical Review** *by the same authors are included in Brief Reports.) A Brief Report may be no longer than $3\frac{1}{2}$ printed pages and must be accompanied by an abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Method for discovering broken symmetries from experimental scattering data

J. H. Reid
Department of Physics, University of Nairobi, Nairobi, Kenya

N. N. Trofimenkoff
Heritage Campus, Collège d'Enseignement Général et Professionnel de l'Outaouais, Hull, Quebec, Canada J8X 3Y8

(Received 3 September 1981)

A method based on a modified eigenchannel formalism is proposed for discovering broken symmetries from experimental coupled-channel scattering data. The method suggests that the low-energy $I=\frac{1}{2}$ S-wave πN-ηN coupled-channel system and the low-energy $I=1$ D-wave pp-$n\Delta^{++}$ coupled-channel system may have broken partial-wave symmetries.

It has long been known that the eigenchannel formalism[1] is convenient for describing coupled-channel scattering if the scattering system has an *exact* internal symmetry. In this paper, we propose a method by which a modified eigenchannel formalism can be used to discover either *broken* internal symmetries or *broken* partial-wave symmetries, defined below, from experimental scattering data. Furthermore, to illustrate the possibilities of this method, we investigate briefly πN-ηN, pp-$n\Delta^{++}$, and $\pi\pi$-$K\bar{K}$ coupled-channel scattering in specific partial waves. Our investigation suggests that broken partial-wave symmetries may exist in the $I=\frac{1}{2}$ S-wave πN-ηN system and in the $I=1$ D-wave pp-$n\Delta^{++}$ system.

First, consider scattering of two coupled states $|a\rangle$ and $|b\rangle$ which have *degenerate* thresholds and which have scattering described by the usual coupled-channel partial-wave $\mathscr{S}$ matrix

$$\mathscr{S}=\begin{bmatrix} \eta e^{2i\delta_a} & i(1-\eta^2)^{1/2}e^{i(\delta_a+\delta_b)} \\ i(1-\eta^2)^{1/2}e^{i(\delta_a+\delta_b)} & \eta e^{2i\delta_b}\end{bmatrix}, \tag{1}$$

where δ_a and δ_b are the phase shifts and η is the absorption parameter. The eigenchannel formalism for the same partial-wave scattering process has a diagonal $\mathscr{S}$ matrix $\mathscr{S}^d$ and eigenstates $|\alpha\rangle$ and $|\beta\rangle$ such that

$$\mathscr{S}^d=U\mathscr{S}U^{-1}=\begin{bmatrix} e^{2i\delta_\alpha} & 0 \\ 0 & e^{2i\delta_\beta}\end{bmatrix}, \tag{2}$$

$$\begin{bmatrix}|\alpha\rangle \\ |\beta\rangle\end{bmatrix}=U\begin{bmatrix}|a\rangle \\ |b\rangle\end{bmatrix}, \tag{3}$$

where U is the rotation matrix

$$U=\begin{bmatrix}\cos\epsilon & \sin\epsilon \\ -\sin\epsilon & \cos\epsilon\end{bmatrix}. \tag{4}$$

In Eqs. (2) and (4), the eigenphase shifts δ_α and δ_β and the mixing angle ϵ are given by

$$\delta_{\alpha,\beta}=\tfrac{1}{2}\{\delta_a+\delta_b\pm\arccos[\eta\cos(\delta_a-\delta_b)]\}, \tag{5}$$

$$\epsilon=\tfrac{1}{2}\arctan[(1-\eta^2)^{1/2}/\eta\sin(\delta_a-\delta_b)]. \tag{6}$$

In general, the mixing angle ϵ for each partial wave depends on the scattering energy. However, the following two types of symmetries wherein the mixing angles are constant may occur:

Type A (exact internal symmetry): If the degenerate threshold system has an exact internal symmetry such that $|\alpha\rangle$ and $|\beta\rangle$ are eigenstates of a symmetry operator, *then the mixing angle ϵ is constant and equal for all partial waves* ($\cos\epsilon$ and $\sin\epsilon$ are then Clebsch-Gordan coefficients).

Type B (exact partial-wave symmetry): If the degenerate threshold system has what we henceforth call an exact partial-wave symmetry,[2] then the *mixing angle ϵ is constant for at least one partial* *wave* (different partial waves could have different constant mixing angles).

Hence, whether a degenerate threshold system has an exact symmetry of either type may be determined from experimental scattering data by calculating the partial-wave mixing angles ϵ from Eq. (6); a constant value of ϵ signals the presence of an exact symmetry.

Second, consider scattering of two coupled states $|a\rangle$ and $|b\rangle$ which have *nondegenerate* thresholds because the system has a broken symmetry. The formalism of Eqs. (1) to (6) still applies above both thresholds. However, the mixing angle ϵ will not, in general, be constant because the symmetry is broken; indeed, ϵ is expected to have substantial energy dependence due to kinematical factors in the $\mathscr{S}$ matrix. Hence, searching for the presence of a broken symmetry in a nondegenerate threshold system by calculating ϵ from experimental scattering data and Eq. (6) may not yield an unambiguous signal even if the system does, in fact, possess a broken symmetry. Therefore, we propose that the presence of some broken symmetries may be signalled not by ϵ but, rather, by another constant mixing angle ξ which diagonalizes not the $\mathscr{S}$ matrix but, rather, a related matrix obtained from the $\mathscr{S}$ matrix by removing kinematical factors known to depend strongly on the masses of the scattered particles. Our proposal may be expected to be valid if the symmetry is broken predominantly by masses rather than by coupling constants.

One way to remove kinematical factors from the $\mathscr{S}$ matrix is to use a K matrix where

$$\mathscr{S}=1+2i\rho(s)^{1/2}K(s)[1-C(s)K(s)]^{-1}\rho(s)^{1/2} \tag{7}$$

in the notation of Refs. 3 and 4. In Eq. (7), $K(s)$ is the K matrix which is real and symmetric, $C(s)$ is a diagonal matrix whose nonzero elements are Chew-Mandelstam functions, and $\rho(s)$ is a diagonal matrix whose nonzero elements are $(2k/s^{1/2})^{2L+1}$ where L is the orbital angular momentum, $s^{1/2}$ is the center-of-mass energy, and k is the three-momentum of each channel. The K matrix may be diagonalized with a rotation matrix V, analogous to U, such that

$$K^d=VKV^{-1}=\begin{bmatrix} K^d_\alpha & 0 \\ 0 & K^d_\beta \end{bmatrix} \tag{8}$$

and, conversely,

$$K=\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}=\begin{bmatrix} K^d_\alpha\cos^2\xi+K^d_\beta\sin^2\xi & (K^d_\alpha-K^d_\beta)\cos\xi\sin\xi \\ (K^d_\alpha-K^d_\beta)\cos\xi\sin\xi & K^d_\alpha\sin^2\xi+K^d_\beta\cos^2\xi \end{bmatrix}, \tag{9}$$

where

$$V=\begin{bmatrix} \cos\xi & \sin\xi \\ -\sin\xi & \cos\xi \end{bmatrix}, \tag{10}$$

$$\xi=\tfrac{1}{2}\arctan[2K_{12}/(K_{11}-K_{22})] . \tag{11}$$

In general, the mixing angle ξ is a function of energy; however, some systems may have broken symmetries such that the symmetry breaking occurs predominantly through the kinematical factors $\rho(s)$ and $C(s)$ of Eq. (7) so that ξ is a constant. To discover such broken symmetries from experimental scattering data, we propose a method consisting of the following recipe. First, determine the partial-wave K-matrix elements from the experimental scattering data. Second, use Eq. (11) to calculate the mixing angle ξ for each partial wave. Third, check whether ξ is a constant. If ξ is a constant for at least one partial wave, a broken partial-wave symmetry has been discovered. If ξ is the same constant for all partial waves, a broken internal symmetry has been discovered.

K-matrix elements are usually extracted from experimental scattering data by first choosing the elements K_{ij} to consist of polynomials and/or poles in the variable s, and then fitting parameters in the K_{ij} to reproduce the experimental data. This procedure is often beset with difficulties due to a lack of reliable experimental data. Nevertheless, we describe below a modest investigation of the following three types of K-matrix fits: the first consists of only poles in πN-ηN scattering, the second consists of only polynomials in pp-$n\Delta^{++}$ scattering, and the third consists of a constant plus poles in $\pi\pi$-$K\bar{K}$ scattering.

To study the possibilities of our proposed method in a nontrivial case, we considered low-energy $I=\frac{1}{2}$ S-wave coupled-channel πN-ηN scattering. In this system, the two thresholds are far apart, so that the kinematical factors $\rho(s)$ and $C(s)$ are significantly different, the resonance $N(1535)$ couples almost exclusively[5] to πN and ηN, so that using only two coupled channels is valid at low energies, and the resonance $N(1650)$

couples primarily to πN. We can fit well the experimental $\pi N \to \pi N$ phase shift δ_a and absorption parameter η of Ref. 6 from the πN threshold to the energy $s^{1/2} = 1.8$ GeV with the K-matrix pole parametrization

$$K_{ij} = g_i g_j / (M_\alpha{}^2 - s) + f_i f_j / (M_\beta{}^2 - s) \tag{12}$$

and the approximate nonrelativistic Chew-Mandelstam function $C(s) \approx i\rho \approx ik$. The numerical values for this fit are $M_\alpha = 1.700$ GeV, $M_\beta = 1.535$ GeV, $g_1 = 0.608$ GeV$^{1/2}$, $g_2 = 0.284$ GeV$^{1/2}$, $f_1 = 0.390$ GeV$^{1/2}$, and $f_2 = -0.836$ GeV$^{1/2}$. Equation (11) can be used to show that these numerical values correspond to the *constant* value $\xi = 25°$. Therefore, this fit suggests that there may be a broken partial-wave symmetry in the low-energy $I = \frac{1}{2}$ S-wave πN-ηN system. We consider these results to be only suggestive because the experimental $\eta N \to \eta N$ scattering phase shift is not available and because we are using the ηN channel to mimic the effects of other channels which couple more strongly than ηN at the higher-energy resonance $N(1650)$.

In Ref. 4, the K-matrix polynomial parametrization

$$K_{ij} = a_{ij} + b_{ij} s + c_{ij} s^2 \,, \tag{13}$$

where a, b, and c are parameters to be fitted to experimental data, is used for $J^P = 2^+$ and $J^P = 3^-$ coupled-channel dibaryon pp-$n\Delta^{++}$ scattering; eight numerical fits are presented. Equation (11) can be used to show that

$$\xi \approx \tfrac{1}{2} \arctan \left[\frac{2(94.2 - 43.2s + 5.0s^2)}{-113.0 + 46.5s - 4.9s^2} \right] \tag{14}$$

for solution 2 of Ref. 4 for the $J^P = 2^+$ dibaryon system, so that $-35° < \xi < -25°$ for all energies above the $n\Delta^{++}$ threshold. Equation (14) shows that a small change in the parameters would result in the constant value $\xi = \frac{1}{2}\arctan(-2) = -31.7°$. This result suggests that a broken partial-wave symmetry may exist in the $I = 1$ D-wave pp-$n\Delta^{++}$ system. It should be noted, however, that the fit of Ref. 4 is for a rather small energy range.

In Refs. 7 and 8 the K matrix for the $I = 0$ S-wave $\pi\pi$-$K\bar{K}$ coupled-channel system is parametrized with a constant plus three poles. Equation (11) or Eq. (9) can be used to show that none of the four solutions of Ref. 7 nor the solution of Ref. 8 is consistent with ξ being constant.[9]

Finally, we note that if ξ is not a constant for two coupled channels, then among the possibilities are the following: (1) The system does not possess a broken symmetry. (2) The system has a broken symmetry, but the breaking of the symmetry does not occur predominantly through the kinematical factors $\rho(s)$ and $C(s)$. (3) The two-channel system is part of a larger system consisting of three or more channels, and the larger system has a broken symmetry.

We thank P. Estabrooks for providing us with the parameters of her K-matrix fit to $\pi\pi$-$K\bar{K}$ scattering.

---

[1]See, for example, the excellent discussion in Y. Yamaguchi, Supp. Prog. Theor. Phys. 7, 1 (1959); to a large extent we follow the notation of this article. R. H. Dalitz and R. G. Moorhouse, Proc. R. Soc. London A318, 279 (1970) contains a discussion of the behavior of eigenphases.

[2]Partial-wave symmetries are important in studies of final-state interactions and form factors because the presence of a partial-wave symmetry allows the reduction of the coupled-channel problem for that particular partial wave to two equivalent uncoupled single-channel problems whose solutions are well known. On this subject see, for example, J. H. Reid and N. N. Trofimenkoff, to be published.

[3]J. L. Basdevant and E. L. Berger, Phys. Rev. D 16, 657 (1977).

[4]B. J. Edwards and G. H. Thomas, Phys. Rev. D 22, 2772 (1980); B. J. Edwards, *ibid.* 23, 1978 (1981).

[5]Particle Data Group, Rev. Mod. Phys. 52, S1 (1980).

[6]R. Ayed, P. Bareyre, and G. Villet, Saclay report, 1970 (unpublished); L. D. Roper and R. M. Wright, Phys. Rev. 138, B921 (1965).

[7]P. Estabrooks, Phys. Rev. D 19, 2678 (1979), and private communication.

[8]C. Sorensen, Phys. Rev. D 23, 2618 (1981).

[9]On the other hand, in the rather simple and unorthodox fit of A. N. Kamal and E. D. Cooper, Z. Phys. C 8, 67 (1981), the K matrix is parametrized with two poles, one of which has nonfactorizable residues; changing slightly the parameters (Γ'_{11} from -0.12 to -0.13 GeV and Γ'_{22} from -0.14 to -0.13 GeV) yields a K matrix which can be diagonalized with a constant $\xi = 45°$.