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Soliton bag model

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The soliton bag model of Friedberg and Lee is investigated. We study in detail the numerical solution of the mean-field limit of the model and develop quantum corrections to this approximation. Both the MIT and SLAC bags are recovered as limiting cases of the model. The surface bag excitations coupled to quark-antiquark excitations and their relation to the virtual-meson cloud as well as other aspects of the model are discussed.

I. INTRODUCTION

Static bag models have[1–8] had remarkable success in describing hadronic structure and spectroscopy. Extensions of the pure quark-QCD bag to include meson degrees of freedom have led to a quantitative description of the pion cloud and pion-nucleon scattering.[6,7] The latter models are called "hybrid" or "cloudy" bag models, since the mesons (at least the pions) are treated as elementary particles, and are not structured from quarks.

A description of hadron-hadron interactions requires more than bag statics. As a particular example, consider the N-N interaction. The static energy of six-quark bags has been calculated as a function of a shape, or N-N separation, parameter.[8] Qualitative understanding of the N-N potential was obtained. However, in order to calculate scattering or bound-state quantities, it is usually necessary to insert the "potential" into a Schrödinger, Dirac, or Lippmann-Schwinger equation. This requires a knowledge of the mass parameter which enters in the kinetic-energy term.

Static bag models cannot yield the kinetic mass parameter because they are not associated with a complete Hamiltonian. There exist an expression for the energy, a differential equation, and a boundary condition, but no Hamiltonian.

Experience with atomic and nuclear composite systems presents many prescriptions for calculating the kinetic mass parameter or for calculating dynamics by bypassing the mass-parameter approach, when a complete Hamiltonian is known.[9]

Friedberg and Lee[10] have proposed a model based on a complete Lagrangian or Hamiltonian formulation of the problem, and which therefore admits dynamical solutions of structure and scattering problems. The model contains only a few adjustable parameters, but is extremely rich in the variety of problems it can address.

The heart of the Friedberg-Lee model is the nontopological soliton, or σ, field.[10–12] This is a phenomenological representation of the quantum excitations of the self-interacting gluon field. It is a scalar field. The energy of a uniform system as a function of the σ field strength has two minima, one at zero and a second, deeper minimum at a large finite value identified as the vacuum value. In the absence of quarks, the normal state of the σ field is at the vacuum value. In the presence of quarks, the σ field finds a minimum in the vicinity of zero; the quarks dig a hole in the vacuum. This is the origin of confinement in the model.

Section III contains a description of the soliton model.[10] In addition to quarks and solitons, the model also contains vector gluons, Higgs fields, and counterterms. Since the soliton field can be viewed as a representation of certain gluon excitations, the model is in fact overcomplete. Problems of double-counting, however, will not arise until we include multiple-gluon excitations in our calculations.

The calculational program which emerges has a strong parallel with that of the Bohr-Mottleson unified model[13] of nuclear structure. The soliton field plays the role of a collective, shell-model potential. The quarks are well described as independent particles confined by the scalar soliton potential. One can have excitations of the quarks from their positive-energy states and from their negative-energy states (antiquark excitation). The soliton bag can also be excited in a manner that is quite analogous to surface excitations of nuclei. These excitations obtain quite naturally in the soliton bag model in contrast to other bag-model formulations. The observations of such excitations should provide a test of the soliton model. The full Hamiltonian contains terms which couple quark and surface excitations. These perturbations can be handled by fairly standard techniques.[14] The quark-antiquark excitations can be interpreted as the near meson cloud surrounding a hadron.

In this paper we study the quark-soliton system in the mean-field approximation.[15] Of the four parameters in the model, one is fixed by the nucleon size. Extended studies are made on two families of parameters; each family freezes one other parameter. Certain limiting cases of the parameters lead to the MIT (volume) bag or to a surface bag. The mean-field approximation determines the quark and surface excitation spectra, from which further correction can be calculated. The discussion of some of these corrections can be found in Secs. IV and V. The results of our calculations are presented in Sec. VI.

The soliton model has been studied variationally by Huang and Stump,[10(b)] and the relationship of their work to ours is discussed in Sec. IV.

II. STATIC BAG MODEL

Since the development of quantum chromodynamics (QCD), it has been generally accepted that hadrons (nucleons, mesons) are color-singlet states formed from interacting quarks and gluons. The mathematical structure and some physical consequences of non-Abelian gauge theories, of which QCD is the prototype, are qualitatively understood. However, to date no completely satisfactory explanation for the existence of color confinement has been given.[16]

In order to obtain an understanding of hadronic properties, various *models* of color confinement have been developed. Of these, the most successful have been the bag models.[1–8,17] The basic idea in these models is the confinement of quarks (and gluons) in a cavity of a finite size (the bag). This cavity is ususally taken to be spherically symmetric.[18]

Inside the bag the quarks are assumed to satisfy the bound-state Dirac equation

$$(\vec{\alpha}\cdot\vec{\mathrm{p}}+m\beta)\psi=\epsilon\psi\ , \tag{1}$$

where ψ is the quark wave function. The quark mass is usually taken to be zero except for the strange and charmed quarks.

At the bag surface, the quark wave function saisfies the boundary condition[1]

$$i\vec{\mathrm{n}}\cdot\vec{\gamma}\psi=\psi\ , \tag{2}$$

where $\vec{\gamma}\equiv\beta\vec{\alpha}$ and $\vec{\mathrm{n}}$ is the outward normal to the surface. This is equivalent to the Dirac equation for quarks bound in a scalar potential which is finite inside the cavity and infinite outside. It should be noted that the sharp boundary alone does not confine quarks since the quark energy decreases as the cavity radius increases. In order to obtain confinement it is necessary to include volume and/or surface energy terms. This introduces into the theory two adjustable parameters, which are related to the magnitude of volume and surface energy contributions to the bag energy.

The bag models have been successful in the description of the mass spectra, magnetic moments, and other physical properties of hadrons.[1–8] The more sophisticated versions of these models include explicit quark-pion coupling,[6,7] nonspherical boundary conditions,[8] corrections to include center-of-mass motion[19,22] and QCD effects.

III. SOLITON BAG MODEL

In spite of their successes, the static bag models discussed in the preceding section are necessarily limited in scope because of their noncovariant, non-Hamiltonian formulation. These models cannot be expected to yield an adequate description of hadronic collisions, $N\bar{N}$ annihilation, and many other dynamical phenomena for which QCD analyses are usually limited to lowest order in gluon exchange. Thus it is desirable to develop alternative bag formulations which can be described by a complete Hamiltonian and are manifestly covariant.[19,20,22] One such formalism is the soliton bag model proposed recently by Friedberg and Lee.[10,11] We now briefly outline this model and its pertinent features below.

In the soliton bag model, the QCD theory is approximated by an effective Lagrangian density

$$\begin{aligned}\mathscr{L}=\;&\bar{\underline{\psi}}\not{p}\underline{\psi}-g\bar{\underline{\psi}}\underline{\sigma}\underline{\psi}+\tfrac{1}{2}(\underline{\pi}_\sigma{}^2-|\vec{\nabla}\underline{\sigma}|^2)-U(\underline{\sigma})\\&+\mathscr{L}'(\underline{\phi},\underline{G},\underline{\psi})+\text{counterterms}\ ,\end{aligned} \tag{3}$$

where $\underline{\sigma}$ is a color-singlet scalar field, $\underline{\pi}_\sigma$ is the conjugate momentum, and

$$U(\underline{\sigma})=\frac{a}{2}\underline{\sigma}^2+\frac{b}{6}\underline{\sigma}^3+\frac{c}{24}\underline{\sigma}^4+p\ , \tag{4a}$$

$$\begin{aligned}H=\int d^3x\,[\,&\underline{\psi}^\dagger\vec{\alpha}\cdot\vec{\mathrm{p}}\underline{\psi}+\tfrac{1}{2}(\underline{\pi}_\sigma{}^2+|\vec{\nabla}\underline{\sigma}|^2)\\&+U(\underline{\sigma})+g\bar{\underline{\psi}}\underline{\sigma}\underline{\psi}]\ .\end{aligned} \tag{4b}$$

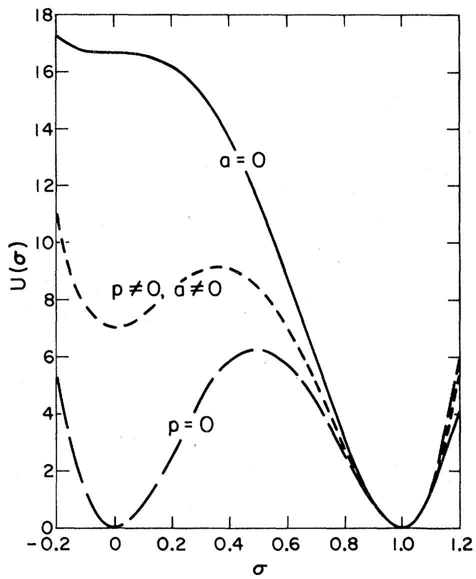

FIG. 1. Soliton self-energy $U(\sigma)$ as a function of σ for three different sets of parameters. Units on the vertical and horizontal axes are arbitrary.

In (3), $\mathscr{L}'(\underline{\phi},\underline{G},\underline{\psi})$ is the remainder of the effective Lagrangian containing the Higgs fields ϕ and the vector-gluon field G. The detailed discussion of this term has been given in Ref. 10.

In this paper, we omit $\mathscr{L}'$, which reduces the number of free parameter to four, namely a, b, c, and g. The quartic form (4) is the most general renormalizable form for the scalar-field self-energy and has the shape illustrated in Fig. 1. The minima in the curves correspond to $\sigma=0$ and

$$\sigma_v=\frac{3}{2c}[-b+(b^2-\tfrac{8}{3}ac)^{1/2}] \tag{5}$$

(a minus sign before the radical yields the local maximum). As in Ref. 10, we take $b<0$ (which is no restriction) and $b^2\geq\frac{8}{3}ac$. The constant p is taken such that $U(\sigma_v)\equiv 0$, i.e.,

$$-p\equiv\frac{a}{2}{\sigma_v}^2+\frac{b}{6}{\sigma_v}^3+\frac{c}{24}{\sigma_v}^4 . \tag{6}$$

We note that in this form the model is colorless and flavor independent; we must put in color and flavor "by hand." This feature is present in all current bag models of hadrons.

We can expand the operator $\underline{\psi}$ as follows:

$$\underline{\psi}=\sum_k c_k\psi_k(r) , \tag{7}$$

where $\{ \psi_k \}$ is an arbitrary, complete orthonormal set of Dirac spinor functions and the c_k are fermion annihilation operators.

For the scalar field, we write

$$\underline{\sigma}=\sigma_0+\underline{\sigma}_1 , \tag{8}$$

where σ_0 is a c-number field. It is convenient to work in the rest frame of the scalar field σ_0. In this frame, $\sigma_0(\vec{r})$ is time independent. To lowest order in σ_1, $\psi_k(\vec{r})$, and $\sigma_0(\vec{r})$ satisfy the coupled differential equations

$$(\vec{\alpha}\cdot\vec{p}+g\beta\sigma_0)\psi_k=\epsilon_k\psi_k , \tag{9}$$

$$-\nabla^2\sigma_0+U'(\sigma_0)=-g\sum_k{}'\,\psi_k^\dagger\beta\psi_k , \tag{10}$$

where

$$U'(\sigma_0)=dU(\sigma_0)/d\sigma_0 .$$

The sum in (10) is over the occupied quark states minus the hole states.

Only the "valence" quark states are needed in solving these equations self-consistently. However, we use Eq. (9) to define a complete set of basis states $\{ \psi_k \}$ in which to expand the quark field operator $\underline{\psi}$. In deriving these equations we have omitted contributions from vacuum polarization. Below we shall assume that these have already been accounted for by the appropriate renormalization of the constants a, b, c, and g. In the following, Eqs. (9) and (10) will be referred to as the mean-field approximation (MFA).[21]

In the ground state, the three valence quarks in the baryon can be in the same space state (e.g., $k=0$). Thus (9) and (10) may be written as

$$(\vec{\alpha}\cdot\vec{p}+g\beta\sigma_0)\psi_0=\epsilon_0\psi_0 , \tag{11}$$

$$-\nabla^2\sigma_0+U'(\sigma_0)=-Ng\psi_0^\dagger\beta\psi_0 , \tag{12}$$

where N is the number of hadronic constituents; for a baryon $N=3$ and for a meson $N=2$. The total energy of the quark–scalar-field system in the approximation given by (11) and (12) is

$$\begin{aligned}\mathscr{E}_0&\equiv N\epsilon_0+\int[\tfrac{1}{2}\,|\,\vec{\nabla}\sigma_0\,|^2+U(\sigma_0)]\,d^3r\\&\equiv N\epsilon_0+\mathscr{E}_{\sigma_0} .\end{aligned} \tag{13}$$

Equation (13) can be rewritten in a different form with the aid of Eqs. (9)–(12). Rafelski[12] has shown that

$$\mathscr{E}_0=\int d^3x\left[a(\sigma^2-{\sigma_v}^2)+\frac{b}{6}(\sigma^3-{\sigma_v}^3)\right] . \tag{13'}$$

Since the form (13′) is based upon the assumption that Eqs. (9)–(12) are satisfied exactly, the use of

(13) and (13′) for comparison in computing $\mathscr{E}_0$ can be used to test the accuracy of the computations.

Equations (11)–(13) are already in a form reminiscent of the standard bag models, where σ_0 is a step function, viz.,

$$\sigma_0 = -m/g, \quad r < R ,$$

$$\sigma_0 = \sigma_v \to \infty, \quad r > R .$$

The structure of (11) and (12) already indicates certain physical features of the present model. First we note that in the absence of quarks, the solution of (12) is $\sigma_0 = \sigma_v$, constant in position, and the energy of the system is zero. However, where $\psi_0^\dagger \beta \psi_0$ is not negligible, $\sigma_0 \neq \sigma_v$. Qualitatively, σ_0 in this region can be made small and negative by a proper choice of a, b, c, and g. Thus quarks will be confined to a small region of space where the scalar field is small. Outside the region of confinement, the quarks have the mass $\approx g\sigma_v$, which is presumably a large number. We note that this does not yield absolute confinement for finite σ_v. Since in construction of bag models we do not expect to be able to describe high-energy processes, this feature of the soliton model is not particularly troublesome. Furthermore, the limit of large $g\sigma_v$ can be carried out after the completion of the calculations, thus ensuring that no spurious results occur due to discontinuities of the bag surface as is the case in the MIT bag model.

It should be pointed out that *within* the approximation (9)–(13) one is *not limited* to the quartic form (7) for $U(\underline{\sigma})$. For example, an interesting (and numerically highly stable) form is

$$U(\underline{\sigma}) = Ce^{-A\underline{\sigma}}(1 + A\underline{\sigma} + B\underline{\sigma}^2) , \tag{14}$$

where A, B, C are adjustable constants. This form of the scalar-field self-energy is similar in shape to the quartic form for small σ. It yields absolute confinement for quarks. However, in contrast to the quartic form (4), the full field theory with $U(\underline{\sigma})$ given by (14) is not renormalizable. Thus we should not attempt to compute quantum corrections and other higher-order effects for this model. On the other hand, the form (14) can be used in Eqs. (8)–(12) to obtain (a) a basic idea about the behavior of a confined system and (b) a basis set to generate higher-order corrections in another, renormalizable theory. In this paper, however, we do not consider this form further.

IV. CORRECTIONS DUE TO $\underline{\sigma}_1$

Inclusion of only σ_0, the c-number part of the soliton field, has led to what is essentially a mean-field approximation (MFA). Deviations from this approximation are generated by $\underline{\sigma}_1$. The MFA already contains important nonlinear effects. If effects due to $\underline{\sigma}_1$ are not great, the separation will be a useful one. We will utilize the MFA to generate a representation in terms of which the corrections can be calculated.

The Hamiltonian (without the terms due to Higgs fields, vector gluons, and counterterms) can be written

$$\begin{aligned} H = \mathscr{E}_{\sigma_0} + \sum_k \epsilon_k [(b_k^\dagger b_k + d_k^\dagger d_k)] \\ + \int d^3r \Bigg\{ \tfrac{1}{2}[\underline{\pi}_1{}^2 + |\vec{\nabla}\underline{\sigma}_1|^2 + U''(\sigma_0)\underline{\sigma}_1{}^2] \\ + \left[\left(\frac{b}{6} + \frac{c}{12}\sigma_0 \right) \underline{\sigma}_1{}^3 + \frac{c}{24}\underline{\sigma}_1{}^4 \right] + g[\underline{\bar{\psi}\psi} - \langle 0 | \underline{\bar{\psi}\psi} | 0 \rangle] \underline{\sigma}_1 \Bigg\} , \end{aligned} \tag{15}$$

where $\langle 0 | \underline{\bar{\psi}\psi} | 0 \rangle = N\bar{\psi}_0\psi_0$. We have introduced the particle operators $b_k \equiv c_k$ for $\epsilon_k > 0$ and $d_k \equiv c_{\bar{k}}^\dagger$, where $k = (\kappa, m, \epsilon)$, $\bar{k} = (-\kappa, -m, -\epsilon)$ so that

$$\underline{\psi} = \sum_{k\,(\epsilon_k > 0)} (b_k \psi_k + d_k^\dagger \psi_{\bar{k}}) . \tag{16}$$

The b_k and d_k are particle and antiparticle annihilation operators.

We can also expand $\underline{\sigma}_1$ in terms of an arbitrary, complete set of functions, e.g., $\{ s_j \}$, as

$$\underline{\sigma}_1 = \sum_j (2\omega_j)^{-1/2}(a_j^\dagger + a_j)s_j , \tag{17a}$$

$$\underline{\pi}_1 = i \sum_j (\omega_j/2)^{1/2}(a_j^\dagger - a_j)s_j , \tag{17b}$$

where the a_j and $a_j^\dagger$ are the usual Bose annihilation and creation operators. The index j is the collection of quantum numbers needed to describe the eigenstates s_j.

The s_j and ω_j can be fixed by requiring the s_j to satisfy the eigenvalue equation

$$[-\nabla^2+U''(\sigma_0(r))-\omega_j{}^2]s_j(\vec{r})=0 \ . \tag{18}$$

Now the Hamiltonian can be written

$$H=\mathscr{E}_{\sigma_0}+\sum_k \epsilon_k(b_k^\dagger b_k+d_k^\dagger d_k) + \sum_j \omega_j(a_j^\dagger a_j+\tfrac{1}{2})+H' \tag{19}$$

with

$$H'=\int d^3r \left[g(\underline{\bar{\psi}\psi}-N\bar{\psi}_0\psi_0)\underline{\sigma}_1 + \left[\frac{b}{6}+\frac{c}{12}\sigma_0\right]\underline{\sigma}_1{}^3+\frac{c}{24}\underline{\sigma}_1{}^4\right] \ . \tag{20}$$

The $\{\psi_k\}$ and $\{s_j\}$ define a basis in terms of which corrections due to H' can be calculated. This is very analogous to the weak particle-surface coupling representation of the Bohr-Mottelson unified model.[13] The representation states and spectra are relatively easy to solve for once the self-consistent $\sigma_0(r)$ has been obtained. Numerous approximation methods are available for handling H', such as perturbation theory or matrix diagonalization in a finite basis. Note that the nonlinear terms ($\underline{\sigma}_1{}^3$ and $\underline{\sigma}_1{}^4$) are not an essential complication.

There are alternative basis sets available which could also be useful. One could, for example, solve static equations of the form (9) and (10) but where, e.g., the σ_0 field is constrained to contain deformation, such as would be appropriate for bag collisions or oscillations. The dynamics could then be included by generator coordinate or other techniques.

A variety of methods is being explored, and will be reported subsequently. The goal of such calculations is a description of such phenomena as the following.

(a) The meson cloud surrounding the nucleon bag. We expect this to appear as σ oscillations as-associated with $q\bar{q}$ excitations.

(b) The dynamics of bag collisions. Various authors have calculated the statics of bag collisions (for MIT-type bags), but the effective-mass parameter must also be determined.

(c) Normal-mode oscillations of the bag, leading to decay. By studying unstable oscillations, one may extract partial widths for decay to other modes.

(d) Hadron form factors.

(e) Hadron spectra.

(f) Properties of deformed bags.

The present model predicts a new type of hadron excitation, namely σ excitation, in addition to quark excitations. These excitations are strongly coupled to each other due to the presence of the coupling term $g\sigma\bar{\psi}\psi$ in the Hamiltonian as had been observed previously by Rebbi[14] who studied surface bag deformations in the MIT bag model. Of the four parameters of the theory one can be fixed by the nucleon size. There are more than enough data available to fix the other three and test the model experimentally (although the data may be insensitive to one or two combinations of parameters).

The computational procedure outlined in Secs. III and IV is but one possible route to solving the problem. An alternative method of solution has been discussed by Huang and Stump[10(b)] who used variational techniques. Their variational wave function is written as the product of soliton and quark wave functions (uncoupled representation). The fermion wave function is quite arbitrary, but the soliton wave function is written as an expansion about a reference c-number field σ_0. The variational procedure is combined with the projection operator techniques to obtain a state with zero total momentum. Renormalization of $U(\sigma)$ is carried out for the specific choice of the variational ansatz. The choice of a stepfunction for $\sigma_0(r)$ yields renormalized values of $a,b,c\rightarrow\infty$ and the valence quarks satisfy Eq. (1). In order to (possibly) obtain finite values of a,b,c after renormalization, the choice of σ_0 must include a finite surface thickness, a case not considered in Ref. 10(b) because of complications arising in the renormalization procedure. Thus the variational calculation does not allow a simple investigation of the model parameter dependence.

In the present approach, renormalization is assumed *a priori* at each step of the calculation. Thus the parameters of $U(\sigma)$ as used in the MFA are renormalized parameters. When quantum corrections to the MFA are included for a finite number of modes, one may be calculating pieces of renormalization terms. Thus at each level one must be prepared to readjust $U(\sigma)$, which is phenomenological anyway. This is similar, in spirit, to the quantization of the MIT bag proposed by Rebbi.[14]

V. OTHER CORRECTIONS

Several other corrections not included in our discussion in Sec. IV are expected to be important. The first is due to the flavor dependence of the quark mass. This requires the inclusion of a mass term $m\bar{\psi}\psi$ in the Lagrangian and, in zeroth order, leads to a modification of Eq. (9) in the form

$$[\vec{\alpha}\cdot\vec{\mathrm{p}}+\beta(g\sigma_0+m)]\psi_k=\epsilon_k\psi_k \tag{21}$$

and the placing of the appropriate flavor labels on the quark fields. Evidently, based on the experience with other bag models, this dependence is required to reproduce hadron mass spectra, in particular for the strange baryons and mesons.

Another important correction is due to the presence of vector gluons. In order to understand this correction, a detailed understanding of gluon-soliton interaction is needed. Furthermore, since the soliton field simulates the major nonlinearity of QCD, one must make sure that no double-counting occurs. The authors in Ref. 10 suggest that we write

$$\mathscr{L}(\underline{G},\underline{\sigma})=-\tfrac{1}{4}(1-\underline{\sigma}/\sigma_v)\underline{G}^a_{\mu\nu}\underline{G}^a_{\nu\nu} -\tfrac{1}{2}h\underline{\sigma}^2(1-\underline{\sigma}/\sigma_v)\underline{G}^a_{\mu}\underline{G}^a_{\mu}\ , \tag{22}$$

where the covariant derivative is

$$\underline{G}^a_{\mu\nu}=\partial_\mu\underline{G}^a_\nu-\partial_\nu\underline{G}^a_\mu+\alpha f_{abc}\underline{G}^b_\mu\underline{G}^c_\nu \tag{22a}$$

and h is the gluon-soliton coupling constant. The a,b,c are the color indices and G is the vector-gluon field which is assumed to interact with the quarks as

$$\mathscr{L}(\underline{\psi},\underline{G})=i\alpha\underline{\bar{\psi}}\frac{\lambda_a}{2}\underline{G}^a_\mu\underline{\psi}\ ,$$

λ_a are the generators of the color SU(3) group, f_{abc} are the SU(3) structure functions, and α is the QCD coupling constant.

If we neglect the nonlinear, non-abelian part of the gluon equations [cf., Eq. (22)], the resultant theory is quite analogous to electromagnetics in media. This has been the approach of bag researchers, who utilize the smallness of the QCD coupling constant at short distances. To this order there is no problem of double-counting the diagrams included in the σ confinement field.

VI. NUMERICAL RESULTS

In order to establish the validity of the proposed model, we study in detail the properties of the numerical solutions of (11) and (12). We define the functions u,v related to ψ_0 for the case $\kappa=-1$ ($s_{1/2}$) by

$$\psi=\begin{bmatrix} u \\ i(\vec{\sigma}\cdot\vec{\mathrm{r}}/r)v\end{bmatrix}\chi_m\ , \tag{23}$$

where $\vec{\sigma}$ is the Pauli matrix and $\chi_m\equiv\binom{1}{0}$ or $\binom{0}{1}$. From here on we drop the subscripts 0 on ψ_0, ϵ_0, and σ_0. Equations (11) and (12) reduce, for the spherically symmetric solutions, to the form

$$\frac{du}{dr}=-(\epsilon+g\sigma)v\ , \tag{24a}$$

$$\frac{dv}{dr}=-\frac{2}{r}v+(\epsilon-g\sigma)u\ , \tag{24b}$$

$$\frac{1}{r}\frac{d^2}{dr^2}(r\sigma)-U'(\sigma)=Ng(u^2-v^2)\ . \tag{25}$$

The normalization condition then reads

$$4\pi\int(u^2+v^2)r^2dr=1\ . \tag{26}$$

The authors of Ref. 10(a) did not obtain a self-consistent solution of Eqs. (24)–(28) but approximated the scalar field in (25) by

$$\sigma\cong-\frac{Ng}{a}(u^2-v^2) \tag{27}$$

and made some other approximations in their solutions. Equations obtained from (24) with the approximation (27) have been solved numerically by Rafelski.[12]

The soliton can be visualized as a gas bubble immersed in a liquid medium. The parameter p can then be interpreted as the gas pressure. The situation in which the bubble is filled uniformly with the quark gas is representative of the MIT bag. In the SLAC bag the quarks are concentrated on the bubble surface, a picture apparently not supported by the experimental nucleon properties. The above visualization can be used to determine the thermodynamic properties of the quark gas in the bubble (i.e., the nucleon).[10]

We choose a length scale (see Appendix A) in such a way that the quantity

$$\langle r_p{}^2\rangle=4\pi\int(u^2+v^2)r^4dr \tag{28}$$

is $(0.83\ \mathrm{fm})^2$, the mean-square charge radius of the proton.[23] Once the scale is chosen, there are only three free parameters. We have studied mostly two families of parameters. These are the $p\rightarrow0^+$ and the $a=0$ cases. Each choice fixes one other parameter and it is sufficient to label our results by the values of the coupling constant g and the con-

TABLE I. Variation of bag properties with increasing parameters for the $p\to 0^+$ bag for two values of g.

c	ϵ_0	μ	g_A/g_V	$\mathcal{E}$	$\epsilon_1-\epsilon_0$
		$g=40$			
1.2×10^4	1.40	2.54	0.79	8.6	4.08
2×10^5	1.62	2.37	0.98	8.9	3.27
4×10^5	1.66	2.32	1.02	10.0	3.23
8×10^5	1.71	2.28	1.05	10.9	3.07
		$g=200$			
1.2×10^4	1.23	2.65	0.58	6.4	6.88
2×10^5	1.27	2.62	0.65	7.5	5.28
4×10^5	1.25	2.60	0.67		4.99
8×10^5	1.34	2.58	0.73		4.41

TABLE II. Variation of the bag parameters as a function of g for several values of c ($p\to 0^+$ bag).

g	ϵ_0	$\mathcal{E}$	μ	g_A/g_V	ρ_q
		$c=1.22\times 10^4$			
15	1.69	9.4	2.31	1.03	volume
40	1.40	8.6	2.54	0.79	
90	1.30	10.8	2.61	0.68	
200	1.23	6.4	2.65	0.58	surface
		$c=2\times 10^5$			
15	1.85	9.6	2.18	1.13	volume
40	1.62	8.9	2.37	0.98	
90	1.39	8.3	2.54	0.79	
200	1.27	7.5	2.62	0.65	
400	1.23		2.65	0.59	surface
		$c=4\times 10^5$			
15	1.91		2.14	1.16	volume
40	1.66	10.0	2.32	1.02	
90	1.44		2.50	0.84	
200	1.29		2.61	0.67	
400	1.24		2.65	0.60	surface
		$c=8\times 10^5$			
15	2.09	9.0	1.99	1.22	volume
40	1.71	10.9	2.28	1.05	
90	1.54		2.42	0.92	
200	1.34		2.58	0.73	
400	1.25		2.64	0.62	surface
MIT			2.20	1.09	volume
SLAC			2.65	0.57	surface
Experiment			2.79	1.25	volume

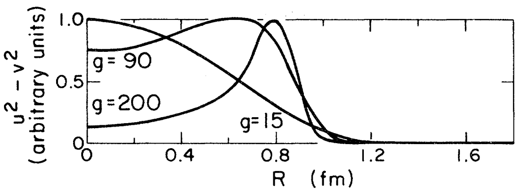

FIG. 2. Quark density u^2-v^2 versus radius for $g=15$, 90, and 200, $p\to 0^+$.

stant c, since these do not scale with length. We compute the values $\epsilon_0 \equiv \epsilon(1s_{1/2})$, $\epsilon_1 \equiv \epsilon(2s_{1/2})$, and the bag energy, which for the spherical solution is

$$\mathscr{E}=N\epsilon+4\pi\int_0^\infty dr\, r^2\left[U(\sigma)+\frac{1}{2}\left[\frac{d\sigma}{dr}\right]^2\right] . \tag{29}$$

We also calculate the proton magnetic moment

$$\mu_p=\frac{8\pi}{3}\int_0^\infty r^3 uv\, dr , \tag{30}$$

and the (axial-vector)/(vector) coupling-constant ratio

$$g_A/g_V=\frac{20\pi}{3}\int_0^\infty r^2(u^2-\tfrac{1}{3}v^2)dr . \tag{31}$$

Since in the present form the model is flavor independent, the charge radius of the neutron in this model is

$$\langle r_n{}^2\rangle \equiv 0 , \tag{32}$$

and the neutron magnetic moment is

$$\mu_n=-\tfrac{2}{3}\mu_p , \tag{33}$$

as given by the SU(6) algebra. Corrections to these relations arise only when QCD effects are included.

Our results for the two families ($p\to 0^+$ and $a=0$) are summarized in four tables. In all tables

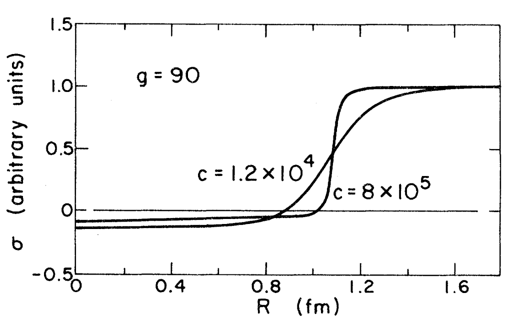

FIG. 3. The soliton field σ versus radius for $c=1.2\times 10^4$ and 8×10^5, $p\to 0^+$.

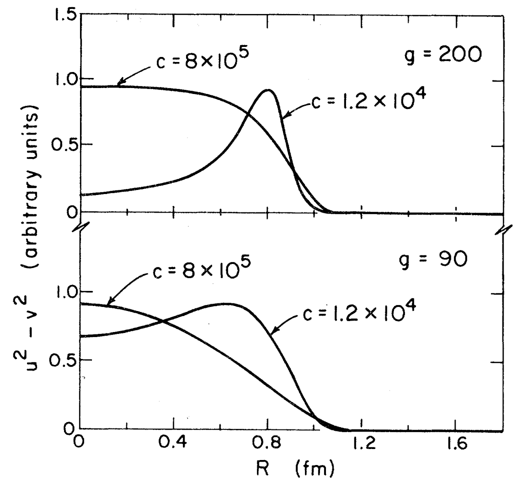

FIG. 4. Quark density u^2-v^2 versus radius for $c=8\times 10^5$ and 1.2×10^4 and $g=90$ and 200, $p\to 0^+$

the units $\hbar=c=1$ are used; lengths are measured in fm.

We consider first the case $p\to 0^+$, corresponding to the relationship among the constants $b^2=3ac$. This is the limiting case for stability of solutions for which $\sigma\to_{r\to\infty}\sigma_v>0$; for $p<0$, the asymptotic solution is $\sigma\to_{r\to\infty}0$. In Table I we list the bag properties as a function of the parameter c for two values of the coupling constant g. The variation of bag properties with the coupling constant for several values of c is given in Table II. Several features emerge from these calculations. First, we note that an increase of the coupling constant produces a continuous change from a volume quark distribution for small g to a surface quark distribution for large g. This change is illustrated in Fig. 2 where we plot the quark charge density u^2-v^2 for three values of g. It should be noted that the shape of the soliton field does not change significantly with g for given c.

The variations of the soliton field and the quark charge density as a function of the radius with increasing c are plotted in Figs. 3 and 4. We see that the transition of the soliton field from its interior to its exterior values becomes more abrupt for large c. This is illustrated in Fig. 3 and the change in the shape of $\sigma(r)$ is quite marked. When c increases, with a fixed value of g, the quark charge distribution u^2-v^2 changes, albeit rather slowly from surface to volume. This is illustrated in Fig. 4 for $g=90$ and $g=200$. The change from volume to surface quark charge density is also evident in the variations of the values of the magnetic moment μ_p and g_A/g_V. We find that for a given g

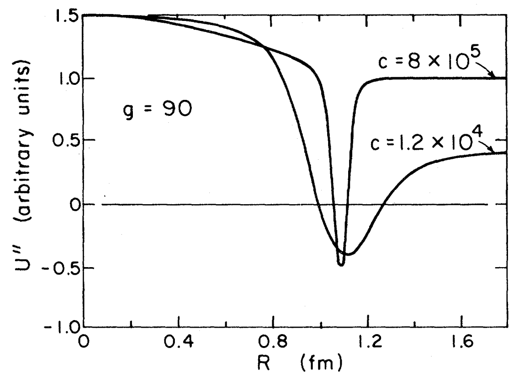

FIG. 5. The soliton "mass" $U''(\sigma)$ versus radius for two values of c, $p \to 0^+$.

the magnetic moment varies with increasing c from $2.65\mu_B$, the SLAC-bag limit, to $2.20\mu_B$, the MIT-bag limit, where μ_B is the Bohr magneton of the proton. Similarly, g_A/g_V varies form 0.56 for small c to 1.10 for large c. This variation is not very marked for the extreme values of g (i.e., 15 and 400) through the range of the parameter c considered, but is quite evident for all other values of g.

Another interesting quantity is the soliton effective mass $U''(\sigma)$ as a function of r. As seen from the analysis of Sec. IV this function determines the nature of the expansion basis $\{s_j\}$. We give the variation of $U''(\sigma(r))$ with c in Fig. 5 for the coupling constant $g=90$. It is evident that the solutions for s_j [Eq. (18)] are essentially localized at the position of the dip in $U''(\sigma)$, with the localization becoming more pronounced as c increases. Thus the low-soliton quantum excitations are confined to the bag surface, much as ripples on the surface of a bubble.

If we approximate the dip in U'' by a parabola, the equivalent harmonic-oscillator frequency for Eq. (18) is

$$\Omega^2 = \frac{1}{2}\left[\frac{d^2}{dr^2}U''(\sigma_0(r))\right]_{r=r_{\min}}, \qquad (34)$$

where $r_{\min}$ is the location of the minimum. The eigenmode energy [see Eqs. (18) and (19)] is

$$\omega_{nl} \simeq \left[(2n+1)\Omega + \frac{l(l+1)}{r_{\min}^2} + U''(\sigma_0)\Big|_{r=r_{\min}}\right]^{1/2}, \qquad (35)$$

where we have replaced the index j by the set (nlm); for spherical σ_0, ω_j is independent of m. In Table III we give the values of $U''(\sigma_0)$ at the minimum, Ω and ω_{00} for several values of c,g. We see that ω_{00}^2 is positive (indicating stability of the MFA solution); and $\omega_{00} \sim 2$ GeV. This is moderately large. Ω is generally much larger than ω, so that excitations of well states will lie very high. We speculate that multiple surface excitations, corresponding to ω_{0l}, nearly equally spaced in energy, should be experimentally observable. Evidently, the value of ω_{00} depends on the values of the parameters and measurements of these excitations could provide a test of the model. Indeed such types of excitation exist in the nuclear liquid drop model, and here represent the simplest example of a collective bag excitation.

The other family of parameters we consider is characterized by $a=0$; $U(\sigma)$ has an inflection point at $\sigma=0$, and only one minimum (see Fig. 1). We vary b,c,g subject to the bag size constraint. Our numerical results for this family are summarized in Table IV. We again find that as g increases, the character of the bag changes from volume confinement to surface confinement. Similarly, increasing c leads to a sharpening of the interior-exterior transition in the soliton field, as in the $p \to 0^+$ case. An example of the $a=0$ solution is given in Fig. 6, where we plot $u^2 - v^2$, σ, and $U''(\sigma)$ as functions of r. An interesting result of the $a=0$ calculations is that the bag energy is significantly smaller than for the $p \to 0^+$ case. For $g=30$, the volume type confinement bag, the bag energy is ~ 1400 MeV, about 300 MeV higher than the average mass of

TABLE III. The soliton effective-mass parameters and eigenmode energy ($p \to 0^+$ bag).

c	g	$U''_{\min}$	Ω	ω_{00}	$r_{\min}$ (fm)
2×10^5	40	-335	422	9.3	1.06
4×10^5		-624	752	11.3	1.06
8×10^5		-1135	1222	9.3	1.05
2×10^5	200	-323	421	9.9	1.00
4×10^5		-478	621	11.9	1.00
8×10^5		-1091	1350	16.1	1.03

TABLE IV. Bag properties with $a=0$ as a function of g and c.

c	g	ϵ_0	$\mathscr{E}$	μ	g_A/g_V	$\epsilon_1-\epsilon_0$
10^4	15	1.63	7.71	2.36	0.976	2.00
	40	1.34	6.81	2.59	0.702	4.75
	90	1.27	6.41	2.64	0.605	6.66
	200	1.23	5.63	2.66	0.574	9.32
	400	1.22	5.65	2.67	0.560	19.0
2×10^4	15	1.69	7.63	2.30	1.03	1.72
	40	1.36	6.83	2.57	0.729	4.51
	90	1.26	6.30	2.64	0.610	6.61
	200	1.16	5.75	2.66	0.582	8.57
	400	1.21	5.86	2.66	0.566	12.32
4×10^4	15	1.79	7.58	2.24	1.08	1.35
	40	1.38	6.78	2.55	0.761	4.26
	90	1.27	6.24	2.63	0.627	5.95
	200	1.23	5.98	2.66	0.580	8.00
	400	1.28	5.36	2.66	0.565	10.64

the $(np\Delta)$ combination and very close to the centroid of the 56-plet, 1316 MeV. The $p\rightarrow0^+$ bag gives the mass of 1800 MeV, about 400 MeV higher than the $a=0$ bag for comparable values of g and c. On the other hand, in the $a=0$ bag the ratio $\sigma_v/\sigma_{\text{interior}}$ is somewhat smaller than in the $p\rightarrow0^+$ case, indicating somewhat weaker confinement.

We have not been able to find a set of parameters which gives a bag energy roughly equal to that of the proton. In fact, the quark contribution alone gives $N\epsilon\approx1$ GeV, just as for the MIT bag. Since this contribution is controlled by the bag size and the soliton contribution is always positive, we cannot reproduce the proton mass without considering additional corrections to energy or assuming a size for the bag which is much greater than the experimental nucleon form factors seem to suggest.[24] Our analysis however indicates that the

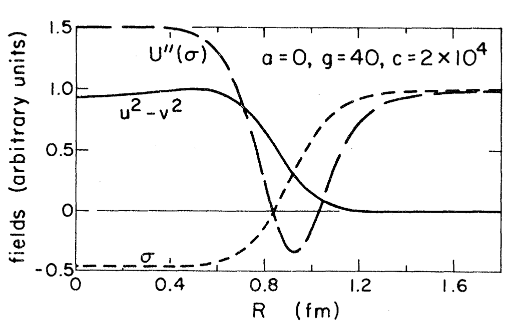

FIG. 6. The $a=0$ bag results. Note the magnitude of $\sigma(r)$ for small r as compared to the $p\rightarrow0^+$ case in Fig. 3.

model can give the correct relative position of the N^* resonance, roughly 600 MeV above the proton. Also, as shown in Ref. 10 the soliton model yields the correct mass formula for a number of particles in lowest order in α.

VII. SUMMARY AND CONCLUSIONS

We have investigated the soliton bag model of Ref. 10. This model is explicitly covariant and does not contain an explicit statement of sharp, nondynamical bag boundary. The mean-field approximation to the model has been solved numerically over a range of model parameters. The results indicate a great flexibility in the choice of bag parameters. We find that these values cannot be fixed by the nucleon size and magnetic moment, the value of g_A/g_V, and the position of the first excited state, although the effects of the parameter changes on the nucleon properties are readily predictable. This suggests that the variation of these parameters may be used to obtain correctly other more complicated hadron properties. For example, we may attempt to fix these parameters from the investigation of nucleon-nucleon scattering, pion-nucleon interaction, etc. In the future, this model should be useful to study center-of-mass corrections, quantum effects, and corrections due to gluon degrees of freedom. These studies are particularly important since the soliton model reduces to the standard bag models in certain limits, yet does not contain explicitly any discontinuous bag boundaries, thus allowing an independent test of various, often intuitive, results obtained previously (e.g., those related to the bag formation dynamics). Work along these lines is under way and will be reported in future publications.

ACKNOWLEDGMENTS

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APPENDIX A: SCALING PROPERTIES OF THE (MFA) SOLUTION

The solutions to Eqs. (11) and (12) have several interesting scaling properties which allow us to go from one given solution to another by changing the length scale, the number of hadronic constitutents, etc. Below we outline some of these properties and discuss their consequences.

1. Length scaling

Length scaling is particularly useful if we wish to normalize our results to a bag of given radius. Let us write

$$r=\lambda\hat{r}\ , \tag{A1}$$

where λ is a dimensionless scale constant. In order to preserve normalization, the quark wave function is

$$\hat{u}=\lambda^{3/2}u\ , \tag{A2}$$

$$\hat{v}=\lambda^{3/2}v\ , \tag{A3}$$

and the σ field becomes

$$\hat{\sigma}=\lambda\sigma\ . \tag{A4}$$

Inserting (A1)—(A4) into our original equations we find new parameters

$$\hat{a}=\lambda^2 a\ , \tag{A5}$$

$$\hat{b}=\lambda b\ , \tag{A6}$$

$$\hat{c}=c\ , \tag{A7}$$

$$\hat{g}=g\ , \tag{A8}$$

such that our results are of the same form with all quantities replaced by their careted counterparts. This leads to the scaling of energy, viz.,

$$\hat{\mathscr{E}}=\lambda\mathscr{E}\ , \tag{A9}$$

$$\hat{\epsilon}=\lambda\epsilon\ . \tag{A10}$$

The magnetic moment scales in the same way as length, i.e.,

$$\hat{\mu}=\mu/\lambda\ , \tag{A11}$$

and the value of g_A/g_V is scale invariant. We note that this scaling does not change the ratio σ_0/σ_v, but affects the values of the effective quark mass $g\sigma$ inside and outside the bag.

2. Quark-number scaling

An alternative scaling allows us to change the number of hadronic constituents. Let us write

$$\sigma=\alpha\hat{\sigma}\ . \tag{A12}$$

In order to preserve the form of the original equations when N is replaced by $\hat{N}$ we must take

$$\alpha\equiv N/\hat{N} \tag{A13}$$

and the various quantities in the theory scale as

$$\hat{a}=a\ , \tag{A14}$$

$$\hat{b}=\alpha b\ , \tag{A15}$$

$$\hat{c}=\alpha^2 c\ . \tag{A16}$$

The bag energy does not scale simply and the contributions due to the quarks and the σ field must be considered separately. Let

$$\mathscr{E}=\mathscr{E}_\sigma+N\epsilon\ . \tag{A17}$$

Then

$$\hat{\mathscr{E}}_\sigma=\mathscr{E}_\sigma/\alpha^2=\left[\frac{\hat{N}}{N}\right]^2\mathscr{E}_\sigma \tag{A18}$$

and

$$\hat{\mathscr{E}}=\hat{\mathscr{E}}_\sigma+\hat{N}\epsilon\ . \tag{A19}$$

If $\mathscr{E}_\sigma << N\epsilon$, this scaling suggests that the meson mass is approximately $\frac{2}{3}$ of the baryon mass, as borne out by calculations.

APPENDIX B: NUMERICAL METHODS

The coupled differential equations for a quark in the $\kappa=-1$ state are

$$\frac{du}{dr}=(\epsilon+g\sigma)v\ ,$$
$$\frac{dv}{dr}=\frac{2}{r}+(\epsilon-g\sigma)u\ , \tag{B1}$$

and that for the soliton field in spherical symmetry is

$$-\frac{1}{r}\frac{d^2}{dr^2}(r\sigma)+U'(\sigma)+gN(u^2-v^2)=0\ . \tag{B2}$$

Equations (B1) and (B2) were solved alternately until consistency was obtained using the methods described below. Each successive solution of (B1) and (B2) is termed "a cycle."

For a given (approximate) σ, an iterative eigenvalue solving scheme was invoked to find u, v, and ϵ. Let the nth-iteration results be denoted by $u^{(n)}$, $v^{(n)}$, and $\epsilon^{(n)}$. We replace ϵ by $\epsilon^{(n)}$ in (B1) and integrate by Runge-Kutta techniques from the inside [$u^{(n)}(0)$ is arbitrary, $v^{(n)}(0)=0$] to some matching radius R. The equations are also integrated from the outside [$u^{(n)}(R_{\max})$ and $v^{(n)}(R_{\max})$ are small numbers] to the same matching radius. The $u^{(n)}$ and $v^{(n)}$ are renormalized so that $u^{(n)}$ is continuous at R. Then

$$\epsilon^{(n+1)}-\epsilon^{(n)}=\frac{u^{(n)}(R)[v^{(n)}(R+0)-v^{n}(R-0)]}{\int_0^{R_{\max}} r^2dr[(u^{(n)})^2+(v^{(n)})^2]}\ . \tag{B3}$$

The convergence is rapid once one is in the vicinity of the true solution. Even if $\epsilon^{(n)}$ is not close, the RHS of (B3) gives the correct sign of the correction. One can use the sign of the correction to repeatedly halve the distance between bounds until the numerical correction is valid. Iteration is continued until the change in ϵ is less than the desired tolerance. (The method was extended to excited states by requiring a specified number of modes.) The quark wave function is normalized to unity before entering the soliton solver.

Equation (B2) is nonlinear and inhomogeneous, subject to the boundary conditions

$$\frac{d\sigma}{dr}(0)=0,\ \ \sigma(R_{\max})=\sigma_v\ . \tag{B4}$$

We employ the following iterative method.[25] Let $\sigma^{(n)}(r)$ be the nth approximation to the solution of (B2) for fixed u and v. Set

$$r\sigma^{(n+1)}=r\sigma^{(n)}+y^{(n+1)}\ . \tag{B5}$$

To first order in $y^{(n+1)}$, $y^{(n+1)}$ satisfies the linear inhomogeneous equation

$$-\frac{d^2}{dr^2}y^{(n+1)}+U''(\sigma^n)y^{(n+1)}=\frac{d^2}{dr^2}(r\sigma^{(n)})-rU'(\sigma^{(n)})-rNg(u^2-v^2) \tag{B6}$$

subject to the boundary conditions $y(0)=y(R_{\max})=0$. Equation (B6) can be solved by a variety of standard methods. We used the Numerov method, reducing the differential equation to a tridiagonal matrix equation and inverting. The iteration procedure is continued until $\int y^2dr$ is less than the required tolerance.

It frequently happens that convergence to self-consistency (after a quark-soliton cycle) is slow or is not achieved; sometimes cycles oscillate alternately between two forms. This is remedied by insertion of a convergence factor f which leads to the redefinition of the $(i+1)$ *cycle:*

$$\sigma(i+1)\rightarrow f\sigma(i+1)+(1-f)\sigma(i)\ . \tag{B7}$$

We find that $f=0.5$ results in fairly rapid convergence in nearly all cases. To start the cycle, the first guess for σ is chosen to be a suitable Wood-Saxon form. This is fed into the quark eignevalue solver, which yields the first cycle for u, v. The subsequent cycles are then fully determined. The speed of convergence depends crucially on the quality of the first guess. A good guess can result in convergence in five cycles or less.

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